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Forest Mensuration and Biostatistics


Department of Forestry and Environmental Science School of Earth and Environmental Science


Uttarakhand Open University Haldwani, Nainital (U.K.)

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## Unit 1: Introduction to Forest Mensuration

## Unit Structure

### 1.0 Learning Objectives

### 1.1. Introduction

1.2. Definition of Forest Mensuration
1.3. Objects
1.3.1. Basis for sale
1.3.2. Basis for management
1.3.3. Measurement for research
1.3.4. Measurement for planning
1.4. Scope of forest mensuration
1.5. Accuracy
1.5. Scales of measurement (nominal, ordinal, interval and ratio scale).
1.6. Units of measurement
1.6.1 Systems of Measurement
1.6.2 Units of Length
1.6.3 Units of Area
1.6.4 Units of Volume
1.6.5 Units of Weight
1.6.6. Conversion factors
1.7. Accuracy Implied in Expression of Measurement

### 1.0 Learning Objectives

After going through this unit you shall be able to:

- Understand about Mensuration.
- Major objectives of mensuration.
- Scope and scales of measurement.
- Accuracy and scales of measurement.


### 1.1. Introduction

## Mensuration in Ancient India

Mathematics, as a discipline, was highly developed in ancient India. Mensuration, which was an essential part of this discipline, is derived from the Latin word 'mensura' meaning
'measure' It is, thus, concerned with the determination of lengths, areas and -volumes. Excavations in the Indus valley have disclosed various 'types of weights current in India in 3000 B.C. The term Pramana, meaning 'measure', was classified into four types, namely, maw (measure of capacity or volume), tula mana (measure of weight), avamana (linear measure) and kala pramana (measure of time). For linear measurements, different parts of human body were regarded as units, viz., angul, balisht and hath. A man's stride known as prakarma was often used for measurement of distances smaller than yojan. In the same way, anjali, meaning two hands joined together as a cup, was used as a measure of capacity. Such measures are used even today in rough measurements. Later on, these simple practical units were further developed. Sulva sutra, portion of Dharmasutras, refers to a variety of linear and area measurements. Kautilya in his Arthshastra describes several kinds of balances, there-by showing the appreciation for the need of accuracy in weighing even in those early times. He has also described gorut ( 12 sq km ) as the area unit for forests. Cubic measures were also used for measuring out _grains. Manu, Yajnavalkya and later Vijnaneswara followed almost the system of Kautilya with slight variations. Aryabhatta I (born in 476 A. D.) after whom the first Indian satellite was named, in his Aryabhatika has allotted a chapter on mensuration called Ganitapada. Yarahamihira refers to mensuration in Brahmasputasiddhanta ( 628 A.D.) of Brahmagupta.

### 1.2. Definition of Forest Mensuration

1.2 Forestry, in the broadest sense, is a management activity' management has to take into account mans desire to use forests and. their produce. Forest Mensuration is that branch of forestry which' deals with the determination of dimensions (e.g" diameter, height, volume, etc.), form, volume, age and increment of single trees, stands-o ce wools, either standing or after "felling. It includes measurements of felled and standing trees, sawn wood and round logs and various other products referred to as minor forest products such as fuel, charcoal, bark, fruits, etc. Forest mensuration, therefore, concerns with linear, area, volume and weight measurements. While volume is the most important measurement from the point of view of forest management, weight, which is also referred to as biomass, is being increasingly used now in place of volume.

### 1.3. Objects

Forest mensuration serves the following objects:

### 1.3.1. Basis for sale

Forest mensuration forms the basis of every transaction involving sale of forests or their products. In olden days when supplies were ample, and the value of timber or any other forest produce were low, measurements may not have been necessary but with the increasing gap between demand and supply and the rising prices of timber or other products, it has assumed great importance. Before an sale, the forest department prepares estimates of the quantity o timber or other produce contained in various sale lots and the value which each 44 is likely to fetch. Similarly, the contractors or other purchasing agencies estimate the material they would receive from the lots and the maximum price they could pay to the forest department to earn a reasonable profit after meeting working costs. This has resulted in developments of methods of measurements and calculations which give reasonably accurate estimates of timber and other forest produce in the lots and their prices.

### 1.3.2. Basis for management

With proper management, forests, can meet the demand of timber and other forest produce in perpetuity. For this purpose, a knowledge of the quantity of timber stand:: ing in forest and the increment it is putting on every year or in a period of years is necessary. This led .to the development of methods of preparing forest inventories using satellite imageries, aerial photographs and various optical and electronic techniques so that forests could be harvested without depleting the capital.

### 1.3.3. Measurement for research

In order to find out which treatment and method of management would maximize production, it is necessary to layout experiments to compare the results of different treatments. This required the laying out of sample plots and their periodic measurements.

### 1.3.4. Measurement for planning

The forester is concerned not only with the demands on forests at present but also with those of the future. With the demands on forests at present but also with those of the future. With the steeply increasing population and rising standards of living every year, the gap between the demand and supplies of timber and other forest products is widening fast and therefore,- the forest manager has .plan to reduce this gap For this purpose, fast-growing species have to be introduced. This requires not only reasonably accurate estimates of future yields but also the cost of production so that such projects may be financially found. Thus, forest mensuration supplies basic statistical data to plan for the future and serves as a yardstick to check the viability of projects.

