Semiconductor Devices, Analog and Digital Electronics

BLOCK – II OPERATIONAL AMPLIFIERS

UNIT –10: OPERATIONAL AMPLIFIER – FREQUENCY RESPONSE

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Content

- Ideal op-Amp as Low pass filter
- Ideal op-Amp as high pass filter
- Ideal op-Amp as band pass filter
- Real op-Amp frequency response

Objective

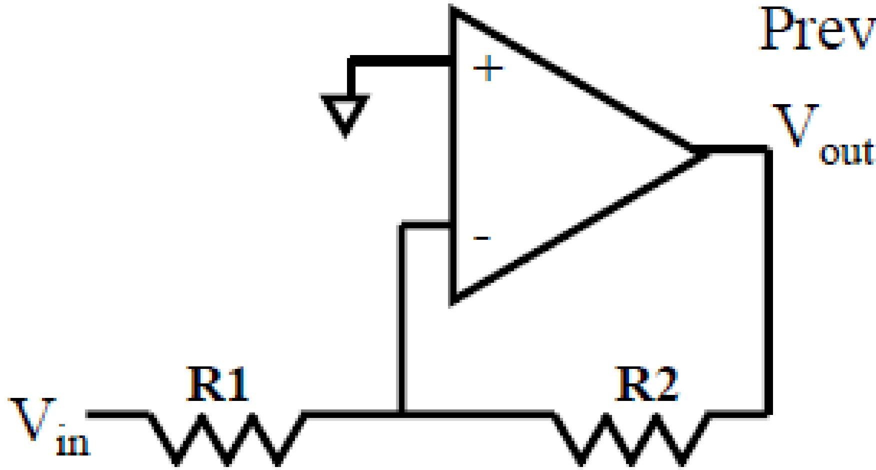
After studying this unit, you should be able to-

1. Define Ideal op-Amp as Low pass filter
2. Define Ideal op-Amp as high pass filter
3. Define Ideal op-Amp as band pass filter
4. Understand the Basic Concept of Real op-Amp frequency response

Ideal Op Amps Used to Control Frequency Response

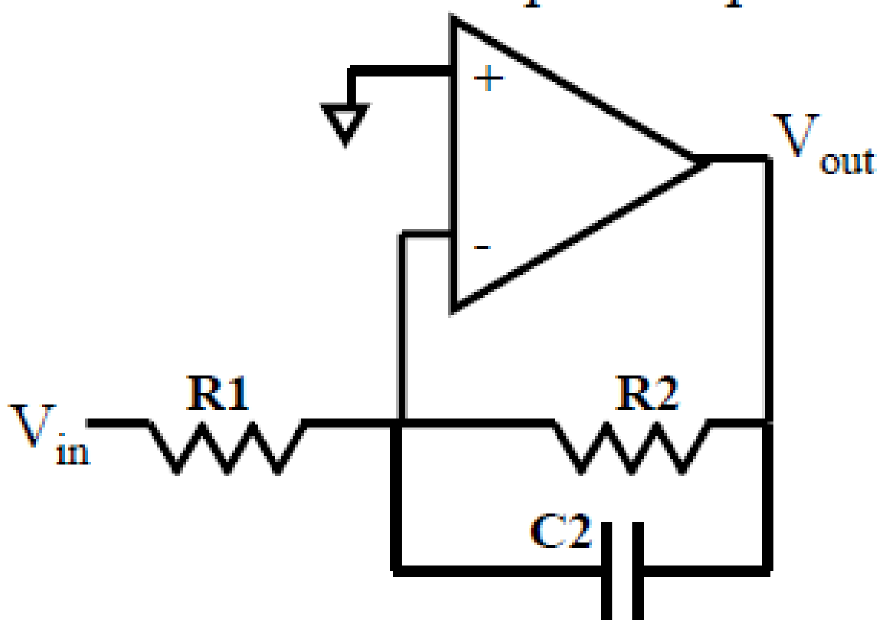
Low Pass Filter

Previously:



$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$$

Now put a capacitor in parallel with R2:



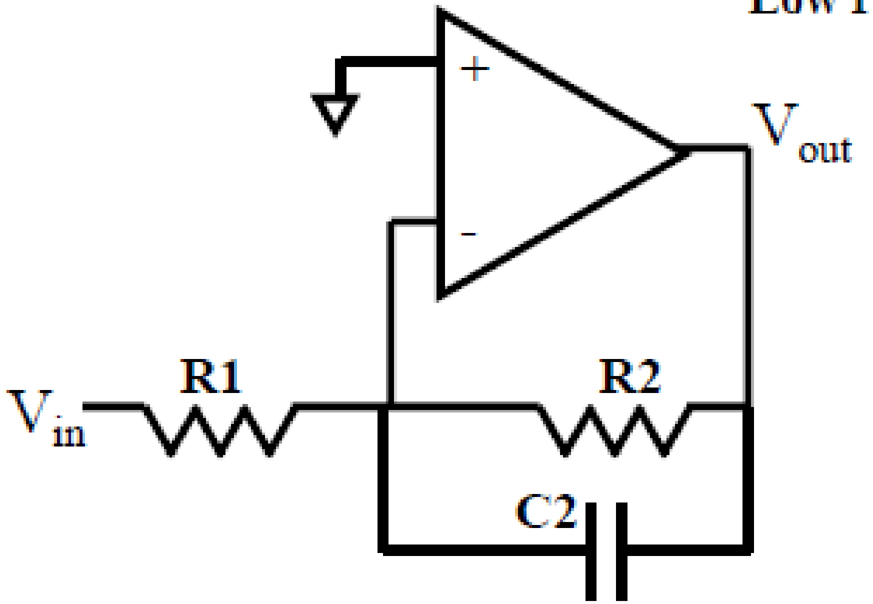
If $s = j\varpi$,

$$\frac{V_{out}}{V_{in}} = -\frac{R_2 \left\| \frac{1}{C_2 s} \right.}{R_1}$$

$$\frac{V_{out}}{V_{in}} = -\frac{1}{R_1}\frac{R_2 \frac{1}{C_2 s}}{R_2 + \frac{1}{C_2 s}} = -\frac{R_2}{R_1}\left(\frac{1}{1 + R_2 C_2 s}\right)$$

Ideal Op Amps Used to Control Frequency Response

Low Pass Filter



$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}\left(\frac{1}{1+R_2C_2s}\right)$$

•At DC (s=0), the gain remains the same as before ($-R_2/R_1$).

