PHY–501

Mathematical Physics and Classical Mechanics

M. Sc. PHYSICS (MSCPHY–12/13/16)

First Year, Examination, 2017

Time : 3 Hours **Max. Marks : 80**

Note : This paper is of **eighty (80)** marks containing **three (03)** Sections A, B and C. Learners are required to attempt the questions contained in these Sections according to the detailed instructions given therein.

Section–A

(Long Answer Type Questions)

Note : Section 'A' contains four (04) long answer type questions of nineteen (19) marks each. Learners are required to answer *two* (02) questions only.

1. Describe recurrence formula for $H_n(x)$ and to show that $H_n(x)$ is a solution of Hermite equation. Also find the value of $\int_{-\infty}^{\infty} e^{-x^2} H_2(x) H_3(x)\, dx$.

2. (a) Express the functions :

$$f(x) = \begin{cases} l \text{ when} & |n| \le l \\ 0 & |n| > l \end{cases}$$

as a Fourier integral.

Hence evaluate :

$$\int_0^{\infty} \frac{\sin \lambda \cos \lambda x}{\lambda}\, d\lambda$$

(b) Find the Laplace transform of (1 + sin 2*t*).

3. State and explain the Hamilton's principle of least action. Derive Lagrange's equation from Hamilton's principle. When string of a pendulum is elastic then set up Lagrange's equation for small oscillation where motion is considered to a vertical plane.
4. Derive an expression for the Gregory-Newton forward and backward difference interpolation formula.

Section–B

(Short Answer Type Questions)

Note : Section 'B' contains eight (08) short answer type questions of eight (08) marks each. Learners are required to answer *four* (04) questions only.

1. Show that :

$$\frac{d}{dx}\ J_n^2 + J_{n+1}^2 \ = 2\left(\frac{n}{x} J_n^2 - \frac{n+1}{x} J_{n+1}^2\right)$$

2. Explain the derivative of two-dimensional wave equation.
3. Show that $\dfrac{\partial A_\lambda}{\partial x_\mu}$ is not a tensor although A_λ is a covariant tensor of rank one.
4. Surface as a sphere is a two-dimensional Riemannian space. Find the fundamental metric tensor.
5. Discus in brief equation of canonical transformation.
6. Explain Hamilton-Jacobi theory and show that the transformation $q = \sqrt{2}\sin Q$ and $p = \sqrt{2}p\cos Q$ is canonical.

7. Derive an expression for the Stirling interpolation formula.
8. Explain numerical solutions of ordinary differential equation with an example.

Section–C

(Objective Type Questions)

Note : Section 'C' contains ten (10) objective type questions of one (01) mark each. All the questions of this Section are compulsory.

1. The integral $\int_0^n P_n(\cos\theta)\sin 2\theta\, d\theta$, $n > 1$ where $P_n(x)$ is the Legendre polynomial of degree n equal to :
   (a) 1
   (b) $\frac{1}{2}$
   (c) 0
   (d) 2

2. If $J_{n+1}(x) = \frac{2}{x} J_n(x) - J_0(x)$, then n is :
   (a) 0
   (b) 2
   (c) –1
   (d) None of the above

3. In the Fourier transform the value of $\int_0^\infty e^{-x^2} dx$ is :

(a) $\frac{\pi}{2}$

(b) $\frac{\sqrt{\pi}}{2}$

(c) $\frac{\pi}{\sqrt{2}}$

(d) $\sqrt{\frac{\pi}{2}}$

4. If L{F(t)} = $\overline{f}(s)$, then L{t F(t)} is :

(a) $\overline{f}\,'(s)$

(b) $-\overline{f}\,'(s)$

(c) $\overline{f}\,'(s)+\overline{f}(s)$

(d) $s\overline{f}\,'(s)+\overline{f}(s)$

5. If A^μ and B_μ components of contravarient and covariant tensors, what is the nature of the quantity $A^\mu B_\mu$:?

(a) A covariant tensor of rank 2

(b) Mixed tensor of rank 1

(c) Rank zero scale

(d) A mixed tensor of rank 2

6. The Christoffel symbols of the first kind $[ij, k]$ is :

(a) $\frac{1}{2}\left[\frac{\partial g_{jk}}{\partial x^i}-\frac{\partial g_{ik}}{\partial x^j}-\frac{\partial g_{ij}}{\partial x^k}\right]$

(b) $\frac{1}{2}\left[\frac{\partial g_{jk}}{\partial x^i}-\frac{\partial g_{ik}}{\partial x^j}+\frac{\partial g_{ij}}{\partial x^k}\right]$

(c) $\frac{1}{2}\left[\frac{\partial g_{jk}}{\partial x^i}+\frac{\partial g_{ik}}{\partial x^j}-\frac{\partial g_{ij}}{\partial x^k}\right]$

(d) $\frac{1}{2}\left[\frac{\partial g_{jk}}{\partial x^i}+\frac{\partial g_{ik}}{\partial x^j}+\frac{\partial g_{ij}}{\partial x^k}\right]$

7. In the case of canonical transformation :

(a) The form of the Hamilton equation is preserved.

(b) The form of Lagrange equations is preserved.

(c) Hamilton's principle is satisfied in old as well as in the new co-ordinates.

(d) The form of the Hamilton's equations may or may not be preserved.

8. The equation $\frac{d}{dt}\left(\frac{\partial L}{\partial q_i}\right)-\frac{\partial L}{\partial q_i}=0$ where L = T – V is :

(a) Lagrangian equation for conservative system.

(b) Lagrangian equation for non-conservative system.

(c) Equation of motion of harmonic oscillator.

(d) None of these

9. For Lagrange's interpolation formula if f (0.5) = 4.56 and f (0.8) = 5.07, the value of f (0.55) is :

(a) 4.645

(b) 6.326

(c) 4.000

(d) 5.326

10. Runge-Kutta method for second order, first degree linear differential equation is :

(a) $\frac{dy}{dx} = f\left(x, y, \frac{d^2y}{dx^2}\right)$

(b) $\frac{d^2y}{dx^2} = f\left(x, y, \frac{dy}{dx}\right)$

(c) $\frac{dy}{dx} = f\left(x, y, \frac{dy}{dx}\right)$

(d) $\frac{d^2y}{dx^2} = f\left(x, y, \frac{dx}{dy}\right)$