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WORKSHOP
ON
"LECTURES AND VIRTUAL PRACTICAL DEMONSTRATION"

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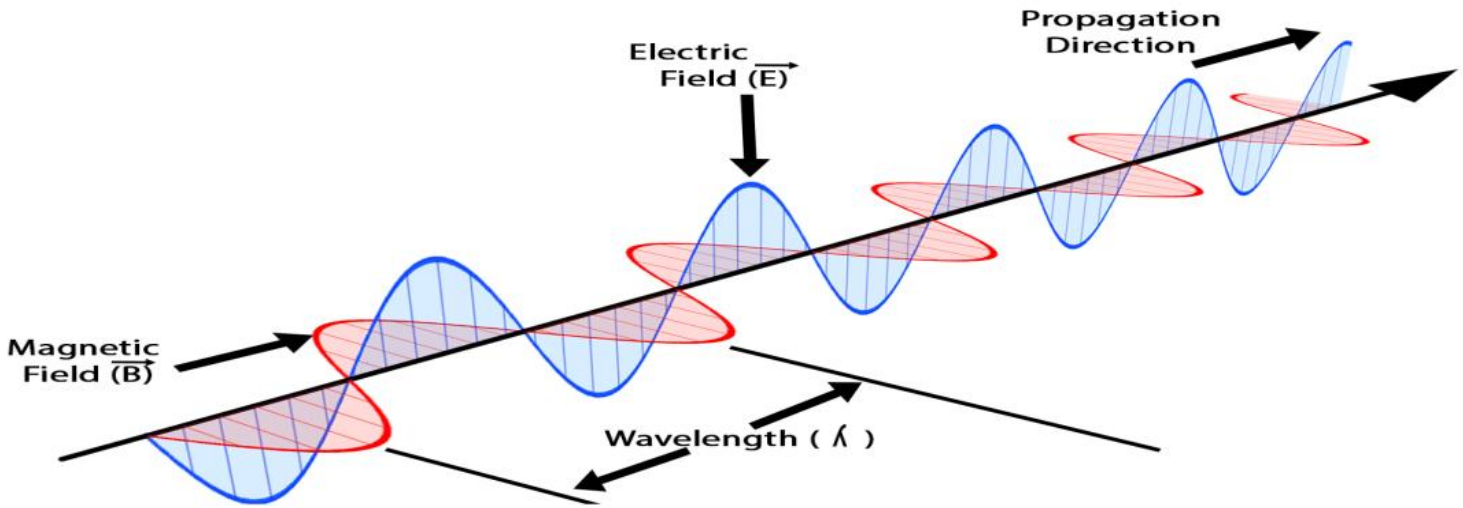
INTEREFERENCE OF LIGHT

In 1680 Huygens proposed the wave theory of light. But at that time, it was not clear about the nature of light wave, its speed and way of propagation. In 1801 Thomas Young performed an experiment called Young's double slit experiment and noticed that bright and dork fringes are formed which is called inference pattern. At that time it was a surprising phenomenon and is to be explained.

After the Maxwell's electromagnetic theory it was cleared that light is an electromagnetic wave. In physics, interference is a phenomenon in which two waves superimpose on each other to form a resultant wave of greater or lower or of equal amplitude. When such two waves travel in space under certain conditions the intensity or energy of waves are redistributed at certain points which is called interference of light and we observe bright and dark fringes.

WAVE NATURE OF LIGHT

Light wave is basically an electromagnetic wave. Electromagnetic wave consists of electric and magnetic field vectors. The directions of electric and magnetic vectors are perpendicular to direction of propagation as shown in the figure. The electric and magnetic vectors are denoted by E and H and vary with time.



In light, electric vectors (or magnetic vectors) vary in sinusoidal manner as shown in figure. Therefore the electric vectors can be given as

$$E = E_0 \sin(kz - \omega t)$$

Where E = Electric field vector, E_0 = maximum amplitude of field vector, k = wave number (= $2\pi/\lambda$), z = displacement along the direction of propagation (say z axis), ω = angular velocity and t = time.

Before understanding the interference we should understand some terms and properties of light which are related to interference.

Monochromatic Light

The visible light is a continuous spectrum which consist a large number of wavelengths (approximately 3500Å to 7800Å). Every single wavelength (or frequency) of this continuous spectrum is called monochromatic light. However, the individual wavelengths are sufficiently close and indistinguishable. Some time we consider very narrow band of wave lengths as monochromatic light.

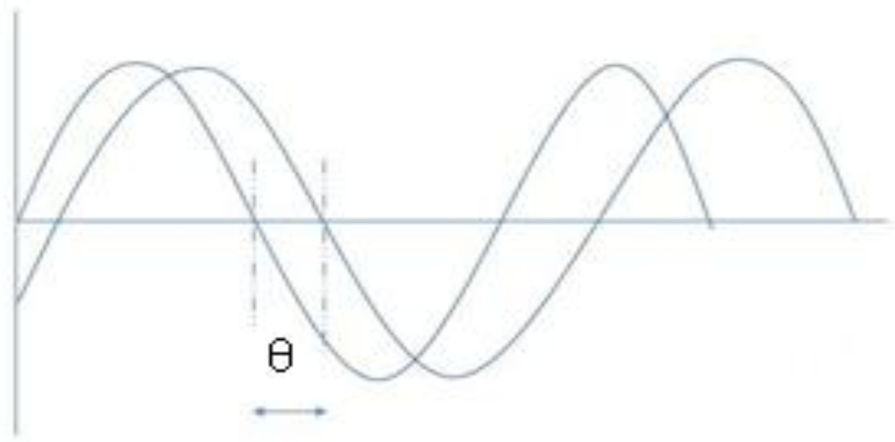
Ordinary light or white light, coming from sun, electric bulb, CFL, LED etc. consists a large number of wave lengths and hence non-monochromatic. But some specific sources like sodium lamp and helium neon laser emit monochromatic lights with wave lengths 589.3 nm and 632.8 nm respectively. It should be noted that sodium lamp, actually emits two spectral lines of wavelengths 589.0 nm and 589.6 nm which are very close together, and source is to be consider monochromatic.

Phase Difference and Coherence

Wave is basically transportation of energy by mean of propagation of disturbance or vibrations. In wave motion through a medium, the particles of medium vibrate but in case of electromagnetic wave the electric or magnetic vectors vibrate form its equilibrium position. The term phase describes the position and motion of vibration at any time. For example if $y= a \sin (\omega t +\theta)$ represents a wave, then the term $(\omega t +\theta)$ represents the phase of wave. The unit of phase is degree or radium. After completion of 360^0 or 2π, the cycle of wave or phase repeats.

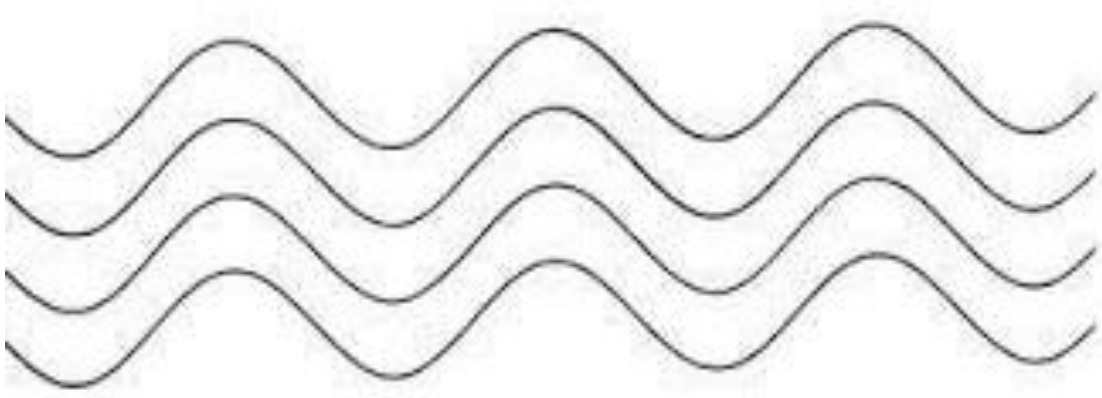
Phase difference

If there are two waves have some frequency then the phase difference is the angle (or time) after which the one wave achieves the same position and phase as of first wave. In the figure, two waves with phase different ϑ are shown.



Coherence

If two or more waves of same frequencies are in same phase or have constant phase difference, those waves are called coherent wave. Figure shows coherent wave with same phase (zero phase difference) and with constant phase difference.



Optical path and Geometric Path

Optical path length (OPL) denoted by Δ is the equivalents path length in the vacuum corresponding to a path length in a medium. Path length in a medium can be considered as geometric path length (*L*). Suppose a light wave travels a path length *L* in a medium of refractive index μ and velocity of light is *v* in this medium, then for a time period *t* the geometric path length *L* is given by

$$L = vt$$

In the same time interval *t*, the light wave travel a distance *Δ* in vacuum which is optical path length corresponding to length *L*. Then

$$\Delta = ct = c\frac{L}{v}$$

Where, c is the velocity of light in vacuum.

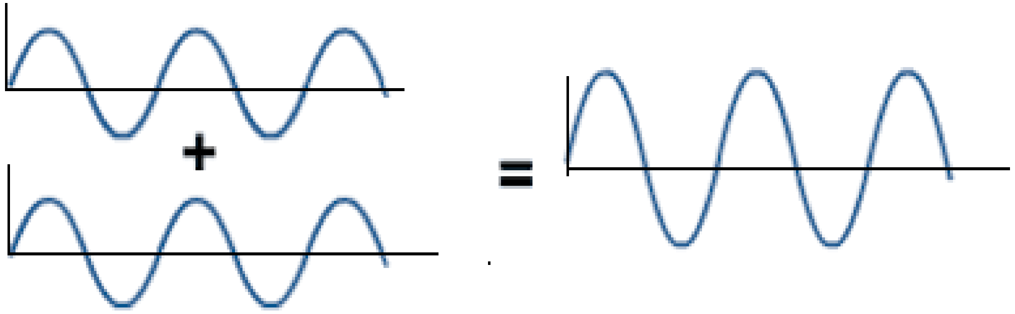
or $$\Delta = \mu L$$

or The Optical path length = μ × (Geometrical path length in a medium).