•At high frequency, $R_2C_2s >> 1$, the gain dies off with increasing frequency,

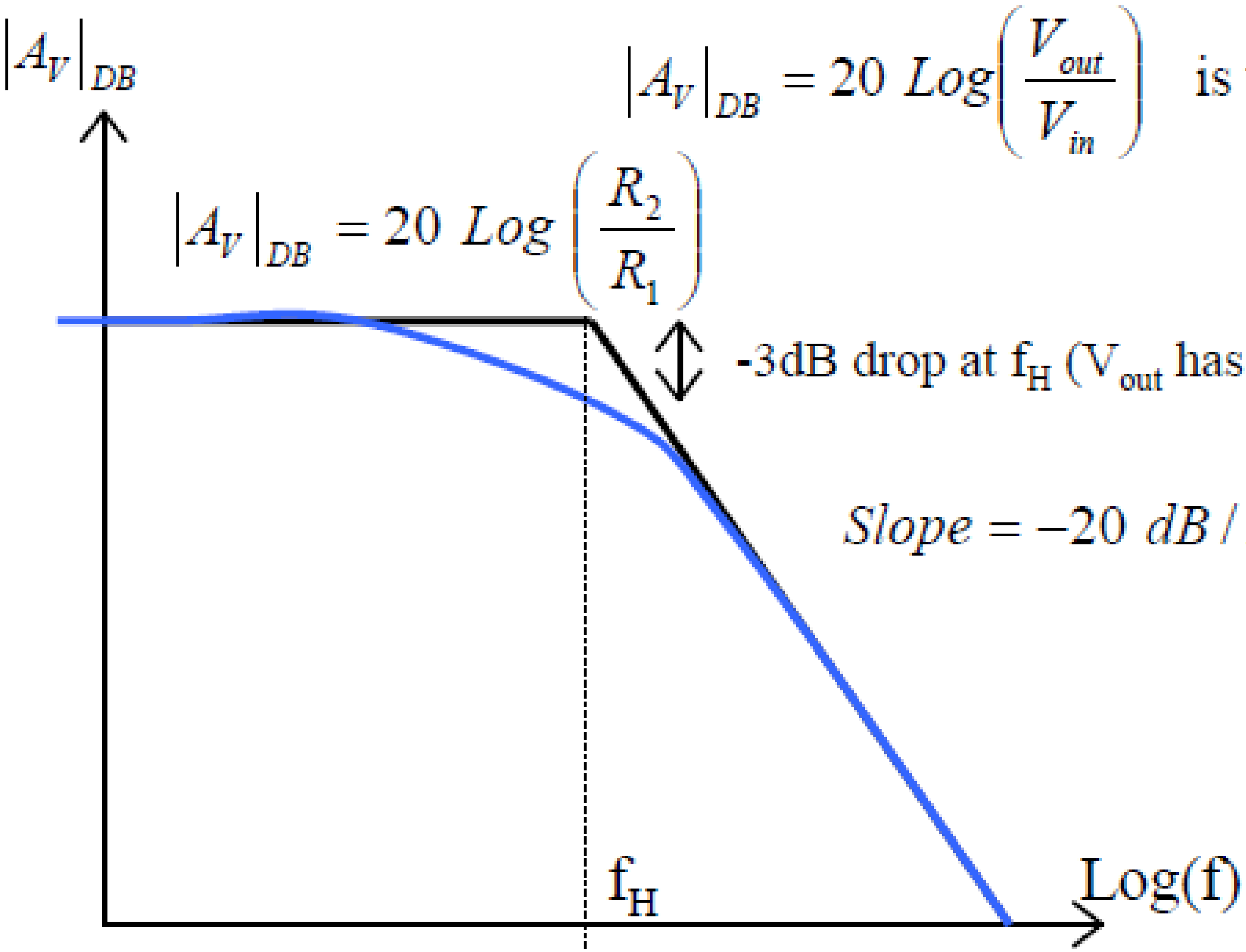
$$\frac{V_{out}}{V_{in}} \approx -\left(\frac{1}{R_1C_2s}\right) = -\left(\frac{\frac{1}{C_2s}}{R_1}\right)$$

$$\frac{1}{R_2C_2} = 2\pi f_H = \omega_H$$

•At high frequencies, more "negative feedback" reduces the overall gain

Ideal Op Amps Used to Control Frequency Response

Low Pass Filter



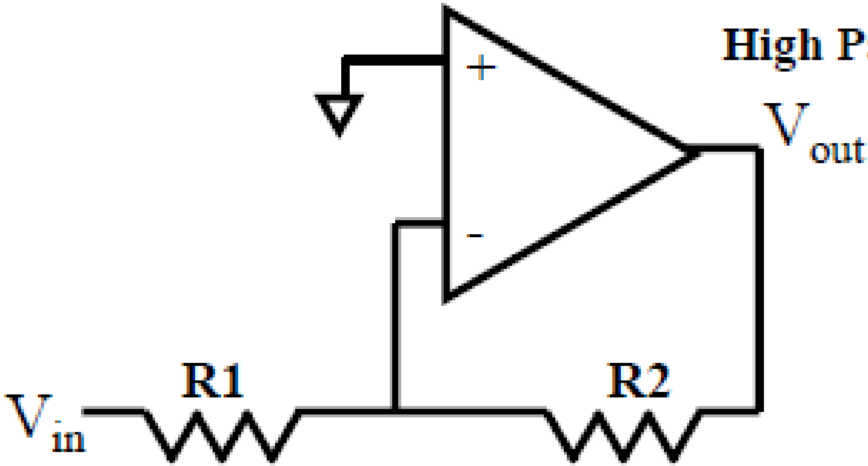
$$f_H = \frac{1}{2\pi\ R_2 C_2}$$

- At DC (s=0), the gain remains the same as before (-R2/R1)
- At high frequency, $R_2C_2s >> 1$, the gain dies off with increasing frequency

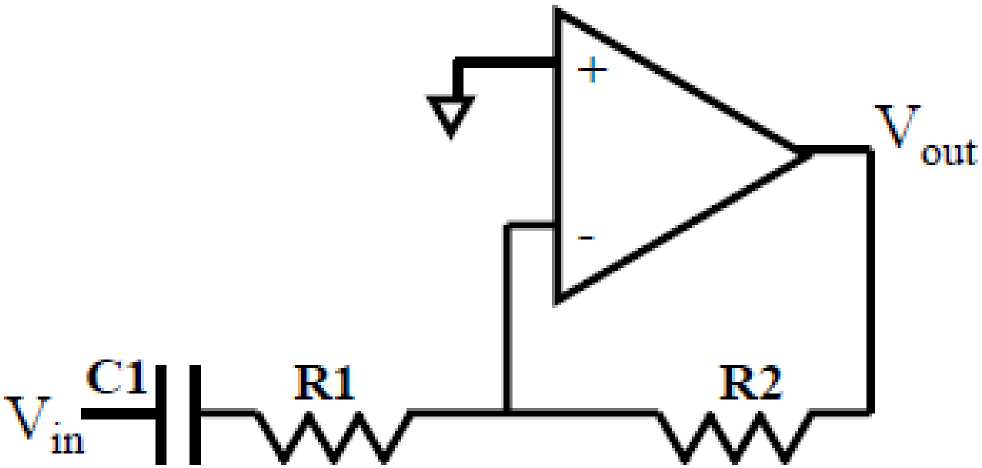
- Implements a "Low Pass Filter": Lower frequencies are allowed to pass the filter without attenuation. High frequencies are strongly attenuated (do not pass).

