Malus Law

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Outline of the Talk

1. Electromagnetic Wave
2. Polarisation
3. Malus Law
4. Practical set-up of Malus Law
5. Observation Table, Graph &Conclusion
6. Sources of error & Precautions

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Electromagnetic Wave

- Maxwell predicted the existence of electromagnetic waves in 1864 and experimentally detected by Hertz in 1887.
- Maxwell formulated a set of equations involving electric and magnetic fields, and their sources, the charge and current densities. These equations are known as Maxwell's equations.
- The Maxwell's equations in vacuum/ free space, without Charge and current is

$$\vec{\nabla} \cdot \vec{E} = 0; \qquad \vec{\nabla} \cdot \vec{B} = 0;$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}; \qquad \vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t};$$

where $\mu_0 = 4\pi 10^7 \frac{H}{m}, \varepsilon_0 = 8.85 \times 10^{-12} \frac{F}{m}$

Electromagnetic Wave

- The field E and$B$ in two curl equations can be separated. This is done by taking the curl of last two equation. The result is

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2};$$
$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2};$$

then we obtain

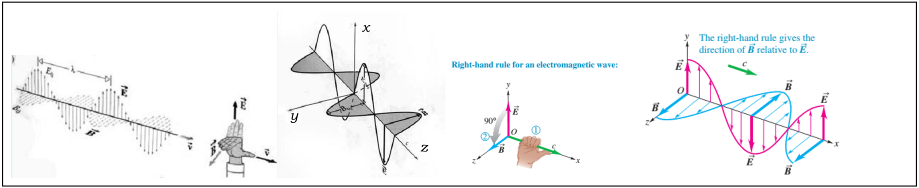
$$\nabla^2 \vec{B} = \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2};$$
$$\nabla^2 \vec{E} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2};$$

where$c = (\mu_0 \varepsilon_0)^{-1/2}$

- The most important prediction to emerge from Maxwell's equations is the existence of electromagnetic waves, which are (coupled) timevarying electric and magnetic fields that propagate in space. The speed of the waves, according to these equations, turned out to be very close to the speed of light($3 \times 10^8 m/s$),

Characteristics of electromagnetic waves in vacuum

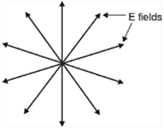
- The wave is transverse: Both $\vec{E}$ and $\vec{B}$ are perpendicular to the direction of propagation of the wave and to each other.Namely, the underlying oscillation (in this case oscillating electric and magnetic fields) is along directions perpendicular to the direction of propagation. This is in contrast to longitudinal waves, such as sound waves, in which the oscillation is conned to the direction of propagation.
- Unlike mechanical waves, which need the oscillating particles of a medium such as water or air to be transmitted, electromagnetic waves require no medium. What's "waving" in an electromagnetic wave are the electric and magnetic fields.



- There is a definite ratio between the magnitudes of $\vec{E}$ and $\vec{B}$: $E = cB$.
- The wave travels in vacuum with a definite and unchanging speed c.

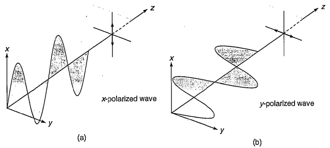
Unpolarized and Polarized Light

- An ordinary light source consists of a very large number of randomly oriented atomic emitters. Each excited atom radiates a polarized wave-train for roughly $10^{-8}s$.
- If these changes take place at so rapid a rate as to render any single resultant polarization state indistinct, so the wave is referred to as natural light, or unpolarized light, or randomly polarized light.
- So in general most light sources in nature emit unpolarized light(EM wave) i.e., light consists of many wave trains whose directions of oscillation are completely random. Light is generally nether completely polarized nor completely unpolarized. More often, it is partially polarized.



Types of polarization

- Polarization occurs with all transverse waves, suppose if we move one end of a string up and down then a transverse wave is generated [Fig. (a)]. Each point of the string executes a sinusoidal oscillation in a straight line (along the x-axis) and the wave is linearly polarized wave. It is also known as a plane polarized wave because the string is always confined to the $x - z$ plane. The displacement for such a wave can be written in the form
$$x(z,t) = acos(kz - \omega t + \phi_1); y(z,t) = 0$$
where a represents the amplitude of the wave and $\phi_1$is the phase constant; the $y-$coordinate of the displacement is always zero.
- The string can also be made to vibrate in the y-z plane [Fig.(b)] for which the displacement would be given by
$$y(z,t) = acos(kz - \omega t + \phi_2); x(z,t) = 0$$
- In general, the string can be made to vibrate in any plane containing the z-axis. So Light is polarized when its electric fields oscillate in
a single plane, rather than in any direction perpendicular to the direction of propagation.