In case of interference we always calculate optical path for simplification of understanding and mathematical calculations.

PRINCIPLE OF SUPERPOSITION

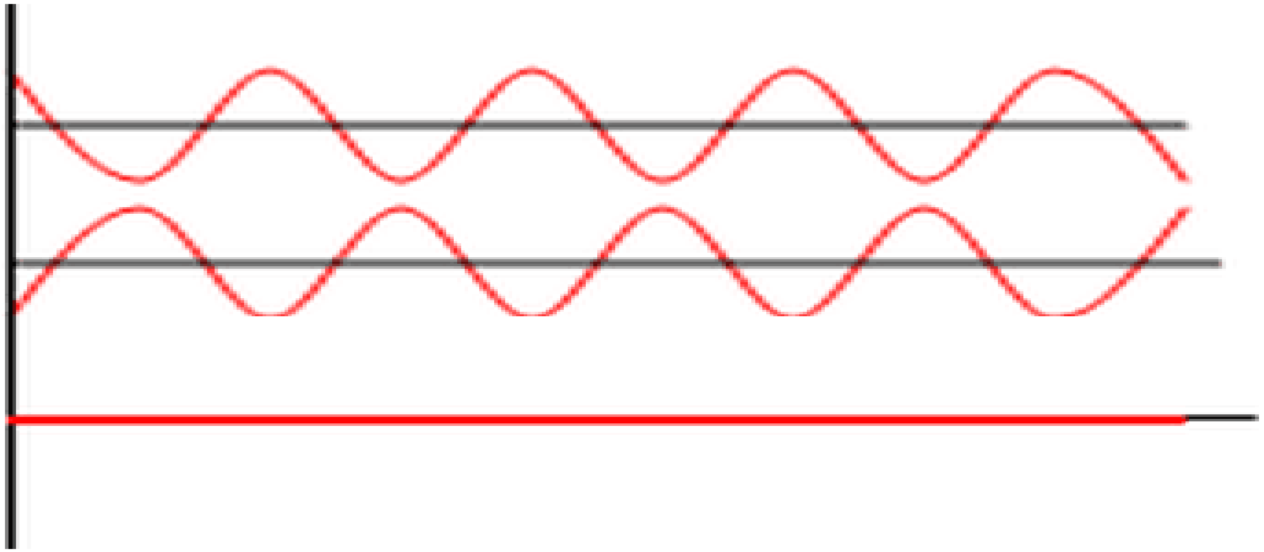
According to Young's principle of superposition, if two or more waves are travelling and overlap on each other at any point then the resultant displacement of wave is the sum of the displacement of individual waves (figure 4.4). If two waves are represented by $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin (\omega t+\delta)$. Then according to principle of superposition, the resultant wave is represented by $y = y_1 + y_2$



INTERFERENCE

When two light waves of some frequency, nearly same amplitude and having constant phase difference travel and overlap on each other, there is a modification in the intensity of light in the region of overlapping. This phenomenon is called interference.

The resultant wave depends on the phases or phase difference of waves. The modification in intensity or change in amplitude occurs due to principle of superposition. In certain points the two waves may be in same phase and at such point the amplitude of resultant wave will be sum of amplitude of individual waves. Thus, if the amplitudes of individual waves are a_1 and a_2 then the resultant amplitude will be $a = a_1 + a_2$. In this case, the intensity of resultant wave increases ($I \propto a^2$) and this phenomena is called constructive interference. Corresponding to constructive interference we observe bright fringes.



On the other hand, at certain points the two waves may be in opposite phase as shown in figure. In these points the resultant amplitude of waves will be sum of amplitude of individual waves with opposite directions. If the amplitudes of individual waves are a_1 and a_2 then the resultant amplitude will be $a = a_1 - a_2$ and the intensity of resultant wave will be minimum. This case is called destructive interference. Corresponding to such points we observe dark fringes. Figure depicts two waves of opposite phase and their resultant.

Theory of Superposition

Let us consider two waves represented by $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin(\omega t + \delta)$. According to Young's principle of superposition the resultant wave can be represented by

$$y = y_1 + y_2$$

$$= a_1 \sin \omega t + a_2 \sin(\omega t + \delta)$$

$$= a_1 \sin \omega t + a_2 (\sin \omega t \cos \delta + \cos \omega t \sin \delta)$$

$$= (a_1 + a_2 \cos \delta) \operatorname{Sin} \omega t + (a_2 \sin \delta) \cos \omega t \qquad \ldots\ldots. \ (4.1)$$

Let $$a_1 + a_2 \cos \delta = A \cos \emptyset \qquad \ldots\ldots. \ (4.2)$$

and $$a_2 \sin \delta = A \sin \emptyset \qquad \ldots\ldots. \ (4.3)$$

Where A and Ø are new constants, then above equation becomes

$y = A\cos\emptyset \sin\omega t + A\sin\emptyset \cos\omega t$

or $$y = A\sin(\omega t + \emptyset) \qquad \ldots\ldots\ (4.4)$$

This is the equation of the resultant wave. In this equation y represents displacement, A represents resultant amplitude, Ø is the phase difference.

From equation (4.2) and (4.3) we can determine the constant A and Ø. Squaring and adding the two equations, we get,

$A^2 = a_1^2 + a_2^2\cos^2\delta + 2\,a_1a_2\cos\delta + a_2^2\sin^2\delta$

or $$A^2 = a_1^2 + a_2^2 + 2\,a_1a_2\cos\delta \qquad \ldots\ldots\ (4.5)$$

On dividing equation (4.3) by eq (4.2), we obtain,

$$\frac{\sin\emptyset}{\cos\emptyset} = \tan\emptyset = \frac{a_2\sin\delta}{a_1 + a_2\cos\delta} \qquad \ldots\ldots\ (4.6)$$

Condition for Maxima or Bright Fringes

If $\cos\delta = +1$ then $\delta = 2n\pi$ where $n = 0, 1, 2, 3$......(positive integer numbers).

Then, $A^2 = a_1^2 + a_2^2 + 2a_1a_2 = (a_1+a_2)^2$

Intensity, $I = A^2 = (a_1+a_2)^2$ .... (4.7)

Therefore, for $\delta = 2n\pi = 0, 2\pi, 4\pi$......, we observe bright fringes.

In term of path difference Δ

$$\Delta = \frac{\lambda}{2\pi л} \times \text{phase difference} = \frac{\lambda}{2\pi л} 2n\pi$$

or $\Delta = n\lambda = \lambda, 2\lambda, 3\lambda$... etc. .... (4.8)

Condition for Minima or Dark Fringes

If $\cos\delta = -1$ or $\delta = (2n-1)\pi = \pi, 3\pi, 5\pi \ldots\ldots$

Then $A^2 = a_1^2 + a_2^2 - 2\,a_1a_2 = (a_1 - a_2)^2$

Intensity, $I = A^2 = (a_1 - a_2)^2$ .... (4.9)

Therefore if phase difference between two waves is $\delta = (2n-1)\pi = 0, 3\,\pi, 5\,\pi \ldots$ etc. is the condition of minima or dark fringes.

Now path difference, $\Delta = \frac{\lambda}{2\pi} \times Phase\ difference$

or $\Delta = \frac{\lambda}{2\pi} \times (2n-1)\pi = \frac{(2n-1)}{2}\lambda = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots\ldots$ .... (4.10)

Intensity Distribution

The intensity (I) of a wave can be given as $I = (½)\ \epsilon_o\ a^2$ where a is the amplitude of wave, and ϵ_0 is the permittivity of free space. If we consider two waves of amplitudes a_1 and a_2 then at the point of maxima

$I_{max} = (a_1+a_2)^2 = a_1^2+a_2^2+2a_1a_2$

If $a_1 = a_2 = a$ then $I = 4a^2$. Therefore, at maxima points the resultant intensity is more than the sum of intensities of individual waves.

Similarly the intensity at points of minima

$I_{min}= a_1^2+a_2^2 - 2a_1a_2 = (a_1 \sim a_2)^2$

If $a_1= a_2=a$ then $I_{min}=0$. Thus the intensity at minima points is less than the intensity of any wave.

The average intensity I_{av} is given as

- $I_{av} = \frac{\int_0^{2\pi} I d\delta}{\int_0^{2\pi} d\delta} = \frac{\int_0^{2\pi} (a_1^2 + a_2^2 + 2a_1 \; a_2 \, Cos \; \delta) d\delta}{\int_0^{2\pi} d\delta} = \frac{(a_1^2 + a_2^2) 2\pi л}{2\pi л} = a_1^2 + a_2^2$
- If $a_1 = a_2 = a$ then $I_{av} = 2a^2 = 2I$
- Therefore, in interference pattern energy (intensity) $2a_1a_2$ is simply transferred from minima to maxima points. The net intensity (or average intensity) remains constant or conserved.

CLASSIFICATION OF INTERFERENCE

The interference can be divided into two categories.