Ideal Op Amps Used to Control Frequency Response

High Pass Filter



$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$$



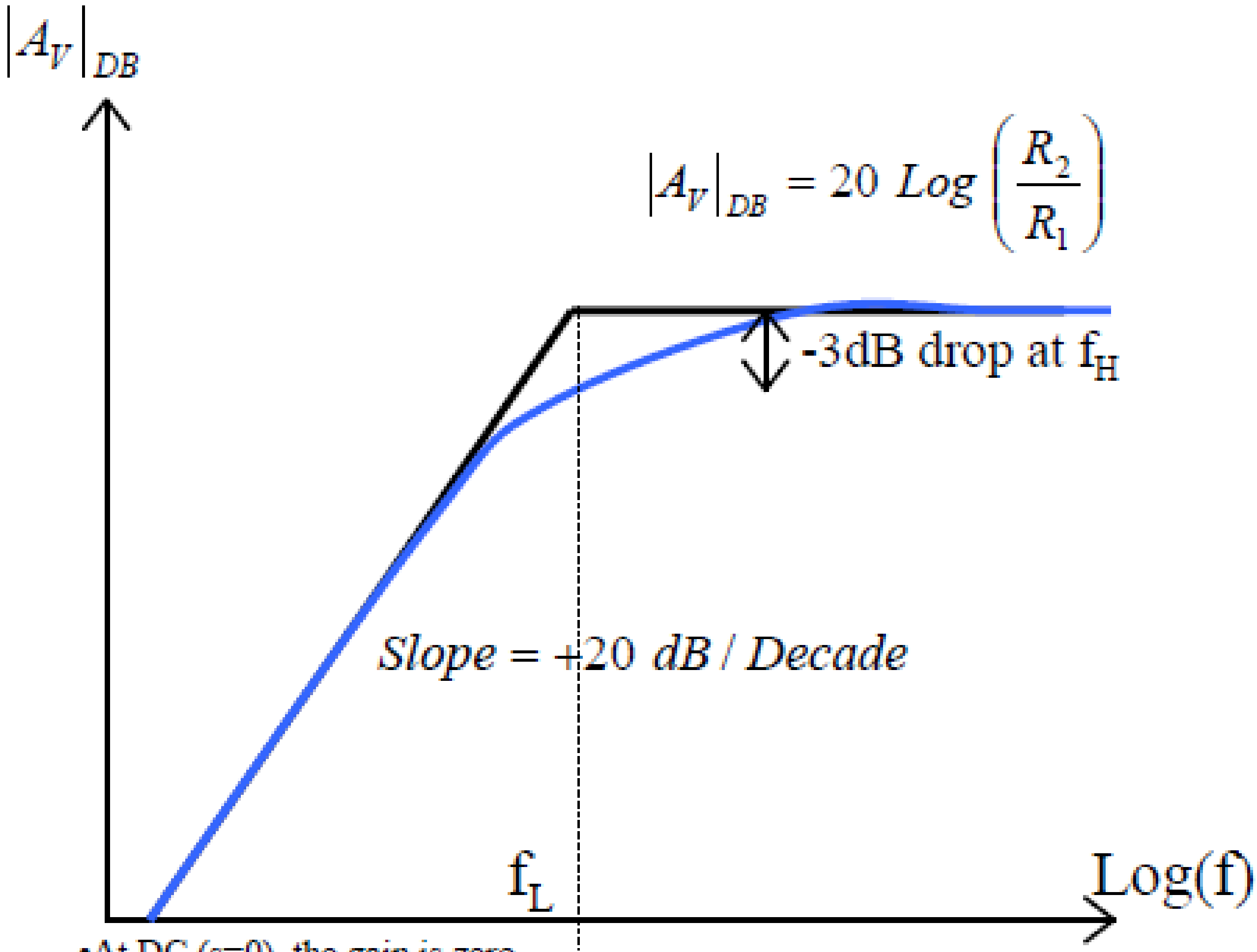
$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1 + \frac{1}{C_1 s}}$$

$$\frac{V_{out}}{V_{in}} = -\frac{R_2 C_1 s}{1 + R_1 C_1 s}$$

- At DC (s=0), the gain is zero.
- At high frequency, $R_1C_1s \gg 1$, the gain returns to it's full value, $(-R_2/R_1)$

Ideal Op Amps Used to Control Frequency Response

High Pass Filter



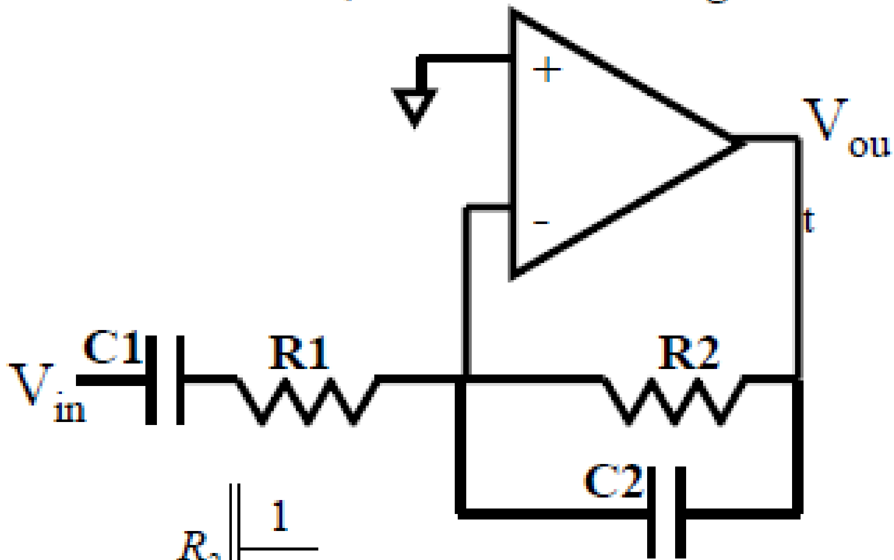
$$\frac{V_{out}}{V_{in}} = -\frac{R_2 C_1 s}{1 + R_1 C_1 s}$$

$$f_L = \frac{1}{2\pi\ R_1 C_1}$$

- At DC (s=0), the gain is zero.
- At high frequency, $R_1C_1s \gg 1$, the gain returns to it's full value, $(-R_2/R_1)$
- Implements a "High Pass Filter": Higher frequencies are allowed to pass the filter without attenuation. Low frequencies are strongly attenuated (do not pass).