(a) A linearly polarized wave on a string with the displacement confined to the x-z plane;
(b) A linearly polarized wave on a string with the displacement confined to the y-z plane.

Types of polarization

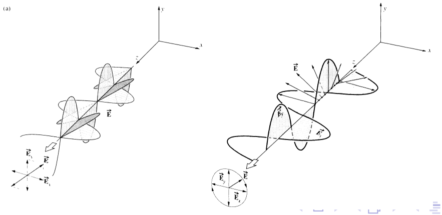
- Linear Polarization : Two orthogonal plane waves with same phase but possibly different amplitudes.
- Let us consider the propagation of two linearly polarized electromagnetic waves (both propagating along the z-axis) with their oscillating electric vectors. The electric fields associated with the waves can be written in the form

$$E_1 = \hat{x} E_{ox} \cos(kz - \omega t)$$
$$E_2 = \hat{y} E_{oy} \cos(kz - \omega t + \varepsilon)$$

where E_{ox} and E_{oy} represent the amplitudes of the waves; ε the relative phase difference between the waves. if ε is zero or an integral multiple of $\pm 2\pi$ the waves are said to be in phase,The resultant of these two waves would be given by

$$E = E_1 + E_2 = (\hat{x} E_{ox} + \hat{y} E_{oy}) \cos(kz - \omega t)$$

This equation tells us that the resultant is also a linearly polarized wave with its electric vector oscillating along the same axis.



Types of polarization

- Circular Polarization : Two orthogonal plane waves with 90 deg phase shift but same amplitudes.
- Another case when both constituent waves have equal amplitudes (i.e., $E_{0x} = E_{0y} = E_0$), and in addition, their relative phase difference $\varepsilon = -\pi/2 + 2m\pi$, where m = 0, ± 1, ±2,.... Accordingly

$$E_1 = \hat{x} E_o cos(kz - \omega t)$$
$$E_2 = \hat{y} E_o sin(kz - \omega t)$$

- the consequent wave is

$$E = E_0[\hat{x} E_o cos(kz - \omega t) + \hat{y} E_o sin(kz - \omega t)]$$

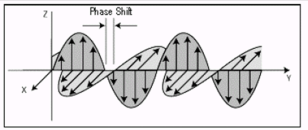
- such type of wave is called Circular Polarized wave

Types of polarization

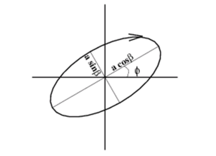
- Elliptical Polarization : (Linear + circular polarization = elliptical polarization) Possibly any degree of phase shift with different amplitudes. In elliptical light the resultant electric-field vector E will rotate, and change its magnitude, as well. In such cases the endpoint of E will trace out an ellipse, if we rearrange the above equations we the more general equation like this

$$\left(\frac{E_1}{E_{0x}}\right)^2 + \left(\frac{E_2}{E_{0y}}\right)^2 - 2\left(\frac{E_1}{E_{0x}}\right)^2\left(\frac{E_2}{E_{0y}}\right)^2 cos\varepsilon = sin^2\varepsilon$$

- If $E_{0x} \neq E_{0y} \neq E_0$ and $\varepsilon = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, ...$ we get elliptical polarized light.