### 1.4. Scope of forest mensuration

Forest mensuration has a very wide scope. It concerns not only the foresters, contractors, sawyers and transport labour working in the forest but also the sawmillers and wood using public in the cities, towns and villages. It comes into play every time the wood is sold, converted or used. While it deals with the measurement of cubical contents of trees and crops at present, it also helps the forecast of yields at any time in their future life. It applies not only to standing trees and crops but also to felled timber and its subsequent conversion till it is used in some work.

With the widening of horizon of forestry, the scope of forest mensuration has also widened. The present day forest mensurationist has to concern himself with the measurement problems of wild life management, watershed management, insect and disease incidence, recreation, tourism and, in fact, many of the mensurational aspects of multiple-use forestry. Traditional measurement problems, however, remain a major responsibility. In short, forest mensuration should be regarded as the branch of forestry which provides foundations of measurement principles applicable to any forest measurement problem. The application of statistical theory and the use of electronic computer for data processing have brought about revolutionary changes in forest measurement problems. Forest mensuration should
make full use of these tools but its principles must be based on sound biological knowledge.

### 1.5. Accuracy

Even though mensuration is a branch of mathematics, forest mensuration does not attempt to secure absolute mathematical accuracy. For the purpose of sale, a very high degree of accuracy in estimation of volume of timber in a lot, is neither necessary nor desirable. Though insistence of high degree of accuracy in forest mensuration results in reduction of waste and exercises a control on the methods of working of forests, there is a limit beyond which economic gains, which may accrue from the adoption of elaborate methods, are incommensurate with the time and money spent on them. Thus, forest mensuration aims at reasonable or relative occuracy, i.e maximum accuracy which is profitable and possible to obtain in practice. The following are some of the reasons which compel foresters to be content with relative accuracy:
(a) Characteristics of trees: The e shapes of trees differ very greatly from one another. Some trees have short stems and big crowns while others have long stems and small crowns. They also differ widely in lengths and sizes of stems and their forms. The cross sections of stems of trees do not correspond strictly to any geometrical form. The characteristic of having cross-sections of stems deviating from circular shape without conforming to any geometrical form introduces error in calculation of cross sectional areas as they are assumed to be of some definite geometric shape, The variation in the degree of eccentricity (rum tree to tree further complicates the problem. The swelling near the base of the trees, !billings, buttresses and thickness of hark introduce so many variables that no single formula can correctly estimate the quantity of timber available from a standing tree.
(b) Varying methods and conditions of felling and conversion - A standing tree has to be felled and converted to obtain the timber it contains. The relation which the volume of timber in a standing tree bears to the converted material depends not only on the skill and care with which felling and conversion arc done but also on local conditions. When felling and conversion arc done unskillfully involving a lot of waste as happens in the rough
squaring of timber in the hills and when local conditions are such that conversion and export of all material are not economic, it is no use aiming at a high degree of accuracy in measurements.

## (c) Instruments and conditions in which they are used

The instruments used for making measurements of trees and the conditions in which they are used impose a physical limit to the degree of accuracy that can be attained. For instance, Abney's level with it vernier gives only approximate angles of elevation and depression not only because of the natural obstructions in sighting the top and base of the tree respectively but also because the instrument is likely to shake while being used without a stand. Measurement of distance from the observer to the tree further contributes to the inaccuracy and thus the measurement of height of a tree is, at best, an approximation which is subject to errors.

## (d) Personal bias of the estimator

Predictions of volume of trees and crops in future have to be based on the growth of trees and crops supposed to be growing in similar conditions or have to be projected for future assuming that the conditions of growth in the past will continue to remain the same in the future as well. In both these cases, personal bias of the estimator influences the judgment considerably and consequently the estimate.

## (e) Biological character of the forest

The forest is a living entity and its growth is a function of several factors acting simultaneously. The result of these complex factors cannot be predicted with mathematical accuracy. Trees growing in a crop are affected not only by climatic, edaphic and topographic conditions but also by biotic factors and therefore growth functions of trees can, at best, be rough estimates.

## (f) The use to which the measurements are to be put

The accuracy to be aimed at, also depends upon the use to which the data are to be put. When the value of the produce is low, as in the case of fuel or miscellaneous forests, it would be a sheer waste of time and money to measure the dimensions with the same meticulous rare and accuracy as in the case of valuable timber forests. Even in the
valuable species, the same degree of accuracy is not necessary in case of immature forests as in the case of areas with mature crops nearing harvest time. Similarly, the routine forest measurements do not deserve the same degree of accuracy with which measurements in a sample plot are carried out for statistical or research work.

## (g) Cost

Cost is another important factor that governs the accuracy to be aimed at. In estimating volume of a stand, detailed measurements of each tree would make the cost prohibitive and as standing timber is not the finished product, its volume and value are affected by the skill and care of the crew employed in felling and conversion. It is, therefore, not desirable to spend too much money and time on accurate measurement of volume of standing trees. The degree of accuracy will depend on the objectives of the estimate.

### 1.5. Scales of measurement (nominal, ordinal, interval and ratio scale).

Different scales of measurement may be used for measuring tree and stand characteristics.

