14/1 Roll No. …………………..

MAT–506

Analysis and Advanced Calculus

M. Sc. MATHEMATICS (MSCMAT–12)

Second Year, Examination, 2017

Time : 3 Hours **Max. Marks : 80**

Note : This paper is of **eighty (80)** marks containing **three (03)** Sections A, B and C. Learners are required to attempt the questions contained in these Sections according to the detailed instructions given therein.

Section–A

(Long Answer Type Questions)

Note : Section ‘A’ contains four (04) long answer type questions of nineteen (19) marks each. Learners are required to answer *two* (02) questions only.

1. Let p be a real number such that $1 \le p < \infty$ and denoted by l_p^n the space of all n-tuples $x = (x_1, x_2, ... x_n)$ of scalars. Show that l_p^n is a Banach space under the norm

$$\|x\|^p = \left[\sum_{i=1}^{n} |x_i|^p \right]^{\frac{1}{p}}$$

2. Let N be a non-zero normed linear space and let S = $x \in \mathrm{N} : \|x\| \le 1$ be a linear subspace of N. Prove that N is an Banach space iff S is complete.

3. If M is a proper closed linear subspace of a Hilbert space H, then there exists a non-zero vector Z_0 in H such that $Z_0 \perp M$.

4. State and prove the implicit function theorem.

Section–B

(Short Answer Type Questions)

Note : Section 'B' contains eight (08) short answer type questions of eight (08) marks each. Learners are required to answer *four* (04) questions only.

1. Let N be a normed linear space and $x, y \in N$. Then $\big| \|x\| - \|y\| \big| \leq \|x - y\|$

2. If B and B' are Banach spaces and if T is a continuous linear map of B into B', then T is an open map.

3. Let N and N' be normed linear spaces and let S be a linear transformation of N into N'. Then T is continuous either at every point of N or at no point of N.

4. If x and y are any two vectors in a Hilbert space H, then $|(x, y)| \leq \|x\| \|y\|$.

5. If S is non-empty subset of a Hilbert space H, then $S \cap S^{\perp} = \{\bar{0}\}$.

6. An operator T on a Hilbert space H is self adjoint iff (Tx, x) is real for all x.

7. Let $f(x, y) = \dfrac{x^2 y}{x^4 + y^2}$, for $x \neq 0, y \neq 0$ and $f(0, 0) = 0$.

   Show that the partial derivatives f_x, f_y exist everywhere in the region $-1 \leq x \leq 1$, $-1 \leq y \leq 1$, although $f(x, y)$ is discontinuous in (x, y) at the origin.

8. Explain existence and uniqueness of solutions of ordinary differential equation of the type $x' = f(t, x)$.

Section–C

(Objective Type Questions)

Note : Section 'C' contains ten (10) objective type questions of one (01) mark each. All the questions of this Section are compulsory.

Fill in the blanks.

1. In a normed linear space, every convergent sequence is a .... sequence.
2. A normed linear space which is complete as a metric space is called ....
3. If N is a finite dimensional normed linear space, then N is ....
4. In Hilbert space $(x, \alpha y)$ = ... $\forall x, y \in \mathrm{H}$.
5. A closed convex subset C of a Hilbert space H contains a unique vector of .... norm.
6. Let x and y be vectors in Hilbert space H. Then x is orthogonal to y if (x, y) = ...
7. The adjoint operator $\mathrm{T} \to \mathrm{T}^*$ on B(H) has the property T^{**} = ...
8. If T is a normal operator on a Hilbert space H and f a polynomial with complex coefficients, then the operator $f(\mathrm{T})$ is ....
9. $\lim\limits_{\substack{x \to 0 \\ y \to 0}} y \sin \dfrac{1}{x} =$ ....
10. By mean value theorem we have :

$$|f(b) - f(a)| \leq (b-a) \operatorname*{lub}_{a<x<b} |......|$$