Ideal Op Amps Used to Control Frequency Response

Band Pass Filter (combination of high and low pass filter)



$$\frac{V_{out}}{V_{in}} = -\frac{R_2 \| \frac{1}{C_2 s}}{R_1 + \frac{1}{C_1 s}}$$

Low Pass **High Pass**

$$\frac{V_{out}}{V_{in}} = -\frac{\frac{R_2 \frac{1}{C_2 s}}{R_2 + \frac{1}{C_2 s}}}{R_1 + \frac{1}{C_1 s}} = -\underbrace{\left(\frac{1}{1 + R_2 C_2 s}\right)}_{\text{Low Pass}}\underbrace{\left(\frac{R_2 C_1 s}{1 + R_1 C_1 s}\right)}_{\text{High Pass}}$$

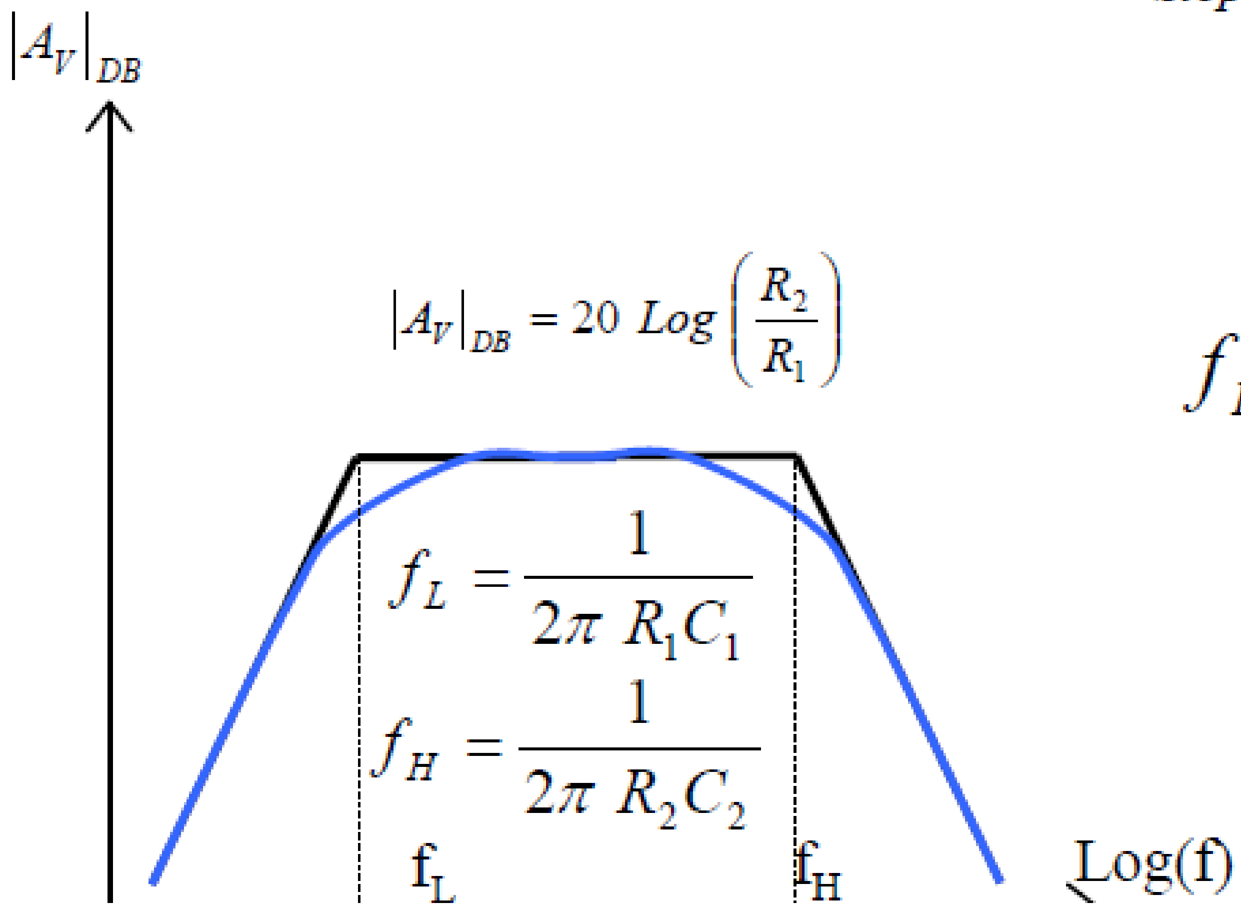
Ideal Op Amps Used to Control Frequency Response

Band Pass Filter (combination of high and low pass filter)

$$\frac{V_{out}}{V_{in}} = -\left(\frac{1}{1+R_2C_2s}\right)\left(\frac{R_2C_1s}{1+R_1C_1s}\right)$$