Division of Wavefront

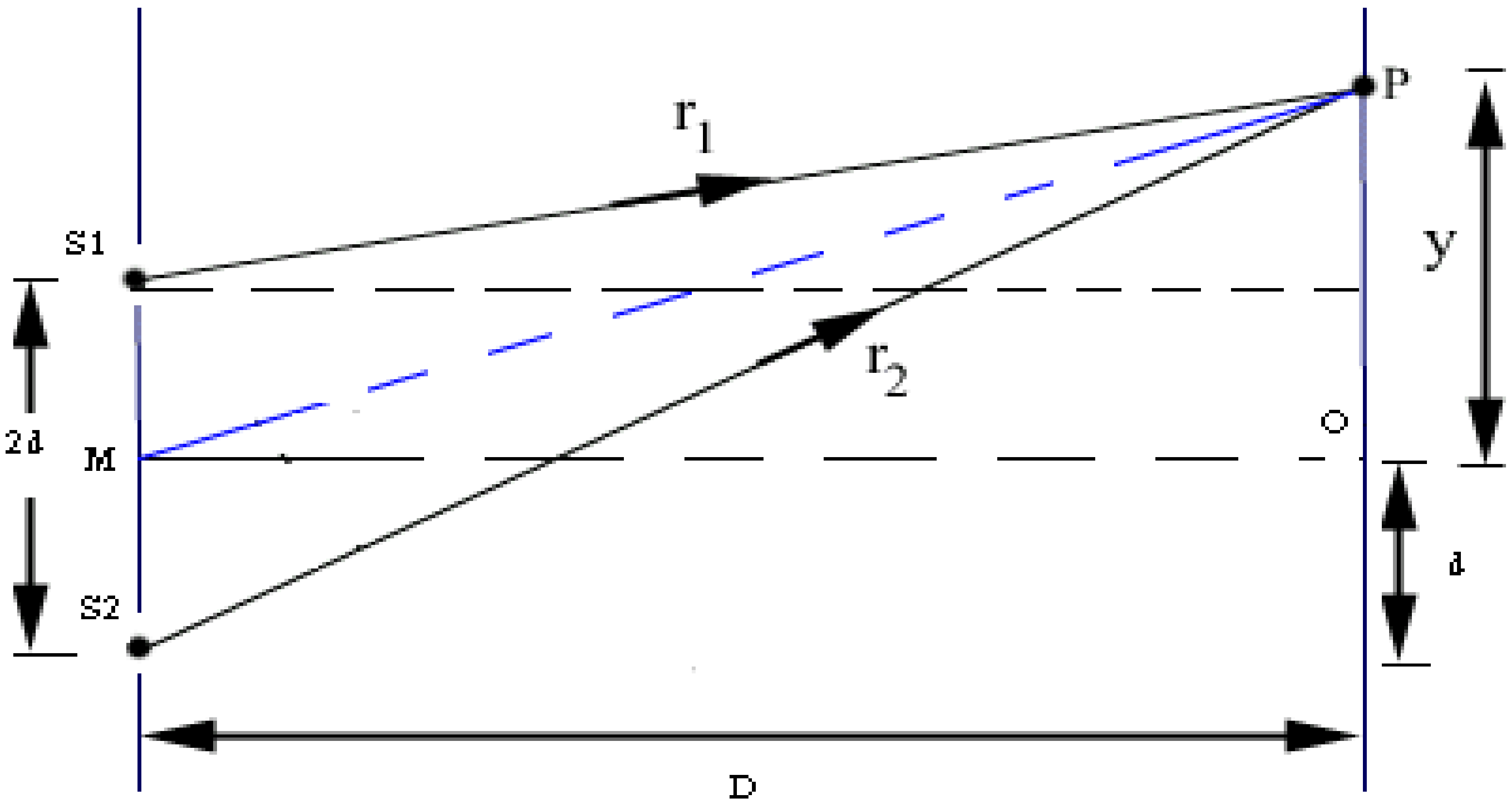
In this class of interference, the wave front originating from a common source is divided into two parts by employing mirror, prisms or lenses on the path. The two wave front thus separated traverse unequal paths and are finally brought together to produce interference pattern. Examples are biprism, Lloyd’s mirror, Laser etc.

Division of Amplitude

In this class of interference the amplitude or intensity of incoming beam divided into two or more parts by partial reflection and refraction. Examples are thin films, Newton’s rings, Michelson interferometer etc.

YOUNG'S DOUBLE SLIT EXPERIMENT

In 1801, Thomas Young performed double slit experiment in which a light first entered through a pin holes, then again divided into two pinholes and finally brought to superimpose on each other and obtained interferences. Young's performed experiment with sum light. Now the experiments are modified with monochromatic light and efficient slits.



P
r1
S1
y
r2
2d
M
O
S2
d
D

Figure shows the experimental setup of double slit experiment. S_1 and S_2 are two narrow slits illuminated by a monochromatic light source. The distance between two slits S_1 and S_2 is 2d. The two waves superimposed on each other and fringes are formed on the screen placed at a distance D from the centre of slits M. Let us consider a point P on the screen which is y distant from O. The two rays S_1P and S_2P meet at point P and produce interference pattern on screen.

Mathematically, path difference between rays S_1 P and S_2P is given as

$$\Delta = S_2P - S_1P \qquad \text{....... (4.11)}$$

$$S_2P^2 = D^2 + (y+d)^2 = D^2[1 + (y+d)^2 / D^2]$$

$$S_2P = D[1 + (y+d)^2 / D^2]^{1/2}$$

$$= D[1 + \frac{1}{2}(y+d)^2 / D^2] \qquad [\because (1+x)^n = 1 + nx + \cdots \ldots]$$

or $$S_2P = D + (y+d)^2 / 2D \qquad \text{....... (4.12)}$$

Similarly

$$S_1P^2 = D^2 + (y-d)^2$$

$$S_1P = D\,[1 + (y-d)^2 / D^2]^{1/2}$$

$$= D\,[1 + \frac{1}{2}(y-d)^2 / D^2]$$

$$= D + (y-d)^2 / 2D \qquad \text{....... (4.13)}$$

Using equation (4.12) and (4.13), the path difference becomes

$$\Delta= D + \frac{(y+d)^2}{2D} - D - \frac{(y-d)^2}{2D} = \frac{2yd}{D} \qquad \text{....... (4.14)}$$

For the position of bright fringes path difference

$\Delta = n\lambda$ (where n=1, 2, 3........)

or $\frac{2yd}{D} = n\lambda$

or $y = \frac{nD\lambda}{2D}$

Since the expression consists of integer *n*, i.e., *y* is a function of *n*. Thus it is better to use y_n in place of *y* and we can write,

$$y_n = \frac{nD\lambda}{2D} \qquad \text{....... (4.15)}$$

Where n = 1, 2 ... etc. represents the order of fringe

On putting the value of n=1, n=2 etc. we get the bright fringes at positions $y_1= \frac{D\lambda}{2D}$, $y_2= \frac{2D\lambda}{2D}$ etc. Similarly for the position of dark fringes, the path difference should be

$$\Delta= \frac{(2n-1)\lambda}{2}$$

or $$\frac{2yd}{D} = \frac{(2n-1)\lambda}{2}$$

or $$y_n = \frac{(2n-1)}{2}\frac{D\lambda}{2D} \quad .... (4.16)$$

If we place the value of n = 1, 2, 3 … we get the positions of dark fringes at $y_1 = \frac{1}{2}\frac{D\lambda}{2D}$, $y_2= \frac{3}{2}\frac{D\lambda}{2D}$, $y_3 = \frac{5}{2}\frac{D\lambda}{2D}$...... etc.

Fringe Width: Distance between two consecutive bright or dark fringes is called fringe width denoted by ω (sometimes β). In case of bright fringes, fringe width

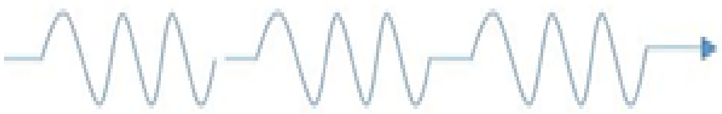
$$\omega = y_{n+1} - y_n = (n+1)\,\frac{D\lambda}{2D} - n\,\frac{D\lambda}{2D} = \frac{D\lambda}{2D}$$

Similarly, in case of dark fringes

$$\omega = y_{n+1} - y_n = \frac{2(n+1)-1}{2}\,\frac{D\lambda}{2D} - \frac{(2n-1)-1}{2}\,\frac{D\lambda}{2D} = \frac{D\lambda}{2D}$$

COHERENCE LENGTH AND COHERENCE TIME

In case of ordinary light source, light emission takes place when an atom leaves it excited state and come to ground state or lower energy state. The time period for the process of transition from an upper state to lower state is about 10^{-8} s only. Therefore an excited atom emits light wave for only 10^{-8} s and wave remains continuously harmonic for this period. After this period, the phase changes abruptly. But in a light source, there are innumerous numbers of atoms which participate in the emission of light. The emission of light by a single atom is shown in figure 4.7. After the contribution of a large number of atoms emitting light photon, a succession of wave trains emits from the light source.



Coherence Length

Coherence length is propagation distance over which a coherent wave maintains coherence. If the path of the interfering waves or path different is smaller than coherent length, the interference is sustainable and we observe distinct interference pattern.

Coherence Time

Coherent time τ_c is defined as the average time period during which the wave remains sinusoidal and after which the phase change abruptly.

CONDITIONS FOR SUSTAINABLE INTERFERENCE

As we studied the different aspects of interference it is clear that under which conditions interference can take place. But for strong interference or sustained interference some more condition may be summarized. The conditions are:

The interfering waves must have same frequencies. For this purpose we can select a single source.

The interfering waves must be coherent. To maintain the coherence, the path difference of two interfering waves must be less than coherence length.

- As fringe width is given by $\omega = \frac{D\lambda}{2d}$. Thus to obtain reasonable fringe width the distance between source and screen D should be large and distance 2d between two sources should be small.
- For good contrast we can prefer the interfering wave of same amplitude. If amplitude of two waves, a_1 and a_2 are same or nearly same than we observe distinct maxima and minima.
- The back ground of screen should be dark.

