Roll No. ……..

MAT–501

Advanced Algebra

M.Sc. MATHEMATICS (MSCMAT-12)

First Year, Examination, 2018

Time : 3 Hours **Max. Marks : 80**

Note : This paper is of **eighty (80)** marks containing **three (03)** Sections A, B and C. Attempt the questions contained in these Sections according to the detailed instructions given therein.

Section–A

(Long Answer Type Questions)

Note : Section 'A' contains four (04) long answer type questions of nineteen (19) marks each. Learners are required to answer *two* (02) questions only.

1. Every finite group G has a composition series.
2. State and prove unique factorization theorem.
3. Let R be an Euclidean ring. Then any finitely generated R-Module N is the direct sum of a finite number of cyclic submodules.
4. If V and v' be vector space over the same field F and $T : V \rightarrow v'$ be a linear transformation. If V is finite dimensional, then

$$\dim V = \text{rank } (T) + \text{Nullity } (T)$$

Section–B

(Short Answer Type Questions)

Note : Section 'B' contains eight (08) short answer type questions of eight (08) marks each. Learners are required to answer *four* (04) questions only.

1. Prove that the intersection of two normal subgroups of a group is a normal subgroup of group.
2. Let G be finite. Show that G is solvable if G in p-solvable for all primes p.
3. Two elements a and b of an integral domain are associated iff one is unit times the other.
4. If an liner product space X is real, show that the condition $|| x || = || y ||$ implies :

$$< x + y , x - y > = 0 .$$

5. Show that a finite field extension of prime degree is a simple extension.
6. Let H be a subgroup of all automorphisms of a field K. Then the fixed field of H is a subfield of K.
7. For any matrix A over a field F, rank (A) = rank (A^T).
8. For any two vectors u and v in an inner product space V,

$$|| u - v || \leq || u || || v ||$$

Section–C

(Objective Type Questions)

Note : Section 'C' contains ten (10) objective type questions of one (1) mark each. All the questions of this Section are compulsory.

Write True/False in the following questions :

1. External direct product and internal direct product of same factors are isomorphic.
2. The center of a group is abelian.
3. S_6 is a solvable group.
4. There are two binary operations defined in an R-module.
5. Each linear functional is a linear transformation.
6. The field C of complex numbers is not an algebraic extension of R.
7. Every field of characteristic zero is perfect.
8. The polynomial $x^5 - 8x + 6$ is not solvable by radical over Q.
9. Each matrix determines a linear transformation.
10. In an inner product space V, a set of orthogonal vectors is always linear dependent.