$$Slopes = {}^{-}\!/_{+}\, 20\ dB\ /\ Decade$$

↕ -3dB drop at f_H



$$|A_V|_{DB} = 20\ Log\left(\frac{R_2}{R_1}\right)$$

$$f_L << f_H$$

$$f_L = \frac{1}{2\pi\ R_1C_1}$$

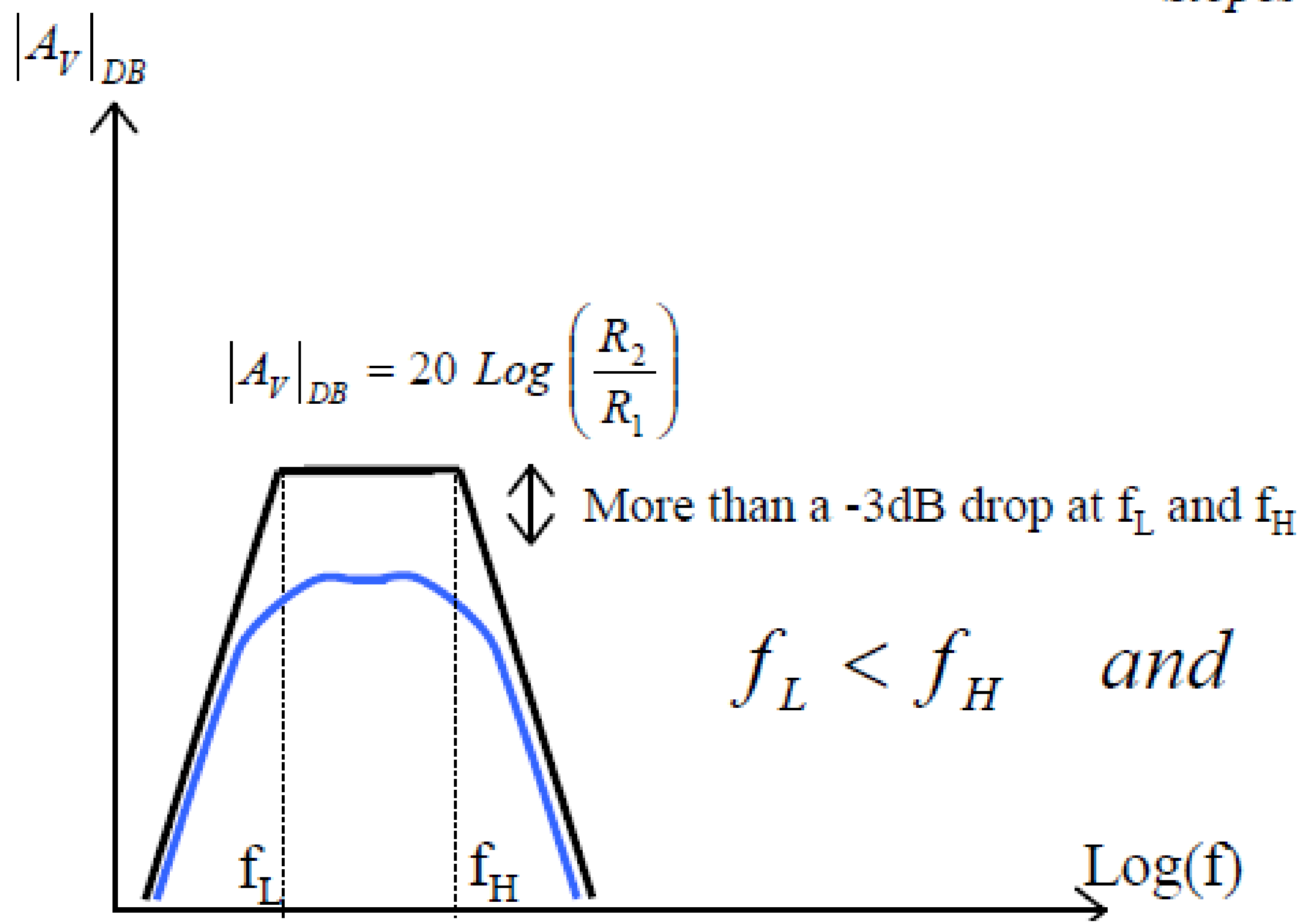
$$f_H = \frac{1}{2\pi\ R_2C_2}$$

Ideal Op Amps Used to Control Frequency Response

Band Pass Filter (combination of high and low pass filter)

$$\frac{V_{out}}{V_{in}} = -\left(\frac{1}{1+R_2C_2s}\right)\left(\frac{R_2C_1s}{1+R_1C_1s}\right)$$

$$Slopes = {}^{-}\!/_{+}\ 20\ dB\ /\ Decade$$



$$f_L < f_H \quad and \quad f_L \rightarrow f_H$$

General Frequency Response of a Circuit

Poles and Zeros

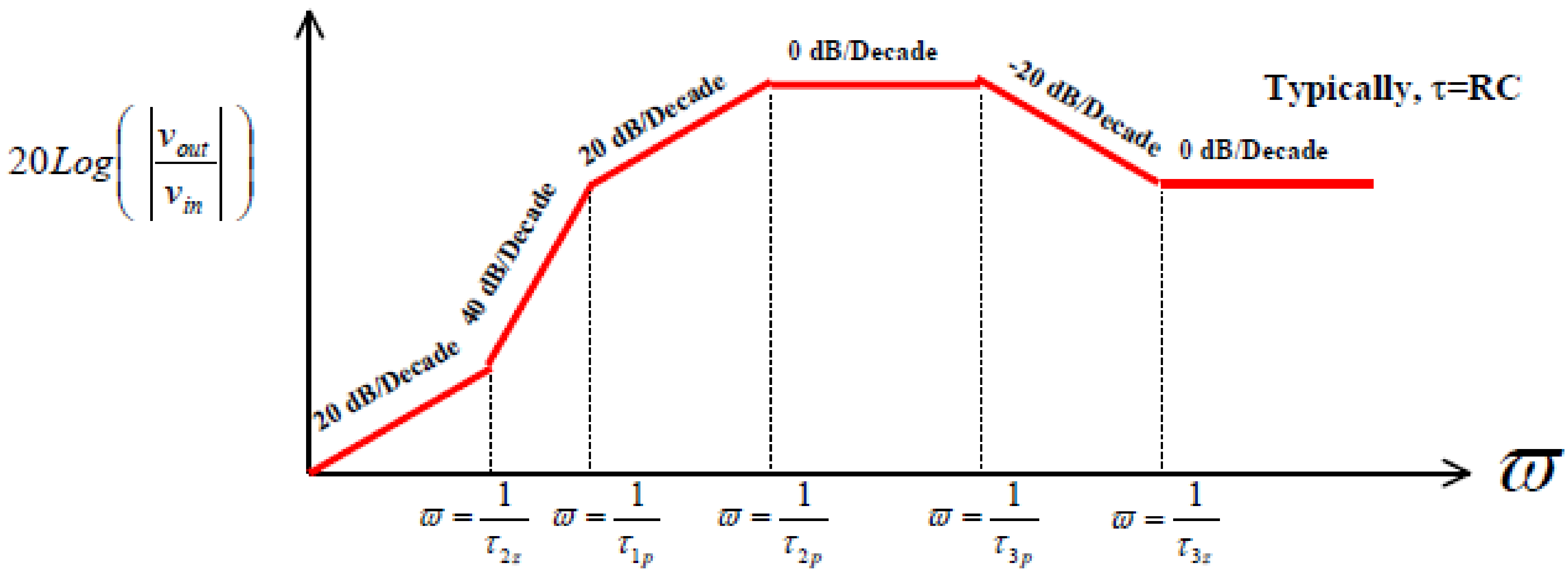
Generally, a circuit's transfer function (frequency dependent gain expression) can be written as the ratio of polynomials:

$$\frac{v_{out}}{v_{in}} = A\frac{(\tau_{1z}s)(1+\tau_{2z}s)(1+\tau_{2z}s)\ldots}{(1+\tau_{1p}s)(1+\tau_{2p}s)(1+\tau_{3p}s)\ldots} \quad \Rightarrow \quad \left|\frac{v_{out}}{v_{in}}\right| = A\frac{(\tau_{1z}\varpi)\sqrt{1-(\tau_{2z}\varpi)^2}\sqrt{1-(\tau_{3z}\varpi)^2}\ldots}{\sqrt{1-(\tau_{1p}\varpi)^2}\sqrt{1-(\tau_{2p}\varpi)^2}\sqrt{1-(\tau_{3p}\varpi)^2}\ldots}$$