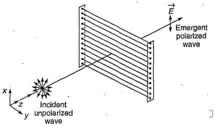


D. Elliptically Polarized Light



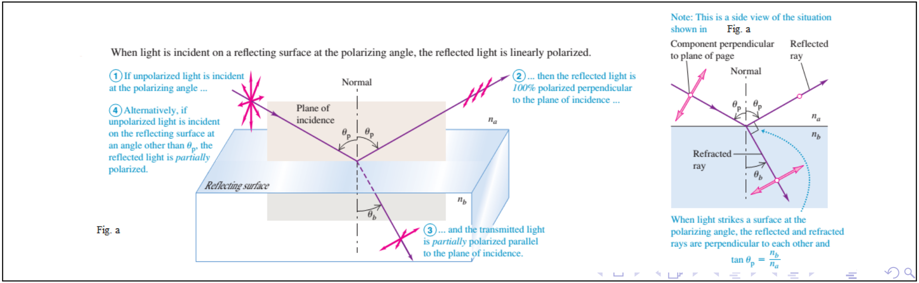
Production of polarized Light

- The Wire Grid Polarizer- The physics behind the working of the wire grid polarizer is probably the easiest to understand. It essentially consists of a large number of thin copper wires placed parallel to each other as shown in Fig.
- When an unpolarized electromagnetic wave is incident on it then the component of the electric vector along the length of the wire is absorbed. This is due to the fact that the electric field does work on the electrons inside the thin wires and the energy associated with the electric field is lost in the Joule heating of the wires.
- On the other hand, (since the wires are assumed to be very thin) the component of the electric vector along the x-axis passes through without much attenuation.
- Thus the emergent wave is linearly polarized with the electric vector along the x-axis. However, for the system to be effective (i.e., the E_y component almost completely attenuated) the spacing between the wires should be $\lesssim \lambda$,. Clearly, the fabrication of such a polarizer for a 3 cm microwave is relatively easy. because the spacing has to be $<$ 3 cm.
- On the other hand, since the light waves are associated with a very small wavelength ($\sim 5x10^{-5}cm$), the fabrication of a polarizer in which the wires are placed at distances $<5x10^{-5}cm$ is extremely difficult. Nevertheless, Bird and Parrish did succeed in putting about 30,000 wires in about one inch.



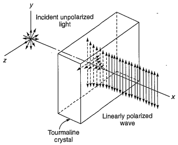
Production of polarized Light

- Polarization by Reflection-When unpolarized natural light is incident on a reflecting surface between two transparent optical materials. For most angles of incidence, waves for which the electric-field vector is perpendicular to the plane of incidence (that is, parallel to the reflecting surface) are reflected more strongly than those for which lies in this plane. In this case the reflected light is partially polarized in the direction perpendicular to the plane of incidence.
- But at one particular angle of incidence, called the polarizing angle $\theta_p (= tan^{-1} n_2/n_1)$ the light for which $\vec{E}$ lies in the plane of incidence is not reflected at all, but is completely refracted. At this same angle of incidence the light for which is perpendicular to the plane of incidence is partially reflected and partially refracted. The reflected light is therefore completely polarized perpendicular to the plane of incidence, as shown in Fig. a.



Production of polarized Light

- In 1812 the British scientist Sir David Brewster discovered that when the angle of incidence is equal to the polarizing angle the reflected ray and the refracted ray are perpendicular to each other.
- Polarization by Double Refraction- When an unpolarized beam enters a dichroic crystal like tourmaline, it splits up into two linearly polarized components. One of the components gets absorbed quickly and the other component passes through without much attenuation.

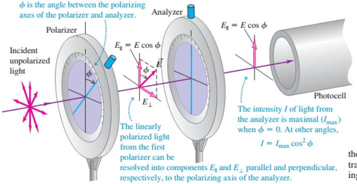


Production of polarized Light

- Polaroids- As already pointed out, it is extremely difficult to fabricate a wire grid polarizer for visible light. However, instead of long thin wires, one may employ long chain polymer molecules that contains atoms (like iodine) which provide high conductivity along the length of the chain.
- A sheet containing such long chain polymer molecules (which are aligned parallel to each other) is known as a Polaroid. When a light beam is incident on such a Polaroid, the molecules (aligned parallel to each other) absorb the component of electric field which is parallel to the direction of alignment because of the high conductivity provided by the iodine atoms; the component perpendicular to it passes through. Thus the aligned conducting molecules act similar to the wires in the wire grid polarizer and since the spacing between two adjacent long chain molecules is small compared to the optical wavelength.
- The Polaroid is usually very effective in producing linearly polarized light. The aligning of the long chain conducting molecules is not very difficult.

Malus Law

- When light falls on a polarizer, the transmitted light gets polarized. The polarized light falling on another Polaroid, called analyzer, transmits light depending on the orientation of its axis with the polarizer. The intensity of light transmitted through the analyzer is given by Malus' law. The law describes how the intensity of light transmitted by the analyzer varies with the angle that its plane of transmission makes with that of the polarizer. The law can be stated in words as follows:
- **The intensity of the transmitted light varies as the square of the cosine of the angle between the two planes of transmission.**



An ideal analyzer transmits only the electric field component parallel to its transmission direction (that is, its polarizing axis).