INTERFERENCE IN THIN FILMS AND NEWTON’S RINGS

In optics any transparent material in a shape of thin sheet of order 1μm to 10 μm is simply called thin film. The material may be glass, water, air, mica and any other material of different refractive index. When a thin film is illuminated by a light, some part of incident light get refracted from the upper surface of film and some part of get transmitted into the film. Some part of transmitted light gets reflected again from the lower surface of thin film. Now the light reflected from upper and lower surface of thin may course interference.

In case of thin film, the maximum portion of incident light is transmitted and a very few part of light reflected form the thin film. Therefore the intensity of reflected light is significantly small. For example if we consider a light beam is reflected from a glass plate of refractive index 1.5 then the reflection coefficient is given by

$$r = \left(\frac{\mu_1 - \mu_2}{\mu_1 - \mu_2}\right)^2 = \left(\frac{1.5-1}{1.5+1}\right)^2 = \left(\frac{0.5}{2.5}\right) = 0.04$$

Thus only 4% of incident light is reflected by the upper surface of glass film and 96% of light is transmitted into the glass plate. Similarly nearly 4% of light is again reflected through the lower surface of glass plate. If we consider the interference due to the light reflected from upper and lower surface of glass plate, the intensity of light will be significantly small.

When white light is incident of thin film, interference pattern is appeared as colourful bands since white light consists different wavelengths, different wavelengths produce interference bands of different colours and thicknesses. Interference in thin films also occurs in nature. Thin wings of many insects and butterflies are layer of thin films. There thin films are responsible for structural colourization which produce different colours by microscopically structured surface, and suitable enough for interference of light.

INTERFERENCE DUE TO PLANE PARALLEL THIN FILM

A plane parallel thin film is transparent film of uniform thickness with two parallel reflecting surfaces. The example is a thin glass film. Light wave generally suffers multiple reflections and refractions at the two surfaces. There are two cases of interference as given below

Interference in Case of Reflected Light

Let us consider a thin film of thickness *t* as shown in figure. A monochromatic light ray SA is incident on a thin film with an angle of incident *i* as shown in figure. The film is made of a transparent material (say glass) of refractive index μ. Some part of light ray reflected at point A along the direction AB and some part of light transmitted into the film along AC direction. The ray AC makes an angle of refraction *r* at point A, and the angle r becomes angle of incident ACN at point C. Some part of light of ray AC again reflected in the direction CD which comes out from the film along the direction DE. The light rays AB and DE come together and they can produced interference pattern on superposition.

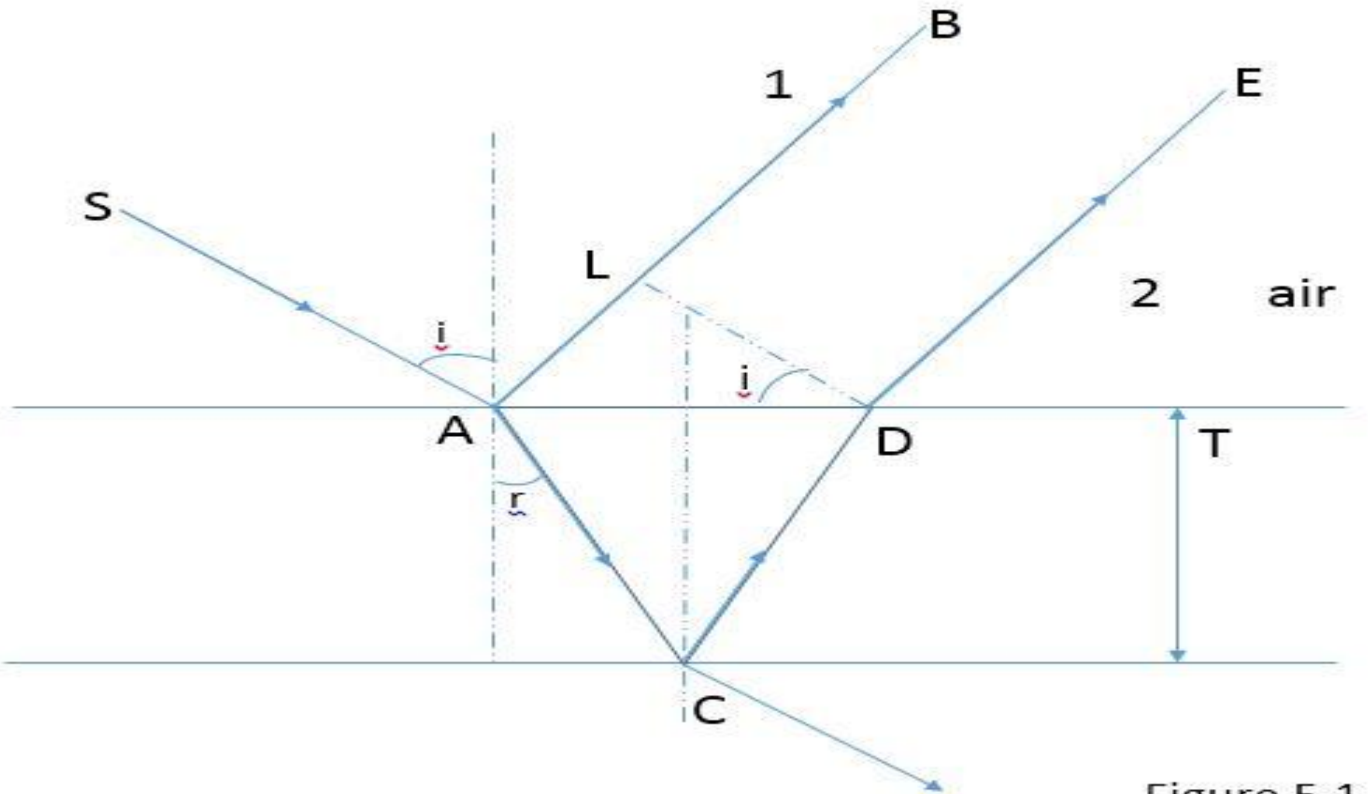


Figure 5.1

The path difference Δ between rays AB and DE is given as

Δ = (AC+DC) in film- AL in air.

Since optical path in air = μ × optical path in a medium

Therefore, path difference Δ can be given as

Δ = μ (AC+ DC) – AL

From figure, we have, cos r = $\frac{t}{AC}$ or AC = $\frac{t}{cos\, r}$ and DC = $\frac{t}{cos\, r}$

Again, AL = AD sin i = (AN+ ND) sin i

= (t tan r + t tan r) sin i = 2t tan r Sin i

$$\Delta = \frac{\mu\, 2t}{\cos r} - 2t.\tan r\, \sin i = \frac{2\mu t}{\cos r} - 2\mu t\,(\sin^2 r)$$

$$= 2\mu \frac{t}{\cos r}(1 - \sin^2 r) = 2\mu t \cos r$$

According to Stock's treatment, if a wave is reflected form a denser medium it involves a path difference of λ/2 or phase difference or π. Therefore, net path difference

$$\Delta = 2\mu t \cos r - \frac{\lambda}{2} \qquad \text{.......} \quad (5.1)$$

Condition of Maxima: For maxima or bright fringes the path difference should be $n\lambda$ where n is integer number given as n= 0,1,2,3 ....

$$\Delta = 2\mu t\, Cos\, r - \frac{\lambda}{2} = n\lambda$$

or $$2\mu t\, Cos\, r = \left(\frac{2n+1}{2}\right)\lambda \qquad \text{....... (5.2)}$$

Thus maxima occur when optical path difference is $\left(\frac{2n+1}{2}\right)\lambda$.

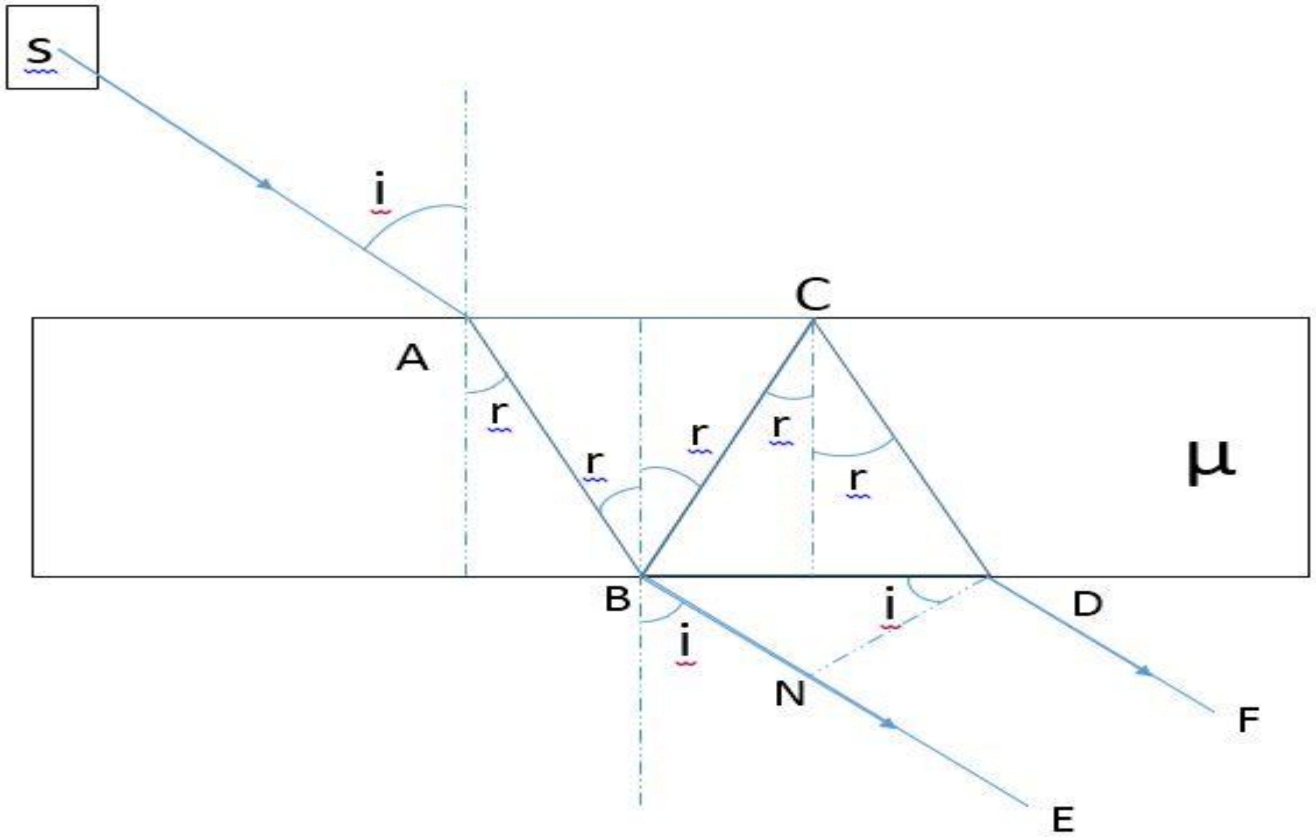
Condition for minima: Minima occur when the path difference is order of $\left(\frac{2n-1}{2}\right)\lambda$. Then