Complex Roots of the numerator polynomial are called "zeros" while complex roots of the denominator polynomial are called "poles"

Each zero causes the transfer function to "break to higher gain" (slope increases by 20 dB/decade)

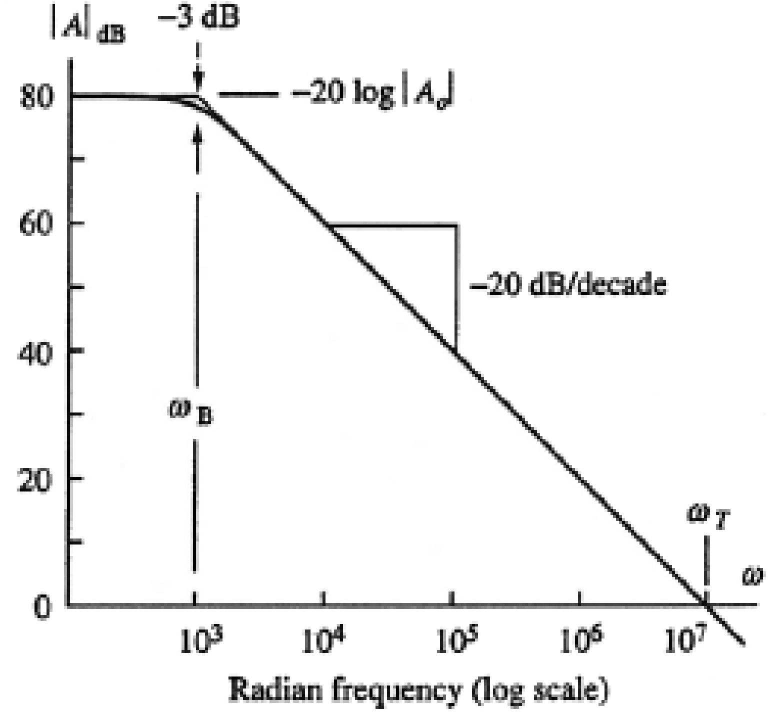
Each pole causes the transfer function to "break to lower gain" (slope decreases by 20 dB/decade)



Real Op Amp Frequency Response

•To this point we have assumed the open loop gain, $A_{Open\ Loop}$, of the op amp is constant at all frequencies.

•Real Op amps have a frequency dependant open loop gain.



$$A_{OpenLoop}(s) = \frac{A_O \varpi_B}{s + \varpi_B} = \frac{\varpi_T}{s + \varpi_B}$$

where,

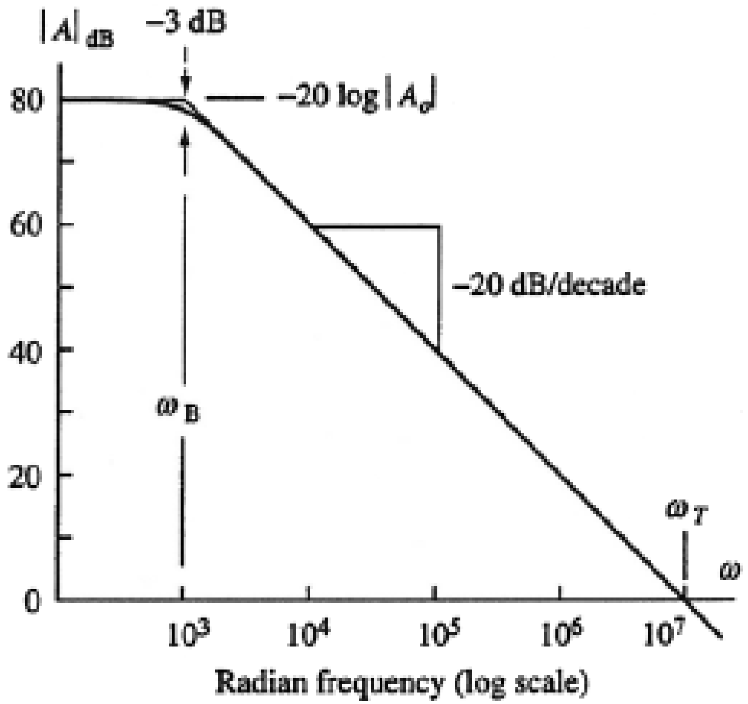
$s = j\varpi$

$A_O \equiv$ Open loop gain at DC

$\varpi_B \equiv$ Open loop bandwidth

$\varpi_T \equiv$ Unity - gain frequency $\left(\text{frequency where} \left|A_{OpenLoop}(s)\right| = 1\right)$

Real Op Amp Frequency Response



$$\left|A_{OpenLoop}(j\varpi)\right| = \frac{A_O \varpi_B}{\sqrt{\varpi^2 + \varpi_B^2}}$$

$$\left|A_{OpenLoop}(j\varpi)\right| = \frac{A_O}{\sqrt{1 + \frac{\varpi^2}{\varpi_B^2}}}$$

At Low Frequencies: $\left|A_{OpenLoop}\right| = A_O$

At High Frequencies: $\left|A_{OpenLoop}\right| \approx \frac{A_O \varpi_B}{\varpi} = \frac{\varpi_T}{\varpi}$

For most frequencies of interest, $\omega >> \omega_B$, the product of the gain and frequency is a constant, ω_T

$$f_T = \frac{\varpi_T}{2\pi} \equiv Gain - Bandwidth\ Product\ (GBW)$$

Real Op Amp Frequency Response

Previously, we found that the closed loop gain for the Non-inverting configuration was (for finite open loop gain):

$$A_{V,ClosedLoop} = \frac{V_{out}}{V_{in}} = \frac{A_{OpenLoop}}{1+\beta\, A_{OpenLoop}}, \text{ where } \beta = \frac{R_1}{R_1+R_2}$$