- The nominal scale, used for attributes, represents the weakest scale of measurement. The observation is assigned to one out of $k$ discrete categories. Species, provenance, forest type and soil type, for example, are discrete variables which cannot be arranged in a certain order.
- The next-strongest ordinal scale is a ranking scale characterized by ordered categories and is used for ranked variables (discrete categorical variables) the scale is characterized by classes of different but unknown width. Forest soils, for example, could be categorized as poor, medium or good, the vital ity of trees as healthy, sick, dying or dead, social tree classes as dominant, co-dominant, dominated and suppressed.
- Almost all forest mensurational characteristics, such as diameter, height, basal area, volume and increments, are continuous variables, measured on a metric scale.
- The metric scale is sometimes subdivided into an interval scale, without a natural zeropoint and a ratio scale, which assumes the existence of a natural zero-point. Temperature represents the classical example of a variable which Graphical Presentation of Data 7 is
measured on an interval scale when expressed in degrees centigrade or Fahrenheit and on a ratio scale when measured in degrees Kelvin.
- In many cases, the ranking scale is converted into a metric scale by assigning numerical values to the class midpoints. Forest soils, for example, might be measured on the basis of soil depth or moisture-storage capacity or by some linear combination of these variables, with different weights being assigned to each of them.


### 1.6. Units of measurement

A prerequisite of any measurement is the selection of a unit of measure. Fixation of the unit of length is, therefore, an essential preliminary to any measurement in forest mensuration because units of area and volume can be derived from it.

### 1.6.1 Systems of Measurement

The following are the two systems of measurement, which" are commonly used.
(a)'British system or F.P.S. System ; and
(b) French or metric system or C.G.S. system.

Prior to the passage of T he Standards of Weights and Measures Act, 1956, British system or F.P.S. system was used in forestry in India. Metric system or C.G.S. system which was introduced in October 1962, is now a legal necessity. Therefore, it has been decided to describe both systems of measurements here and use measurement in metric system in the text with the British system measurement in brackets when reference is to any old table in that system.

### 1.6.2 Units of Length

In the F.P.S. or British system, foot is the unit of length and it is related to other measurements of this -system as follows :

| 12 inches | 1 foot |
| :--- | :--- |
| 3 feet | 1 Yard |
| 66 feet or 22 yards | 1 chain=100 links |
| 10 chains or 220 yards | 1 furlong |
| 8 furlongs or 1760 yards | 1 mile |

In the C.G.S. or metric system, metre is the unit of length and it is related to other measurements of this system as follows :

| 10 millimetres $(\mathrm{mm})$ | 1 centimetre $(\mathrm{cm})$ |
| :--- | :--- |
| 10 centimetres | I decimetre |
| 10 decimetres | I metre $(\mathrm{m})$ |
| 10 metres | I decametre |
| 10 decametres | 1 hectometre |
| 10 hectometres | 1 kilometre $(\mathrm{km})$ |
| 1852 metres | 1 nautical mile (International) |

In forestry, linear measurements are taken in the following cases and the units adopted in both the systems are given below :

| Particulars of linear | In British System | In Metric System |
| :--- | :--- | :--- |
| (i) (a) Total height of a <br> tree | To the nearest whole foot <br> (half or more of a foot to be <br> treated as one) | In metres correct to the <br> first place of decimal |
| (b) Height of first green <br> branch, green branches <br> all round and crown width |  |  |
| (ii) Length of sections of <br> hole for stem analysis and <br> simple plot work. | 10 feet, the odd feet 1 being <br> included in the last section <br> which does not exceed 15 | metres, the last section <br> not exceeding 45 metres |
| (iii) Diameter (of trees or <br> sections) | In inches to the nearest <br> tenth of an inch | In centimetres and to the <br> nearest 1/5 of a <br> centimetre (i.e., to the |
| (iv) Girth | In feet and to the nearest <br> inch | In metres and to the <br> nearest centimetre |
| (v) Height of the stump | To the nearest inch | To the nearest centimetre |
| (vi) Limit of standard stem <br> timber | Down to 8 inches diameter <br> over bark | Down to 20 centimetres <br> diameter over bark |
| (vii) Limits for standard <br> small wood in bole and <br> branches | From under 8 inches <br> diameter over bark to 22 <br> inches diameter over bark | From under <br> centimetres dia-meter <br> over bark to 5 centimetres |
| (viii) Height of seedlings | To the nearest inch | To the nearest centimetre |

### 1.6.3 Units of Area

In the British system, square foot is the unit of area and its relationship with other measurements is as follows:

| 144 Square inches (sq in) | 1 Square foot |
| :--- | :--- |
| 9 Square feet (sq ft) | 1 Square yard |
| 484 Square yards (sq yds) | 1 Square chain |
| 10 Square chains or 4840 sq yds | 1 acre |
| 640 acres | 1 Square mile |

In the metric system, square centimetre is the unit of area and the relationships with other measurements is as follows :

| 100 Square millimetres (sq mm) | 1 Square centimetre |
| :--- | :--- |
| 100 Square centimetres (sq cm)- | 1 Square decimetre |
| 100 Square decimetres | 1 Square metre |
| 100 Square metres (sq m or m2) | 1 are or 1 square decametre |
| 100 arcs or square decametres | 1 hectare or 1 sq hectometre |
| 100 hectares (ha) or square hectometres | 1 square kilometre $\left(\mathrm{km}^{2}\right)$ |

In forest mensuration for cross sectional areas or basal areas of trees square metre (m2) and for land area hectare (ha) are used as basic units.