$$\Delta = 2\mu t\, Cos\, r - \frac{\lambda}{2} = \left(\frac{2n-1}{2}\right)\lambda$$

or $$2\mu t\, Cos\, r = n\lambda \qquad \text{....... (5.3)}$$

Interference in Case of Refracted Light

A light ray SA is incident at point A on a film of refractive index μ as shown in figure 5.2. Some part of light ray reflected at point A and some part of light transmitted into the film along AB. In case of interference due to refracted light we are not interested in the reflected light. At point B some part of light is again reflected along direction BC, then again reflected at point C and finally refracted at point D and comes out form the medium along DF direction. Now the light rays coming along BE and DF are coherent and can produce interference pattern in the region of superposition.


S
i
C
A
r
r
r
r
r
μ
B
i
i
D
N
F
E

In this case path difference Δ is given as

$$\Delta = (BC + CD) \text{ in film} - BN \text{ in air}$$

As Calculated in case of reflection, the path difference comes out

$\Delta = 2\mu t \cos r$

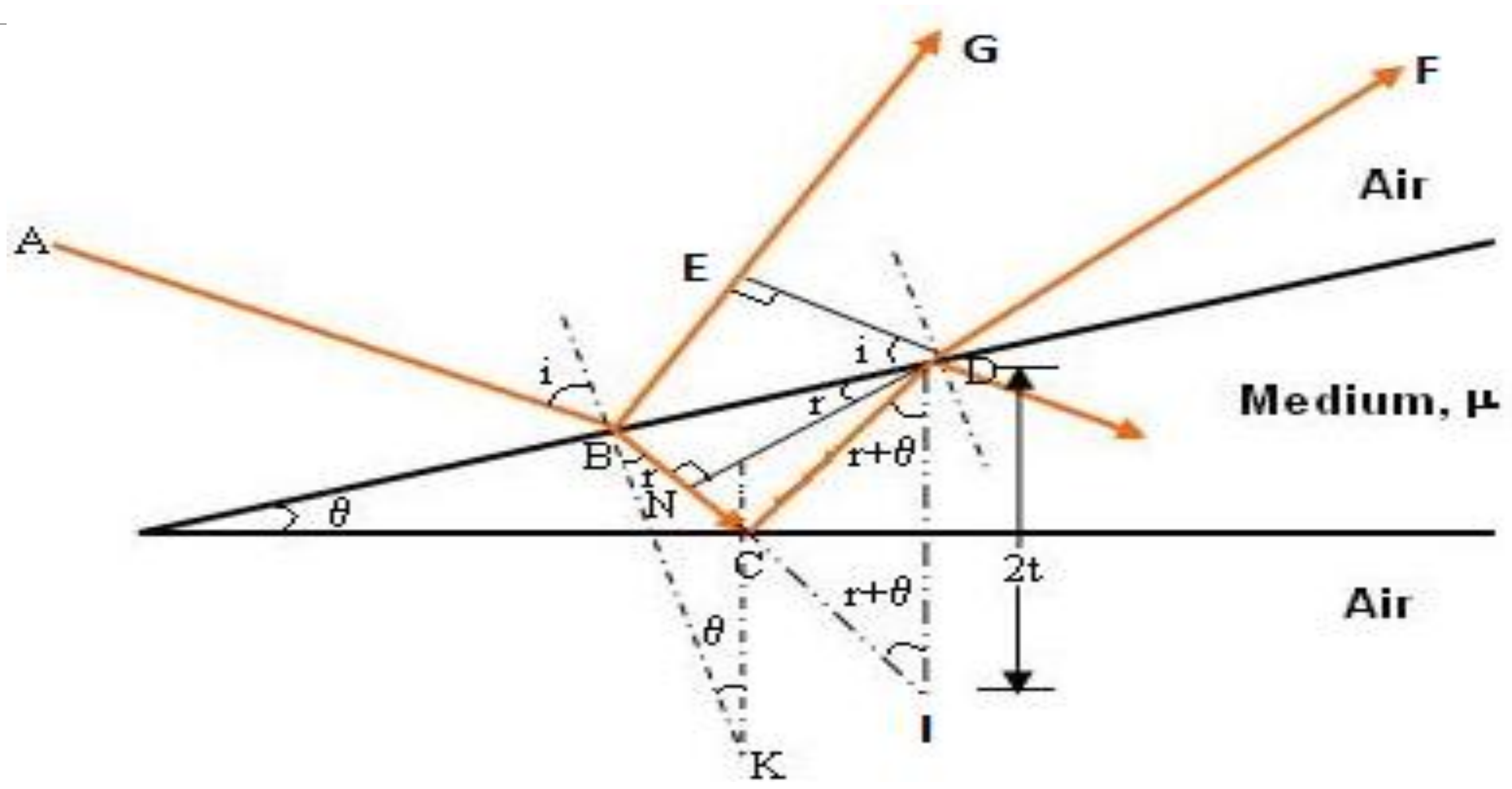
In this case there is no correction according to Stoke's treatment as no wave from rarer medium is reflected back to denser medium. Therefore this is net path difference.

For maxima or bright fringes, $\Delta = 2\mu t \cos r = n\lambda$

For minima or dark fringes, $\Delta = 2\mu t \cos r = (\frac{2n-1}{2})\lambda$

INTERFERENCE IN A WEDGE SHAPED FILM

In a wedge shape film, the thickness of the film at one end is zero and it increases consistently towards another end. A glass wedge shaped film is shown in figure. Similarly a wedge shaped air film can be formed by using two glass films touch at one end and separated by a thin wire at another end.



G
F
Air
A
E
i
i
D
r
Medium, μ
B
r
r+θ
N
θ
C
2t
r+θ
Air
θ
I
K

The angle made by two surfaced at touching end of wedge is called angle of wedge as shown Θ in figure . The angle is very small in order of less than 1^0. Path difference between two reflected rays BE and DF is given by

Δ = (BC+CD) in film – BE in air

= μ (BC+CD) – BE

=μ (BC+CI) – BE ∵ CD= CI

= μ (BN+NI) – BE .... (5.4)

In right triangle Δ BED, $\sin i = \frac{BE}{BD}$

Similarly in Δ BND, $\sin r = \frac{BN}{BD}$

Refractive index μ can be given as

$\mu = \frac{Sin\ i}{Sin\ r} = \frac{BE}{BN}$ or $BE = \mu\, BN$

Putting this value in equation (5.4) we get

Δ = μ (BN+ NI) -μ BN = μ NI .... (5.5)

Now in Δ DNI, $\cos(r+\theta) = \frac{NI}{DI}$

or $cos\ (r + \Theta) = \frac{NI}{2t}$ ⇒ $NI = 2t\ Cos\ (r + \Theta)$

Putting this value in equation (5.5)

Path difference, $\Delta = \mu.\ 2t\ Cos\ (r + \Theta)$ .... (5.6)

Since the light is reflecting from a denser medium therefore according to stokes treatment a path change of λ/2 occurs. Now net path difference

$\Delta = 2t\ Cos\ (r + \Theta) - \lambda/2$ .... (5.7)

For bright fringes the path difference should be in order of Δ = nλ where *n* is an integer (n= 0, 1, 2.......).

$2\mu t\ Cos\ (r + \Theta) - \lambda/2 = n\lambda$

or $2\mu t\ Cos\ (r + \Theta) = (\frac{2n+1}{2})\ \lambda$ where *n*= 0,1,2.....

or $2\mu t\ Cos\ (r + \Theta) = (\frac{2n-1}{2})\ \lambda$ .... (5.8)

Where, *n* = 1, 2, 3.....

For dark fringes path difference should be in order of $\Delta = (\frac{2n-1}{2})\ \lambda$.

$2\mu t\ Cos\ (r + \Theta) - \lambda/2 = (\frac{2n-1}{2})\lambda$

or $2\mu t\ Cos\ (r + \Theta) = n\lambda$ .... (5.9)

Since the focus of points of constant thickness is straight line, therefore the fringes are straight lined in shape.