Using the frequency dependent open loop gain:

$$A_{V,ClosedLoop} = \frac{V_{out}}{V_{in}} = \frac{A_{OpenLoop}}{1+\beta\, A_{OpenLoop}}$$

$$A_{V,ClosedLoop} = \frac{\left(\frac{A_O \varpi_B}{s+\varpi_B}\right)}{1+\beta\left(\frac{A_O \varpi_B}{s+\varpi_B}\right)} = \frac{A_O \varpi_B}{s+\varpi_B\left(1+\beta\, A_O\right)}$$

Low Pass

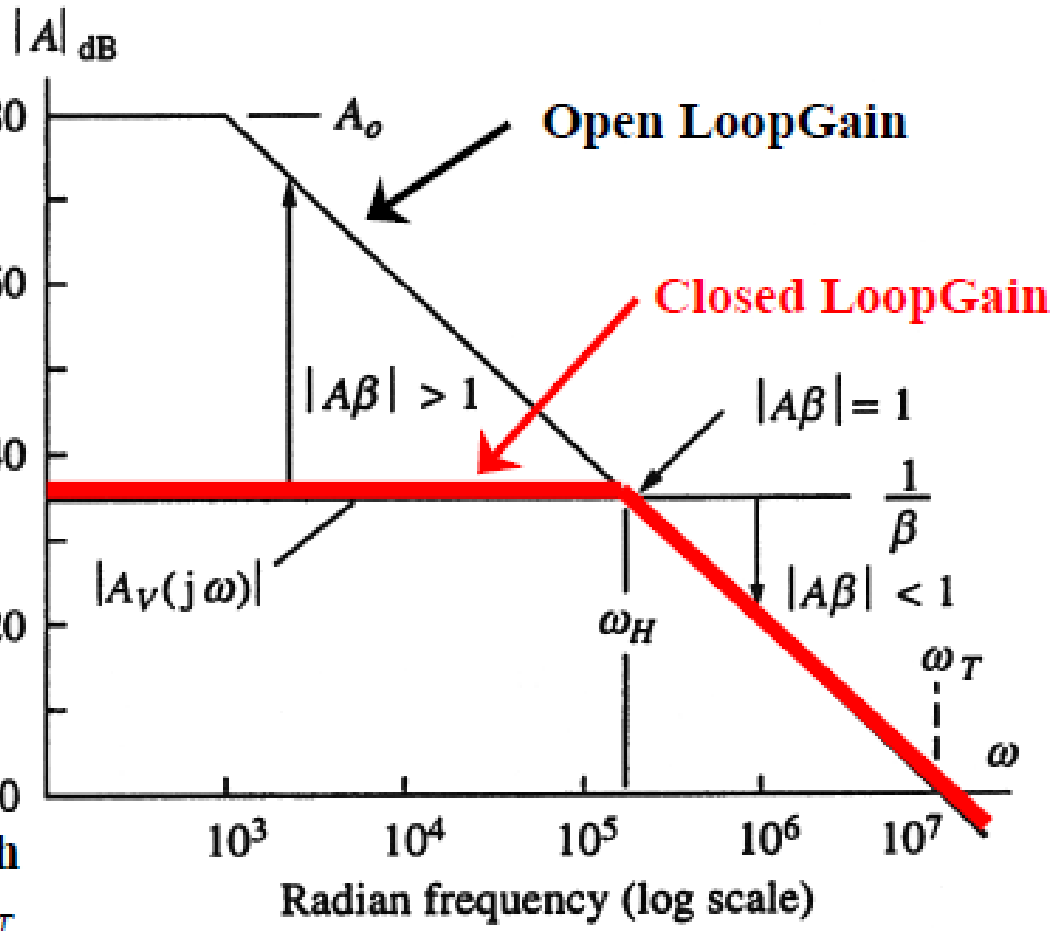
$$A_{V,ClosedLoop} = \frac{\frac{A_O \varpi_B}{\varpi_B\left(1+\beta\, A_O\right)}}{\frac{s}{\varpi_B\left(1+\beta\, A_O\right)}+1} = \frac{\frac{A_O}{\left(1+\beta\, A_O\right)}}{\frac{s}{\varpi_B\left(1+\beta\, A_O\right)}+1} = \underbrace{\left(\frac{1}{1+\frac{s}{\varpi_H}}\right)}_{\text{Low Pass}} A_{V,ClosedLoop}\Big|_{@\,DC}$$

where,

$$\varpi_H \equiv \text{Upper Cutoff Frequency}\left(\text{Closed Loop Bandwith}\right) = \varpi_B\left(1+\beta\, A_O\right)$$