### 1.6.4 Units of Volume

In the British system, cubic foot is the unit of volume and its relation with other measurements is given below:

| 1728 cubic inches (cu in) | 1 cubic foot |
| :--- | :--- |
| 27 cubic feet (cubic ft or cft) | 1 cubic yard |

In the metric system, cubic metre is the unit of volume and it-is related with other measurements as follows:

| 1000 cubic millimetres $(\mathrm{cu} \mathrm{mm})$ | 1 cubic centimetre |
| :--- | :--- |
| 1000 cubic centimetres $(\mathrm{cu} \mathrm{cm})$ | 1 cubic decimetre |
| 1000 cubic decimetres | 1 cubic metre |

In the forest mensuration, cubic metre (m3) is used as the basic unit for tree volumes but as it is a fairly large unit it is not as popular in trade as the cubic foot (ct).

### 1.6.5 Units of Weight

Certain kinds of forest products, e g., fuel, charcoal and minor forest products are generally recorded by weight. Even otherwise, as already stated earlier, weight or biomass is being increasingly used now in place of volume. Therefore, it is necessary to know the basic units of weights ordinarily used. In weights, in addition to the British and metric system, there is also an Indian system. The relationship between different measures of weights in different systems is as follows:

| British system | Metric system | Indian system |
| :--- | :--- | :--- |
| 16 ounces= 1 pound | 10 grams= 1 decigram | 80 tolas=1 seer |
| 28 pounds= 1 quarter | 10 decagrams =1 hectogram | 40 seers=1 maund |
| 4 quarters= 1 hundred weight <br> (cwt) | 10 hectograms=1 kilogram (kg) |  |
| 20 hundred weights =1 ton | 100 kilograms=1 quintal <br> 1000 kilograms <br> 10 quintals---- 1 metric tonne |  |

### 1.6.6. Conversion factors

Conversion factors of some commonly used units are given below:

| F.P.S. to C.G.S. | C.G.S. to F.P.S. |
| :---: | :---: |
| Length <br> 1 inch $=25.4 \mathrm{~mm}$ <br> 1 foot $=30.48 \mathrm{~cm}$ <br> 1 yard $=0.9144 \mathrm{~m}$ <br> 1 mile $=1.609344 \mathrm{~km}$ | 1 centimetre.- 0.393701 inch <br> 1 metre - 1.09361 yards <br> 1 kilometre- 0.62137 mile |
| Area <br> $1 \mathrm{sq} \mathrm{in}=6.4516 \mathrm{sq} \mathrm{cm}$ <br> $1 \mathrm{sq} \mathrm{ft}-9.2903 \mathrm{sq}$ decimeters <br> 1 sq yd -0.83613 sq m <br> 1 acre $=0.40468$ hectares <br> 1 sq mile $=2.58999 \mathrm{sq} \mathrm{km}$ <br> 1 sq ft per acre $-0.230 \mathrm{sq} \mathrm{m} / \mathrm{ha}$ | 1 sq metre= 1.19599 sq yds or 10.7639 sq ft <br> 1 hectare $=2.47105$ acres <br> 1 sq km - 0.386101 sq mile <br> $1 \mathrm{sq} \mathrm{m} / \mathrm{ha}-4.356 \mathrm{sq} \mathrm{ft}$ per acre |
| Volume <br> 1 cu inch $=16.3872 \mathrm{cu} \mathrm{cm}$ I cft - 0.0283 cum | $1 \mathrm{cu} \mathrm{cm}-0.0610 \mathrm{cu}$ in 1 cubic metre- 35.3147 cu ft |


| 1 cft per acre-0.070 cu m$/ \mathrm{ha}$ | 1 cu m per ha-14.291 ct per acre |
| :--- | :--- |
| Weight | 1 kilogram $=2.20462$ pounds |
| 1 pound $=0.4535924 \mathrm{~kg}$ | 1 metric tonne $=0.98420$ ton |
| 1 ton $=1.01605$ metric tonnes |  |

### 1.7. Accuracy Implied in Expression of Measurement

When a pole is said to be 6 meter long, it is not implied that it is precisely of that length. It merely means that it is nearer 'to 6 m than it is to 5 m or 7 m . In other words, unless otherwise 'stated, it is implied that there is inexactness of upto 0.5 m in the measurement. As a general rule, it may be said that the implied standard of accuracy is one half of the unit of measurement. If, therefore, greater accuracy is desired, the unit of measurement should be centimetre rather than metre. If a pole is said to be 6.25 m long, it implies that it is nearer to 6.25 m than it is to 6.24 m or 6.26 m . Thus, in this measurement, error is not more than 0.005 m or 0.5 cm .

The adoption of a unit of measurement that implies a higher degree of accuracy, however, gives no assurance that the measurements are accurate. If accuracy in measurements is to be aimed at, they will have to be done with maximum care and with instruments which are accurate enough.

## Unit 2: Tree Measurement: I

## Unit Structure

### 2.0 Learning Objectives

### 2.1. Introduction

2.2. Standard rules governing measurement at breast height
2.3 Measurements of tree diameter and girth using rulers, callipers and tapes
2.4. Comparison between tape and calliper measurements
2. 5. Finish Parabolic Calliper, Relaskop, Pentaprism.
2.5.1: Calliper
2.5.2. RELASCOPES AND PRISMS
2.5.3. Prisms

### 2.0 Learning Objectives

After going through this unit you shall be able to:

- Standard rules for DBH measurement
- Measurement of diameter through Tape, Calliper and rulers
- Instruments ude for diameter and girth measurement


### 2.1. Introduction

Age, diameter, basal area, total and merchantable height, total and merchantable volume, stem form, bark thickness and growth are important single-tree characteristics, which generate information aboutthe growing stock of stands. They may include other tree characteristics, such as species, crown form, branchiness and damagecaused by insects and pathogens.