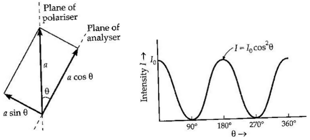
Malus Law

- If $A_0$is the amplitude of the incident light and A_t is amplitude of the light transmitted through the analyzer, then

$$A_t = A_0 cos\theta$$

As we know the $Intensity \propto (amplitude)^2$ then $I_t = A_t^2 = A_0^2 cos^2\theta = I_0^2 cos^2\theta$

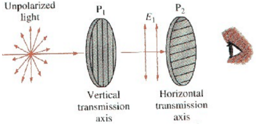
WhereI_t is the intensity of the light transmitted through the analyzer; and I_0^2 is the intensity of the incident plane polarized light.



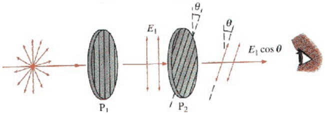
Malus Law

- Consider the two cases

1. If θ is zero, the second polarizer (analyzer) is aligned with the first polarizer, and the value of$cos^2\theta$ is one. Thus the intensity transmitted by the second filter is equal to the light intensity that passes through the first filter. This case will allow maximum intensity to pass through.



2. If θ is 90°, the second polarizer (analyzer) is oriented perpendicular to the plane of polarization of the first filter, and the $cos^2(90^\circ)$ gives zero. Thus no light is transmitted through the second filter. This case will allow minimum (zero) intensity to pass through.



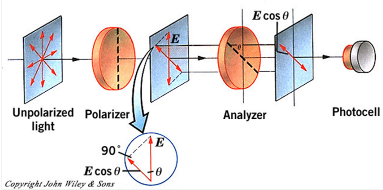
Practical Set-up of Malus law

- Object- To verify the cosine square of (Malus law) for plane polarized light with the help of a photo voltaic cell.
- Apparatus required- Source of light(Bulb/ laser light), optical bench,scale arrangement, polarizer-analyzer pair, photo voltaic cell, output device(moving coil galvanometer/ digital meter).
- Formula used- According to Malus law or cosine square law, when a beam of plane polarized light is incident on the analyzer, then the intensity I of the emergent light is given by

$$I = I_0 cos^2 \phi$$

  where I_0 =intensity of plane polarized light incident on the analyzer and ϕ = angle between planes of transmission of polarizer and the analyzer.
- From this law, when $\phi = 0, I = I_0$, maximum intensity,
- when $\phi = 90, I = 0$ minimum intensity

Practical Set-up of Malus law



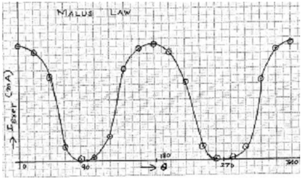
Observation table

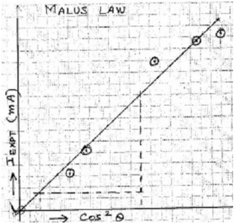
S.No.	Angle through which analyzer is rotated ϕ	Galvanometer deflection θ	$Cos\phi$	$Cos^2\phi$	$\frac{\theta}{Cos^2\phi}$
1	0				
2	10				
3	20				
4	30				
5	40				
6	50				
7	60				
8	70				
9	80				
10	90				

Graph and Conclusion

- The current/deflection (proportional to light intensity), noted for different angles of rotation of the analyzer, follows a cosine curve for 360^0 of rotation, indicating the validity of equation- $I = I_0 Cos^2\theta$.
- The relative intensity of the light emerging from analyzer is maximum at 0° and 180° and it attains minimum value at 90° and 270°. In between it varies as a Cosine function as indicated by the graph.
- Graph between $Cos^2\theta$ on X axis and θ on Y axis curve is

  a straight line, as expected, with unit slope indicating the correctness of the Malus's law.





Sources of error & Precautions

1. Analyzer and Polarizer should be at same horizontal level and position of the polarizer should not be disturbed throughout the experiment.
2. Analyzer must be rotated by small angles$(5^0 - 10^0)$. Changing values abruptly may cause errors.
3. Experiment should be performed in dark room.
4. Photo detector is a very sensitive device. It should be adjusted well (at appropriate height) to receive maximum current.
5. The voltage applied to the source of light should be constant throughout the experiment.

THANKS