Real Op Amp Frequency Response

The closed Loop Amplifier has a lower gain than the Open Loop Amplifier



The closed Loop Amplifier has a higher bandwidth than the Open Loop Amplifier

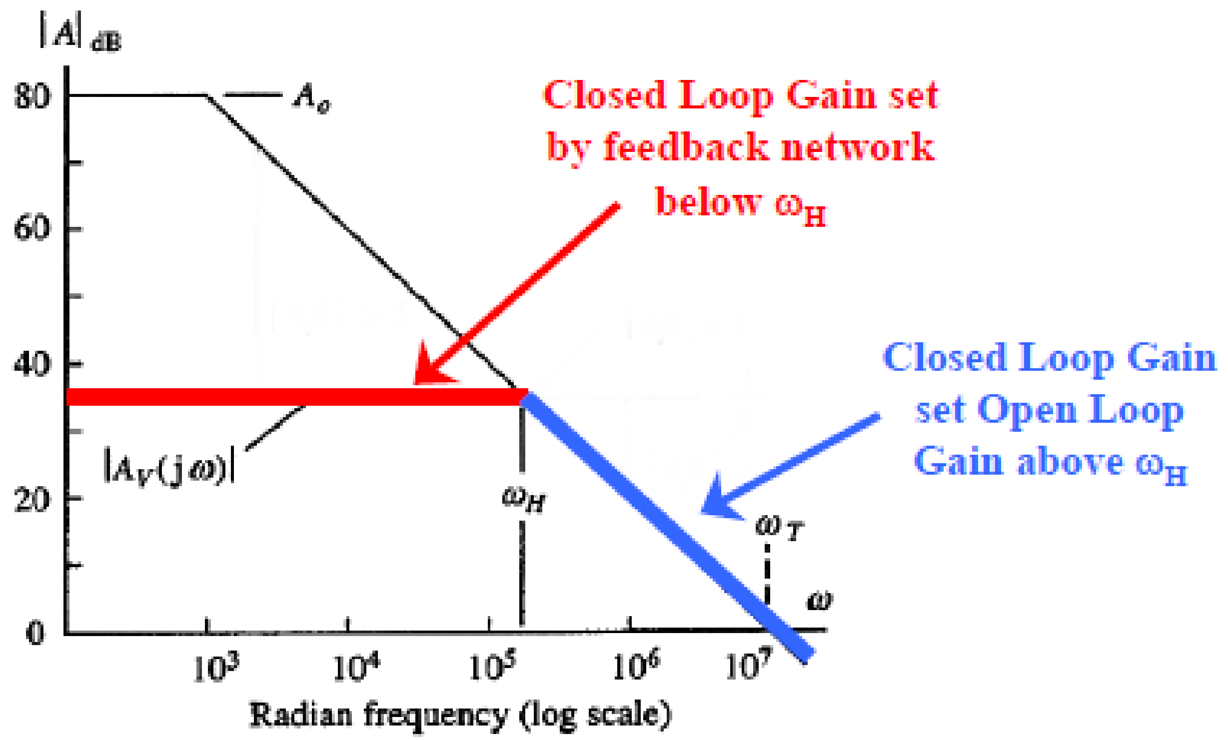
Closed Loop Bandwidth

$$\varpi_H = \varpi_B(1 + \beta A_O) = \frac{\varpi_T}{A_{V,ClosedLoop}\big|_{@DC}}$$

Closed Loop DC Gain

$$A_{V,ClosedLoop} = \frac{A_{OpenLoop}}{1 + \beta\ A_{OpenLoop}}$$

Real Op Amp Frequency Response



$$(Gain\ \ x\ \ Bandwidth)\Big|_{Open\ \ Loop} = (Gain\ \ x\ \ Bandwidth)\Big|_{Closed\ \ Loop}$$

Example: 741 Op Amp is used as a low pass filter with f_L=10kHz. What is the maximum voltage gain possible for this circuit?

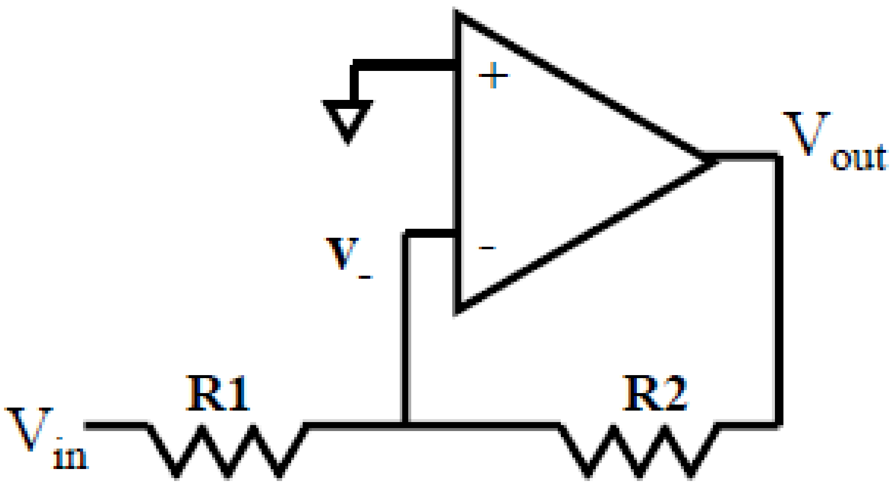
From before, we can write:

$$(200{,}000\ \ x\ \ 5)\Big|_{Open\ \ Loop} = (Gain\ \ x\ 10{,}000)\Big|_{Closed\ \ Loop}$$

$$(Gain)\Big|_{Closed\ \ Loop} = 100\ \ {}^{V}\!/_{V}\ \ Maximum$$

Real Op Amp Frequency Response

For the Inverting Configuration:



By superposition,

$$v_- = v_{out}\frac{R_1}{R_1 + R_2} + v_{in}\frac{R_2}{R_1 + R_2}$$

$$v_- = v_{out}\beta + v_{in}\beta\frac{R_2}{R_1}$$

but,

$$v_{out} = -v_- A_{V,OpenLoop}$$

so,

$$-\frac{v_{out}}{A_{V,OpenLoop}} = v_{out}\beta + v_{in}\beta\frac{R_2}{R_1}$$

$$A_{V,ClosedLoop} = \frac{v_{out}}{v_{in}} = \frac{A_{V,OpenLoop}\beta}{1 + A_{V,OpenLoop}\beta}\left(-\frac{R_2}{R_1}\right)$$

Real Op Amp Frequency Response

Inserting the frequency dependent open loop gain:

$$A_{V,ClosedLoop} = \frac{A_{V,OpenLoop}\beta}{1 + A_{V,OpenLoop}\beta}\left(-\frac{R_2}{R_1}\right)$$

$$A_{V,ClosedLoop} = \frac{\left(\frac{A_O\varpi_B}{s+\varpi_B}\right)\beta}{1+\left(\frac{A_O\varpi_B}{s+\varpi_B}\right)\beta}\left(-\frac{R_2}{R_1}\right) = \frac{A_O\varpi_B\beta}{s+\varpi_B + A_O\varpi_B\beta}\left(-\frac{R_2}{R_1}\right)$$

$$A_{V,ClosedLoop} = \frac{A_O\varpi_B\beta}{s+\varpi_B\left(1+A_O\beta\right)}\left(-\frac{R_2}{R_1}\right) = \frac{\frac{A_O\varpi_B\beta}{\varpi_B\left(1+A_O\beta\right)}}{\frac{s+\varpi_B\left(1+A_O\beta\right)}{\varpi_B\left(1+A_O\beta\right)}}\left(-\frac{R_2}{R_1}\right)$$

$$A_{V,ClosedLoop} = \left(\frac{\frac{A_O\beta}{\left(1+A_O\beta\right)}\left(-\frac{R_2}{R_1}\right)}{\frac{s}{\varpi_B\left(1+A_O\beta\right)}+1}\right)$$

Real Op Amp Frequency Response

$$A_{V,ClosedLoop} = \left(\frac{\frac{A_O \beta}{(1 + A_O \beta)}\left(-\frac{R_2}{R_1}\right)}{\frac{s}{\varpi_B (1 + A_O \beta)} + 1} \right)$$

$$A_{V,ClosedLoop} = \left(\frac{1}{1 + \frac{s}{\varpi_B (1 + A_O \beta)}} \right) A_{V,ClosedLoop}\Big|_{@DC}$$

Closed Loop Bandwidth

$$\varpi_H = \varpi_B (1 + \beta A_O) = \frac{\varpi_T}{A_{V,ClosedLoop}\Big|_{@DC}}$$

Closed Loop DC Gain

$$A_{V,ClosedLoop} = \frac{A_{V,OpenLoop}\beta}{1 + A_{V,OpenLoop}\beta}\left(-\frac{R_2}{R_1}\right)$$

The frequency behavior is the same as for the the Non-Inverting case!

Thanks