### 2.2. Standard rules governing measurement at breast height

The standard position for diameter measurements at standing trees is at breast height, defined at the height of 1.30 m in most countries. But there are many countries where diameter at breast height is measured at different heights. Therefore, it is always good to find out where dbh is measured before engaging in data analysis. In the United States, for
example, dbh is taken at 4.5 feet above ground which is about 1.37 m , and in Korea at 1.20 m . The reason why breast height developed to be a standard measure has probably to do with the ease and convenience of measurement.

However, irregularities and anomalies of tree stems do sometimes prevent the measurement of a diameter at the defined height. For these cases, rules need to be defined how to proceed. This is usually done with graphs describing these specific situations and how to proceed then with the dbh measurements. It is important that all field teams who carry out the measurements follow the same definitions and instructions. In the following, some typical situations are listed and how the dbh measurements are then usually taken:

- Tree on slope: If a tree is in a sloped terrain, the dbh is measured at the standard height above the forest floor on the uphill side of the tree.
- While measuring the breast height from the ground level, we have to pay attention that the level from which is measured is in fact the ground and not the surface of a heap of


Fig 1. Standard rules for DBH measurement woody debris.

- If the tree is leaning, breast height is measured parallel to the lean on the top side of the tree, whereas diameter is measured perpendicular to the longitudinal axis of the trunk.
- In case of buttressed tree, if the height of the buttress is more than 1 m , then the breast height is measured from the point where buttress ends. In this case, reduction
models are sometimes be applied with which the would-be diameter at breast height is estimated from the measurement of a diameter at another height.
- When there is an abnormality at breast height such as crooks, swelling or knots, the dbh is measured above or below the abnormality. Some times, the rule is defined that diameter measurements are then done at equal distances above and below breast height and dbh then estimated by simply taking the mean of the two measurements.
- If a tree bifurcates above breast height, then one measures dbh as usual. If the bifurcation is below breast height, each stem is measured and counted separately.



### 2.3 Measurements of tree diameter and girth using rulers, callipers

 and tapes
## UPPER STEM DIAMETER MEASUREMENT

Diameters of trees at points above the breast-height are often required for precise estimation of volume of a standing tree or for preparation of taper tables. For smaller height, a ladder can be used by the observer. As carrying a ladder is troublesome in forest particularly when the terrain may be hilly, a ruler has been devised to measure diameters up-to 5 or 6 m heights.

The ruler is a simple board of $150 \mathrm{c}-\times 10 \mathrm{~cm} \times 1 \mathrm{~cm}$ with a one meter rod attached to the middle. On 'his rod a detachable handle 2 m long can be fitted. The board is painted white with diameters marked on it with black paint. The diameter marks are not exact linear measurement but slightly corrected to avoid parallax errors. The ruler is usually graduated to read diameters from a horizontal distance of 10 m . In order to use the instrument the observer upper stem diameter stands at a horizontal distance of 10 m from the side of the tree. A helper places the ruler .against the tree at the height of measurement, keeping it perpendicular to the line of sight and the left edge of the ruler in line with the left edge of the stem in relation to observer. The diameter can taken be read on the right hand part of the ruler as shown in Fig. 2.


Fig. 2 Ruler for measuring upper stem diameter

For still greater heights upto 8 m (or even upto 12111 if the observer uses binoculars for reading), Calliper mounted on poles or the help of telescopic ladders can be used. In order to use callipers, the handles of the two arms of calliper are held in two hands.

The movable arm is then tilted inwards so that it can move freely and moved in that position -so that the two arms are separated enough to receive the tree between them without touching. When the tree touches the graduated rule, the movable arm is shifted inwards in the tilted position so that the itree touches the fixed arm and movable arm. In this position the (movable arm is then slowly brought in perpendicular position to the :graduated scale and pressed so as to squeeze out any loose bark as well as ensure that there is no gap between the arms and the tree. The diameter is then read off on the rule.

One such calliper is the finnish parabolic calliper, mounted on telescopic graduated poles. One of the arms of this calliper is straight and the other parabolic. It does not have movable arm like ordinary calliper. The parabolic arm is graduated. In order to measure the diameter of a tree at a particular height, the telescopic graduated pole is adjusted to that height and the calliper held perpendicular to the tree. Then a sight, Parallel to the right arm of the calliper, taken on the parabolic arm, gives the diameter of the tree. For diameter measurements at still greater heights, dendrometers have to be used. They are optical instruments with which diameter measurements can be taken at any point up the stern beyond the reach of foresters equipped with callipers or diameter tapes.

The tree measuring tapes arc generally 3 m long or at the most 5 m long. Butland measuring tapes may be 5 m , $10 \mathrm{~m}, 20 \mathrm{~m}, 30 \mathrm{~m}$ or even upto 50 m long. In British measures, these tapes are usually $25 \mathrm{ft}, 50$ and J 00 ft long.


Fig. 3 Finnish parabolic calliper Such long tapes are generally cased in some leather or plastic case with some winding arrangement so that they are secure inside the case when not in use. The tree measuring tapes, which are usually small, do not
generally have cases to cover them, though some steel tapes 2 or 3 m long are kept in cases with some spring device to wind them back inside when not in use.