According to equation (5.8), for bright fringes

$$t = \frac{(2n-1)\lambda}{4\,\mu\cos(r+\theta)} = \frac{\lambda}{4\,\mu\cos(r+\theta)} = \frac{3\lambda}{4\,\mu\cos(r+\theta)} = \cdots \ldots \ldots \qquad \ldots\ldots. \ (5.10)$$

If x_n is the distance of fringes from the edge (position of n^{th} fringe) then,

$$\tan\theta = \frac{t}{x_n}$$

or

$$x_n = \frac{(2n-1)\lambda}{4\,\mu\cos(r+\theta)\tan\theta} \qquad \ldots\ldots. \ (5.11)$$

Thus, $x_1 = \frac{\lambda}{4\,\mu\cos(r+\theta)\tan\theta}$, $x_2 = \frac{3\lambda}{4\,\mu\cos(r+\theta)\tan\theta}$ ....

Fringe width $\omega = x_{n+1} - x_n$

$$\omega = \frac{2\lambda}{4\,\mu\,\mathrm{Cos}(r+\theta)\tan\theta}) \qquad \ldots\ldots. \ (5.12)$$

If Θ is very small then $\tan\Theta \simeq \Theta$, and $\cos(r+\Theta) \cong r$. Further if we consider normal incidence then $r = 0^0$ then $\cos 0 = 1$ and equation (5.12) becomes

$$\omega = \frac{\lambda}{2\,\mu\,\theta} \qquad \ldots\ldots. \ (5.13)$$

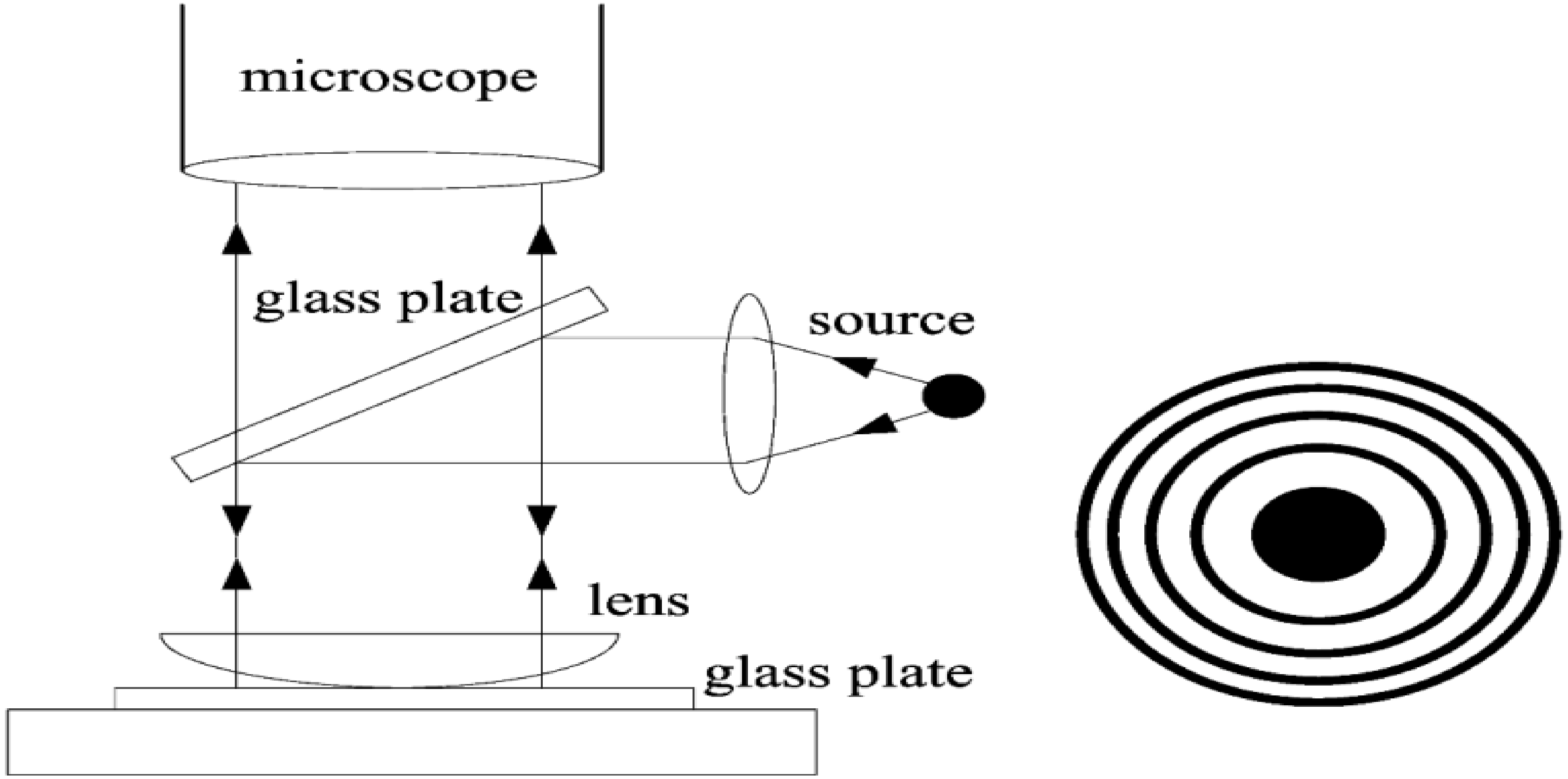
NEWTON'S RINGS

Newton's rings in a special case of wedge shaped film in which an air film is formed between a glass plate and a convex surface of lens. The thickness of air film is zero at the center and increases gradually towards the outside.

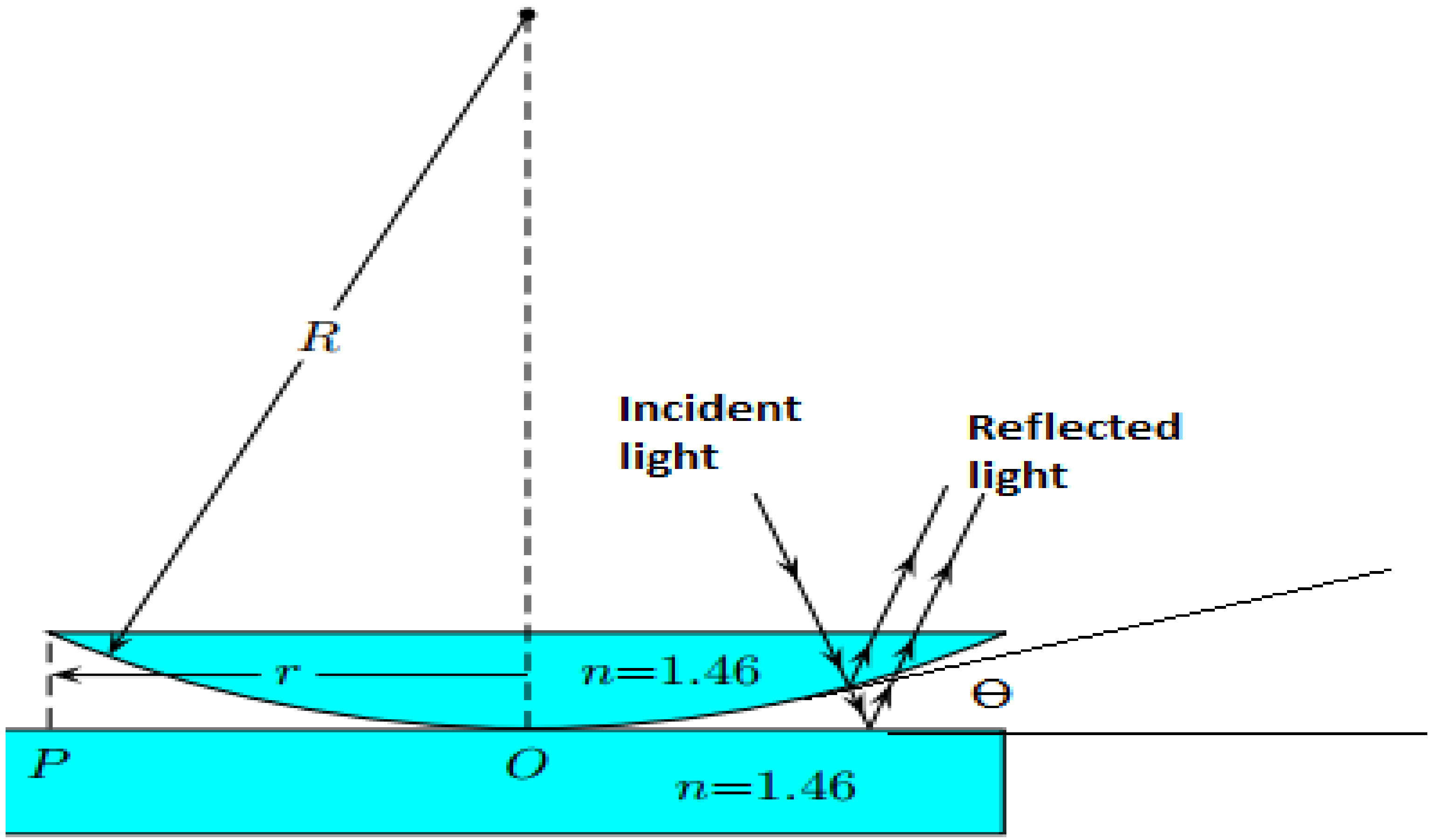
When a plano-convex lens of large focal length is placed on a plane glass plate, a thin air film is formed between the lower surface of plano-convex lens and upper surface of glass plate. When a monochromatic light falls on this film the light reflected from upper and lower surfaces of air film, and after interference of these rays, we get an inner dark spot surrounded by alternate bright and dark rings called Newton's rings. These rings are first observed by Newton and hence called Newton's rings.

Experimental Arrangement for Reflected Light

The experimental arrangement for Newton’s rings experiment is shown in Figure. A beam of light from a monochromatic source S is made parallel by using a convex lens L. The parallel beam of light falls on a partially polished glass plate inclined at an angle of 45^0. The light falls on glass plate is partially reflected and partially transmitted. The reflected light normally falls on the plano-convex lens placed on plane glass plate.