Though tapes are meant for girth measurement, some of them are so graduated as to read diameters instead of girths. This is based on the relationship of diameter and girths. As girth is equal to $\pi d$, each centimeter mark is shown at an interval of 31416 centimeters and such graduations then, read diameters in centimeters directly. Tapes graduated to read diameters are called diameter tapes. Sometimes tree measuring tapes are graduated to read diameters on one side and girths on .the other side.

### 2.4. Comparison between tape and calliper measurements

Tape and calipers are both used to measure girths and diameters of trees and logs in order to calculate their basal areas and from them the volume. It is therefore necessary to see which (If the, two instruments gives, better results in respect of the two items. This can be done by com-paring the errors involved in the measurements by the two instruments, as a result of different sources of error of each of them al shown below:

1. While using the tape and calliper for measuring girths and diameters of logs and trees, the cross-section of the tree is assumed to be circular while it is not so. Therefore by assuming the cross,- section of the tree to be a circle, the basal area is exaggerated because circle is a figure which for a given perimeter or girth encloses the largest area. Since this assumption is made in case of the both instruments. The error in basal area calculated from taped girth is slightly more than that calculated from callipered diameters when calculated by the formula: $\pi\left(\frac{a+b}{2}\right)^{2}$
2. 2. Though the basal area calculated from the callipered diameters by the formula $\pi\left(\frac{a+b}{2}\right)^{2}$ attains the closest approach, to the real basal area, the actual measurement of dimensions $a$ and $b$ by means of a calliper is attended by serious difficulties because of the following reasons
(a) Orientation of the major axis-It is difficult to locate the major axis in the tree and so the diameter is usually under-estimated. Consequently the error in basal areas is of negative character.
(b) Deviation of the angle between two diameters from a right angle- Even if the major axis is correctly oriented, wrong orientation of the minor axis which should be at right angles to the former, results in error in diameter measurement and consequently in the basal area.
(c) Sliding arm is not at right angles to the scale arm-If the sliding arm is not perpendicular to the scale arm, the recorded diameter and consequently the area calculated from it, is inaccurate. Even a slight deviation of $2^{\circ}$ from the right angle position results in an error of about $3.5 \%$ in the basal area.
(d) Scale of the calliper not touching the tree-This is also an important source of error in diameter measurements by calliper.
(e) Devaition of the plane of the calliper from the right angle of the axis of the tree at the time of measurement-This is also a common error in calliper measurement. A deviation of $10^{\circ}$ involves an error of $1.54 \%$ in diameter measurement and about $3.1 \%$ in basal area calculated from it.

The number of sources of error in girth measurements by tape are comparatively much less. The errors in girth measurement in tape are usually due to the following reasons:
(a) Displacement of the tape from the horizontal-Even -though this results in error in girth measurement and consequently in basal area, the percentage of error involved is half of that involved in case of callipered diameter.
(b) Elasticity of tape-The error from this source is negligible.
(c) Expansion of tape-This error is also negligible.

This comparison clearly indicates that the use of calliper is more susceptible to errors than the use of tape and the errors in case of the former are also greater in magnitude.
(3) As the girth is approximately three times the diameter, for a given degree of accuracy in basal area, the permissible absolute error in the measurement of girth will be three times the absolute error in the measurement of diameter. Thus an error of 1 cm in girth will cause the same error in basal area as an error of one third of a centimeter in diameter. Since it is easier to commit an error of one third of a centimeter than to commit an error of one centimeter. Tapes are expected to give more accurate results.
(4) Measurements by tape ensure more consistent results in those by calliper for successive volume determination in a sample plot. The calliper measurements are unreliable because it is difficult to orient their arms. Tong major and minor axes of a tree and secure an angle of 90 " between them. Cross mark eliminates the error due to orientation of one axis but does not provide against the error due to a variation in angle between the two axes from a right angle. Even the same man checking his own diameter measurements on the same day may get different result every time. Personal factors like fatigue, lack of enthusiasm also contribute towards making calliper measurements unreliable.
(5) The probability of the various errors in calliper measure-ments cancelling out one another is often advanced as an argument for the superiority of calliper over tape. As a matter of fact, it is easier to standardize the errors of tape which are always in positive direction than the errors of calliper, which are unsystematic and subject to personal factors.
(6) The tape measurements, apart from being subject to errors due to fewer causes than calliper measurements, have the advantage that they are simpler in practice and do not require the calculation of the mean of the two diameters measured at right angles to each other for each tree.
(7) In the case of a felled tree, it is easier to use a tape than to use a calliper.

It will thus be seen that the tape and calliper have their own advantages and disadvantages. Calliper gives more accurate results but its measurements are more susceptible to errors than those of tape. Tape gives more consistent results but is timeconsuming in ordinary forest practice. So the choice between the two instruments will depend upon the kind and circumstances of work and the prejudices of the men using them.

## 2. 5. Finish Parabolic Calliper, Relaskop, Pentaprism.

### 2.5.1: Calliper

The caliper consists of a fixed arm mounted perpendicularly to a graduated beam and a movable arm, parallel to the former and sliding along the fixed beam (Fig. 1). The
caliper is used to measure stem diameters on felled trees and the over bark breast height diameter of standing trees. In order to minimize instrument errors, a rigid construction of the caliper is imperative. The early wooden calipers were subject to wear and tear, steel calipers are rigid and reliable but heavy and uncomfortable during cold weather. Aluminum calipers have increased in popularity, but they should be regularly checked for their accuracy and, if necessary, calibrated at least once annually. In general, calipers have to meet thefollowing quality specifications:

1. The graduated beam must be perfectly straight, of sufficient length for measuring large-dimension and sturdy trees. To eliminate recording errors the graduations should be clearly visible.
2. The movable and fixed arm should run exactly parallel, two arms of the caliper should be located on a plain. If the movable arm is not at a right angle to the fixed beam (Figure $a-B$ ), the resultant systematic positive error is dependent upon the angle of deviation ( $\alpha$ ) and the diameter of the tree.
3. The point of measurement is consistently incorrectly positioned. The resultant operator bias is two-sided. In research plots, the error may be negligible by permanently marking the breast height position.
4. The graduated beam is not held at an angle of $90^{\circ}$ to the stem.


Fig 1 A. Caliper


Fig 1 B. Incorrect angle of the movable arm

For an angle of deviation of a degrees, the observed diameter is approximately equal to $d(1-\tan (\alpha) / 2)$ and the percentage erroris equal to $50(\tan (\alpha)$. The corresponding error in the estimated basal
area isapproximately twice as high.
5. The operator sometimes tends to exert too much pressure during mea suring, in which case a systematic, negative operator-bias is introduced. 6. On slopes, a positive bias is likely tooccur by not consistently measuring from the uphill position.
7. On terrain with a heavy and tough ground vegetation of grasses and other plants, the point of measurement is usually located above its true position, in which case a negative bias is
introduced. A similar situation may arise during winter, if the ground is covered by snow. In Continuous Forest Inventories, the measuring point is usually permanently marked, partly to establish whether or not a particular tree was measured at the previous occasion. It has the additional advantage of reducing random or systematic errors associated with inaccurately positioning the tape or caliper.
When using calipers for re-measuring permanent sample plots, they should be checked regularly for the occurrence of instrument-related errors. In forest inventories, they are to be calibrated annually. Calipers used to measure research plots require a 1 mm graduation. Those witha 1,2 or 4 cm graduation are adequate and more practicable for management, regional, and national forest inventories. The diameters are marked on the graduated beam, and show the midpoint of the diameter class. When using selfrounding calipers, these midpoints arealways printed at the lowest point of the diameter class.

### 2.5.2. RELASCOPES AND PRISMS

All instruments in this group have in common that the angle subtended between the sampling point and thestem at breast height is evaluated.

## Angle gauges

The early angle gauges consisted of a 50 cm or 1 m long hand-held stick, with a metal blade 1 cm wide for the 50 cm stick and 2 cm for the 1 m stick being mounted on one side. The trees surrounding the sampling point were sighted at breast height in a $360^{\circ}$ sweep. The tree is counted if it subtends an angle which exceeds the critical angle of
the instrument.

## Kramer's dendrometer

The multipurpose instrument incorporates the basic principle of the measuring blade, which is 1 cm wide (Figure 2) and generates such an angle that each tree counted corresponds with $1 \mathrm{~m}^{2}$ basal area per hectare, i.e., it represents a basal area factor (BAF) of 1. When using a width of either 2 or 4 cm ("ор" or "mn" in Figure 3-6), the corresponding BAF are 2 and 4, respectively. The dendrometer is held vertically at a distance of 50 cm to determine the basal area per hectare. The right edge is equipped with a scale for

Fig. 2. Kramer's dendrometer.
 measuring heights, which is similar to the Vorkampff-Laue hypsometer. The observer seeks a position where the top and base of the scale ( $k$ and $h$ in Figure 2) exactly covers thetree and measures the height of the point on the stem which superimposes the mark $i$ on the instrument. Tree height can be calculated by multiplying the distance from $i$ on the tree to tree base by 10. The scale on the left edge indicates the position on the stem, which corresponds with one-fourth of the stem volume. The printed table is based on form heights and used to estimate stand volumes.

## Bitterlich's mirror relascope

The mirror relascope (Figure 3-7) is a small hand-held instrument, which can be used for a variety of purposes:

- Estimation of the basal area per hectare
- Optical distance measurements, adjusted for slope
- Measurement of tree height either for distances of $15,20,25$, and 30 m or for arbitrary distances
- Measurement of upper-stem diameter, from fixed distances • Combined height and diameter measurements
- Estimation of relative form heights, to determine absolute form heights, fac tors, and the volume of the standing tree
- Measurement of slopes

Estimation of Hirata's stand mean height, based on vertical point sampling The instrument is based on the principle of a drum pendulum, which is released when measurements are made. The telescope is equipped with a peephole to be used for viewing the object of measurement and lateral windows to admit light. The instrument is usually held with the right hand, and the left hand may be used to give the instrument extra support and the middle fingers to press the button. Alternatively, the mirror relascope can be mounted on a tripod or monopod, in order to reduce erratic movements of the instrument during view ing, although this restricts the freedom of movement of the operator. The drum pendulum is equipped with a number of measuring bands, which are mounted on roller bearings to ensure that the pendulum is in a vertical position during the measurements. The pendulum wheel is provided with a brake to dampen the movements of the pendulum. A built-in lens projects the magnified mea suring bands onto a mirror. The image is visible in the lower half of the field of vision, with a horizontal line separating the lower from the upper half of the field of vision, which is used to view the object. The width of the measuring bands is adjusted for slope. The adjustment factor is equal to the cosine of the angle of slope. This property is used when measuring upper-stem diameters and when evaluating stems at breast height on sloping terrain. The lower half field of