(a) (b)



R
Incident
light
Reflected
light
r
n=1.46
Θ
P
O
n=1.46

This light reflected from upper and lower surface of the air film form between plane glass plate and plno-convex lens. These rays interfere and rings are observed in the field of view. The figure shows the reflection of light form upper and lower surfaces of air film which are responsible for interference.

Formation of Bright and Dark Rings

As we know the interference occurs due to light reflected from upper and lower surface of air film form between glass plate and plano-convex lens. The air film can be considered as a special case of wedge shaped film. In this case, angle wedge is the angle made between the plane glass plate and tangent from line of contact to curved surface of plano convex lens as shown in figure.

The path difference between two interfering rays reflected by air film

$\Delta = 2\mu t \cos(r + \theta) - \frac{\lambda}{2}$ .... (5.14)

where μ is the refractive index of the air film, t is the thickness of air film at the point of reflection (say point P) r is angle of refraction and θ is angle of wedge.

In this case the light normally falls on the plane convex lens for the angle of refraction r = 0. Further, as we use a lens of large focal length the angle of wedge θ is very small. So $\cos(r+\theta) = \cos\theta = \cos 0^0 = 1$ and thus the path difference

$\Delta = 2\mu t - \frac{\lambda}{2}$ .... (5.15)

At point of contact t = 0, therefore, $\Delta = \frac{\lambda}{2}$

Which is the condition of minima. Hence at centre or at point of contact there is a dark spot.

Condition of Bright Rings or Maxima

The condition for bright rings is path difference $\Delta = n\lambda$ therefore

$\Delta = 2\mu t - \frac{\lambda}{2} = n\lambda$ where n= 0, 1, 2, 3………

or $$2\mu t = \left(\frac{2n+1}{2}\right)\lambda$$

or $$2\mu t = \left(\frac{2n-1}{2}\right)\lambda \qquad \text{……. (5.16)}$$

Where n = 1, 2, 3………

Condition of Dark Ring or Minima

In case of dark rings, the path difference, $\Delta = \left(\frac{2n-1}{2}\right) \lambda$

Where n= 1, 2, 3………

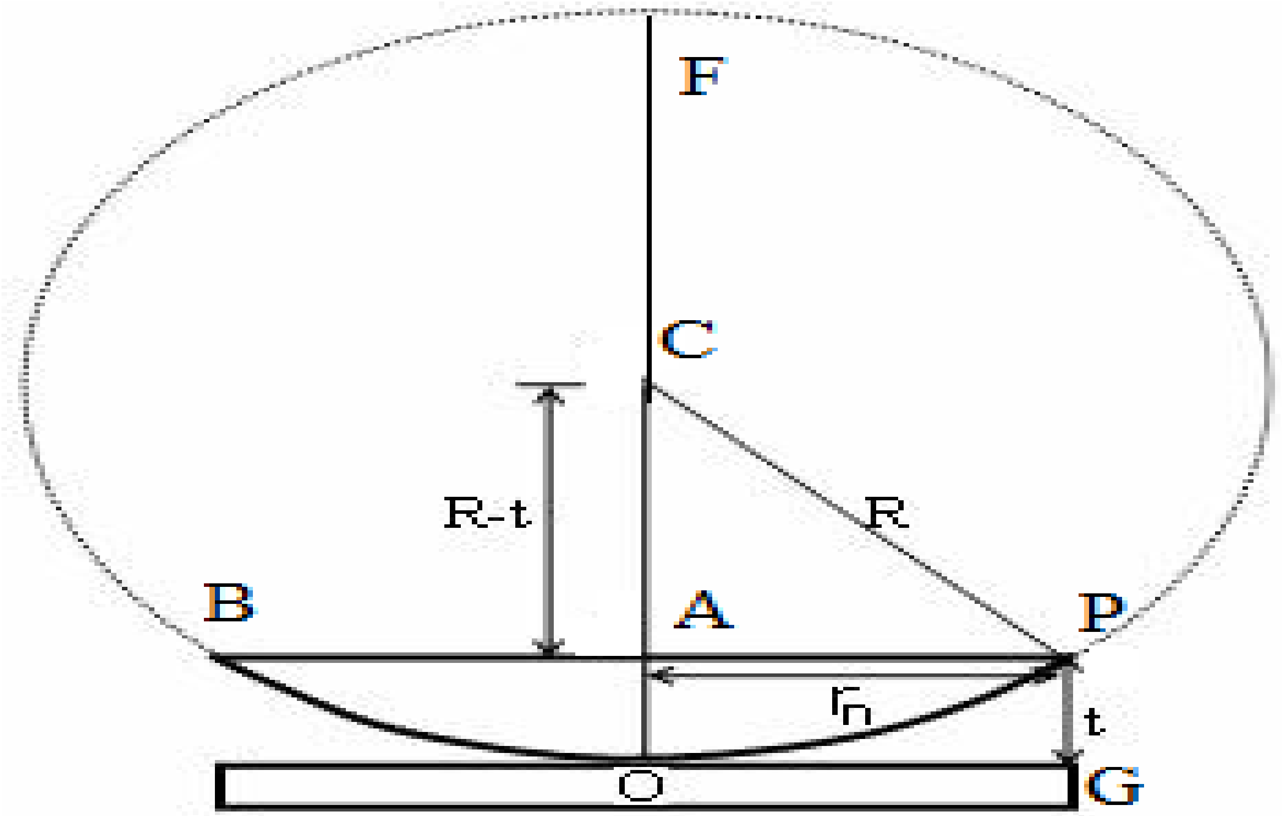
Therefore $\Delta = 2\mu t - \frac{\lambda}{2} = \left(\frac{2n-1}{2}\right)\lambda$

or $2\mu t = n\lambda$ ……. (5.17)

Thus corresponding to n = 1, 2, 3….. we observe first, second third…..etc. bright or dark rings. In Newton's rings experiment the locus of points of constant thickness is a circle therefore the fringes are circular rings.

Diameter of Bright and Dark Rings

In figure 5.9 the plano-convex lens BOPF is place on glass plate G and O is the point of contact. Suppose, C is the centre of the sphere OBFP from which the plano-convex lens is constructed. P is point on the air film at which the thickness of air film is t. At point P, the light is incident and reflected form the upper and lower surface of air film, and rings are formed. AP is the radius of ring passes through point P. According to property of circle



F
C
R-t
R
B
A
P
r_n
t
O
G

AP ×AB = AO× AL

$$r^2 = t \times (2R - t) \quad \because \text{AL=OL-OA}$$

Where R is the radius of curvature of lens.

$$r^2 = 2Rt - t^2$$

Since R is very large and t is very small, we can write

$$r^2 = 2Rt \qquad \text{or} \qquad t = \frac{r^2}{2R}$$

Substituting this value of t in equation (5.16), we get,

$$2\mu \frac{r^2}{2R} = \left(\frac{2n-1}{2}\right)\lambda$$

or

$$r^2 = \left(\frac{2n-1}{2}\right)\frac{\lambda R}{\mu}$$

This expression contains *n, i.e., r* is a function of *n*. Thus it is better to use r_n in place of *r*. If D_n is the diameter of nth bright ring then we have $r = r_n = D_n / 2$ and can write

$$\frac{{D_n}^2}{4} = \frac{\left(\frac{2n-1}{2}\right)\lambda R}{\mu}$$

or

$$D_n{}^2 = \frac{2(2n-1)\lambda R}{\mu} \qquad \text{.......} \quad (5.18)$$

Where n= 1, 2, 3……. Similarly for dark rings

$2\mu t = n\lambda$ or $2\mu \frac{r^2}{2R} = n\lambda$ or $r^2 = \frac{n\lambda R}{\mu}$.

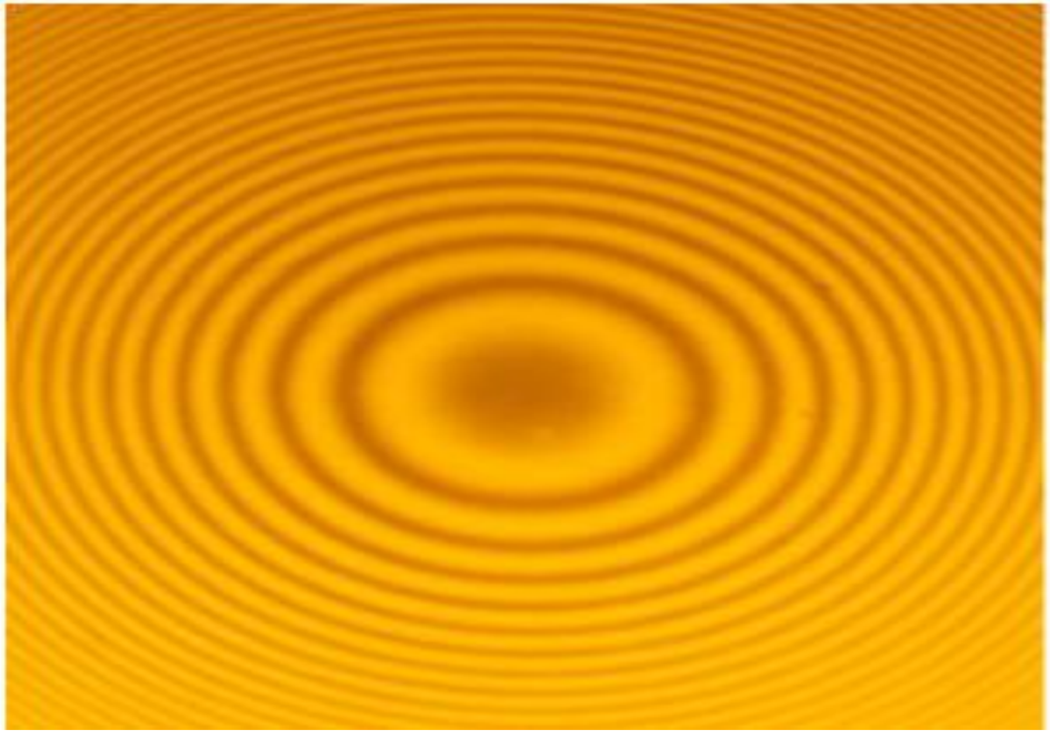
If D_n is diameter of n^{th} dark ring then

$$\frac{D_n^{\,2}}{4} = \frac{n\lambda R}{\mu}$$

or

$$D_n^{\,2} = \frac{4n\lambda R}{\mu}$$ ……. (5.19)

Where n = 1, 2, 3……



The alternate bright and dark rings are formed as shown in figure 5.10. The spacing between two consecutive rings can be given as

$r_{n+1}^{2} - r_{n}^{2} = \left(\sqrt{n+1} - \sqrt{n}\right) \lambda R$ (in case of air film μ =1)

Spacing between 1st and 2nd rings = $\left(\sqrt{2} - \sqrt{1}\right) \lambda R$ = 0.4142 λR

Spacing between 2nd and 3rd rings = $\left(\sqrt{3} - \sqrt{2}\right) \lambda R$ = 0.3178 λR

Spacing between 4th and 3rd rings = $\left(\sqrt{4} - \sqrt{3}\right) \lambda R$ = 0.21 λR

Thus it is clear that the spacing between successive rings decreases with increase in order.

Determination of Wave Length of a Monochromatic Light Source

In Newton's experiment if we use a light source of unknown wave length (say sodium lamp) then we can determine the wavelength of light source by measuring the diameters of Newton's ring.

If D_n is diameter of nth dark ring formed due to air film then

$$D_n^2 = 4n\lambda R$$

Where n is any integer number.

Similarly if $D_{(n+p)}$ is the diameter of $(n+p)^{th}$ ring

$$D_{n+p}^2 = \mu\,(n+p)\,\lambda R$$

Using this equation, we can write

$$D_{n+p}^2 - D_n^2 = 4\,(n+p)\,\lambda R - 4n\lambda R = 4\,P\,\lambda R$$

or $$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$ .... (5.20)

Where p is any integer number and R is radius of curvature of plano-convex lens.

Determination of Refractive Index of a Liquid by Newton's Rings Experiment

In Newton's rings experiment the diameter of n^{th} dark ring in case air film is

$$D_n^2 = 4n\lambda R \qquad (\because \mu = 1)$$

The diameter of $(n+p)^{th}$ ring

$$D^2_{n+p} = 4(n+p)\lambda R$$

If a liquid of refractive index μ is filled between the plane glass plate and convex lens then

$$D^2_n = \frac{4n\lambda R}{\mu} \quad \text{and} \quad D^2_{n+p} = \frac{4(n+p)\lambda R}{\mu}$$

Thus we can write

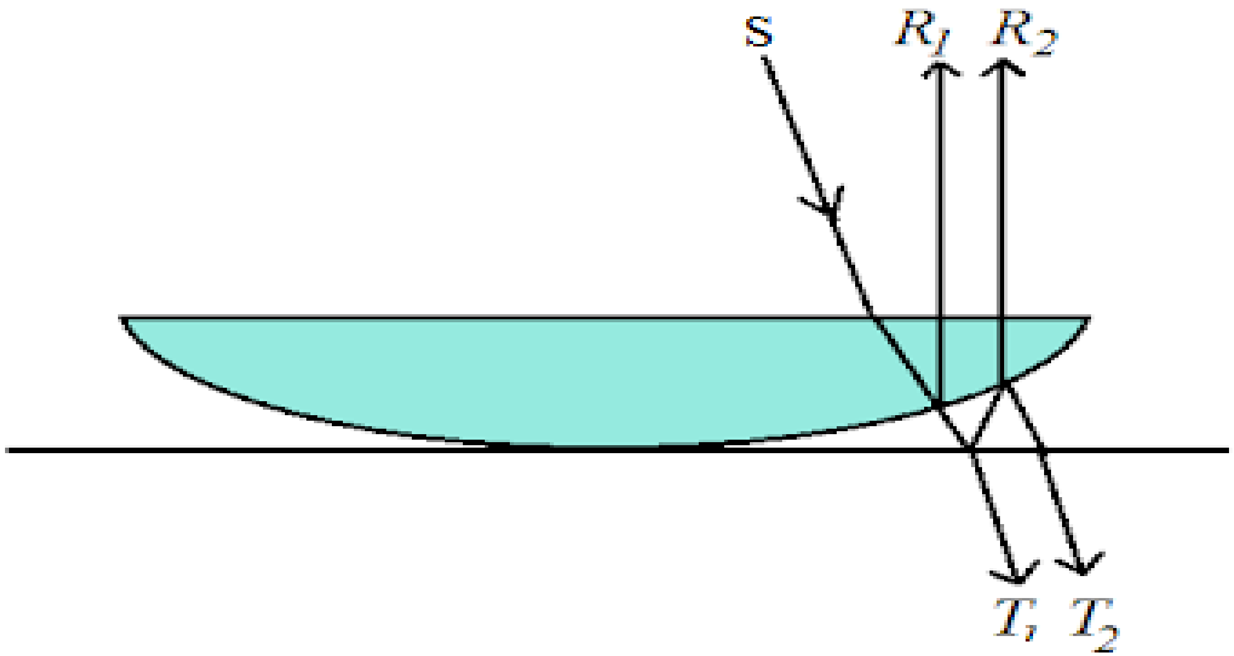
$$\frac{[D^2_{n+p} - D^2_n]air}{[D^2_{n+p} - D^2_n]liquid} = \frac{4p\ \lambda R}{\frac{4p\ \lambda R}{\mu}} = \mu$$

or

$$\mu = \frac{[D^2_{n+p} - D^2_n]air}{[D^2_{n+p} - D^2_n]liquid} \quad \text{.......} \quad (5.21)$$

Newton's Rings in Case of Transmitted Light

The Newton's rings can also be formed in case of interference due to transmitted light as shown in figure. In this case the transmitted rays 1 and 2 interfere, and we can observe the rings in the field of view. In this case the net path difference between the rays is $\Delta = 2\mu t$. since we will not consider the path difference arises due to reflection from denser medium. Therefore this is net path difference.



S
R_1
R_2
T_1
T_2

The condition for maxima (bright rings) is given by

$2\mu t = n\lambda$

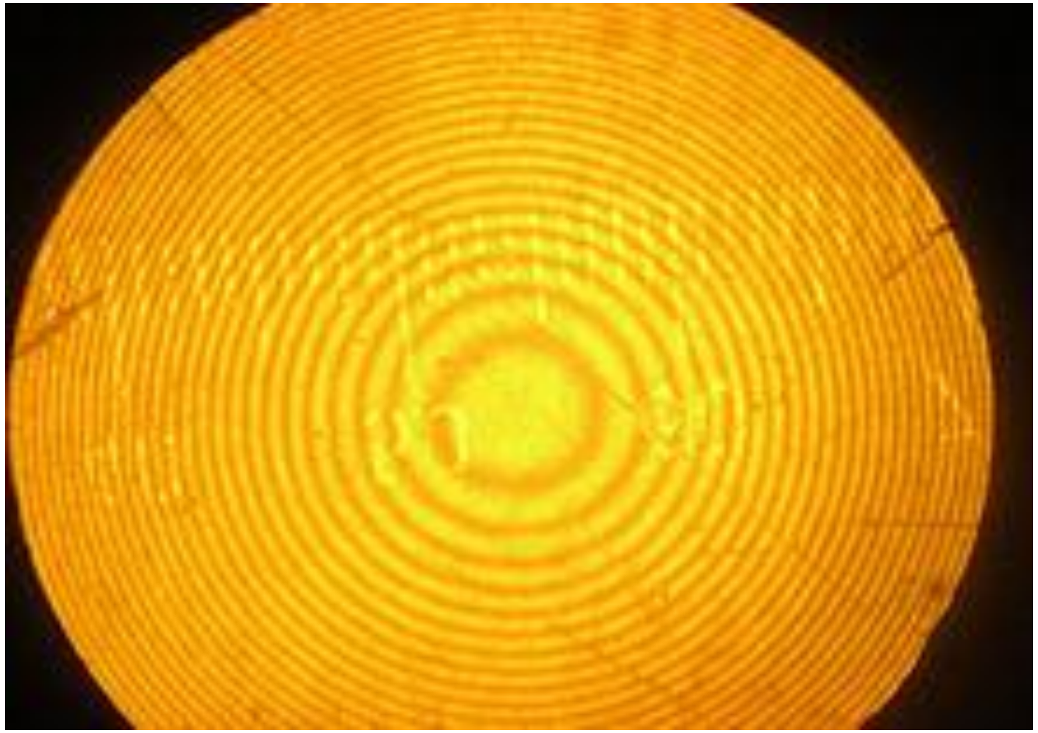
And we know that in case of reflected light, $t = \frac{r^2}{2R}$

$2\mu \frac{r^2}{2R} = n\lambda$

Now if D_n is the diameter of nth bright ring then, $\frac{D_n}{2} = r$ and thus

$Dn^2 = \frac{4n\lambda R}{\mu}$

In case of air film. $Dn^2 = 4n\lambda R$



Similarly in case of minima (dark ring) the diameter nth dark ring is given by

$D_n^2 = 2(2n-1)\lambda R$

We can see that, this is an opposite case of reflected light. In case at point of contact the path difference is zero which is condition corresponding to bright fringe thus the centre point is bright. The rings system in this case is shown in figure.

https://youtu.be/6lULYGC6lAE