Handling


Lowerhalf of the field of vision vision, which reveals a number of white and black bands, is shown in Figure 3-7.***

The "count" bands 1 and 2, which correspond with Zb1 and Zb2 in Figure 3-6, are used to estimate the basal area per hectare for the BAFs 1 and 2 in the metric system. Adding the two white and black bands on the right of Zb 1 gives band 4 (Zb4 in Figure 3-6) to estimate the BAF of 4. The distance bands Ds15, Ds20, Ds25, and DS30 are required for optical distance measurements with the aid of a 2 m vertical staff and correspond with horizontal distances of $15,20,25$, and 30 m from the object. The tangent scale $T \mathrm{~s}$ is used for height measurements, for combined diameter and height measurements and to deter mine Hirata's stand height. They are located to the left of Zb 1 , ( 20 m scale), and between Zb 2 and Zb 4 ( 25 m and 30 m scale).

Estimating the basal area per hectare at breast height
In a $360^{\circ}$ sweep, the number of trees is counted with an apparent diameter, which exceeds one of the selected "count" bands. The number of trees counted gives the estimated basal area in square meter per hectare if band 1 is used, multiplied by 2 and by 4 when using band 2 and 4 , respectively.

Distance measurements with the aid of a horizontal staff Band $\mathrm{Zb4}$ is to be used for measuring horizontal distances. A staff of a fixed length, for example, 80 cm is held against the tree, in a horizontal position. The operator locates the point where band Zb4 exactly covers the 80 cm staff. The distance is found as the product $0.80 * 25$
$=20 \mathrm{~m}$.

## Distance measurements with the aid of vertical staff



Fig. 3-8. Distance measurement with the aid of a vertical staff.
exact distance, the latter is estimated ocularly, ignoring slope. Releasing the pendulum, the relascope is pointed at the halfway point of the horizontal staff and arrested in this position, in order to adjust for slope. The relascope is subsequently rotated counterclockwise at an angle of $90^{\circ}$. The operator moves forward and backward in order to ensure that the lower terminal point of the vertical staff coincides with the lower edge of band 2 and the upper terminal point of the vertical staff coincides with the appropriate distance band.

Measuring tree height
The tangent scales are used in combination with the distance bands. The tan gent scales are provided for each of the horizontal distances 20, 25, and 30 m . The 30 m scale can be used for the measuring distance 15 m by multiplying the recorded tree heights by 0.5 . The height scale for 20 m is found at the far left of the instrument (Ts20), whereas Ts25 and Ts30 are located between ZB4 and ZB2.

## Estimating an upper-stem diameter

Band Zb 1 and the adjoining Zb 4 are used to measure an upper diameter. Band Zb 1 and Zb4 correspond with a ratio object width: horizontal distance of 1:50 and 1:200, respectively. The relascope unit is defined as the band width Zb 4 , so that count-band Zb1 contains 4 and ZB4 contains 8 relascope units. For a horizontal distance of 10 m , one relascope unit corresponds with a width of 5 cm of the object. When measuring upper diameters from a fixed horizontal distance, the diameter is measured in relascope units and then converted to obtain the estimated diameter. For example,


Figure 3-9. Measuring an upper-stem diameter.
(Horizontal distance $=10 \mathrm{~m}$, Relaskop units $(\mathrm{Ru})=12.4$, Diameter $=5 \mathrm{~cm} \cdot 13.5$
when 6.3 relascope units are measured at a distance of 10 m , the estimated upper diameter is 31.5 cm (see Figure 3-8).

## Wide-scale mirror relascope

At a later stage, Bitterlich constructed the wide-scale mirror relascope for measuring upper diameters of large trees and to apply the relascope technique to estimate the basal area per hectare for large BAF (see Figure 3-9). The instrument is equipped with slope scales for degrees $(G)$ and percentage $(P)$, respectively, with four narrow black-white bands to measure upper diameters and to estimate the basal area per hectare for low basal-area factors, with a white band 1 (Zb1), which corresponds with BAF $=1$ and distance factor 50 as well as with five black and five white bands, which correspond with BAF =

The zero mark is located at the right edge of the Zb 1 band. Those units, which are completely covered by the tree to the left and right, are counted and converted into basal area per hectare by using the appropriate conversion factors for those at the left and right of the zero mark, respectively. The instrument is also suitable for measuring distances and upper-stem diameters.

## Bitterlich's telerelascope

The telerelascope represents a vastly improved version of Bitterlich's mirror relascope, primarily to estimate upper-stem diameters and their corresponding heights above the base of the tree, from arbitrary sighting distances. The fol lowing steps are required:

The instrument is mounted on a tripod, either with a movie head adapter or with a micrometer head with a fixed avallactical point, i.e., with a fixed sighting-angle vertex.

The left edge of the tree and that of a white band, which corresponds with one tachymetric unit are aligned. The number of tachymetric units is determined in $1 / 10$ units. This gives the upper-stem diameter in tachymetric units.

A second reading is made on a horizontal base rod, positioned aside the tree on which as many full tachymetric units as possible are read out. The reading on the base rod, divided by the number of full units, gives the base reading, which is multiplied by the number of tachymetric units obtained for the upper stem diameter. The reading obtained on the base
rod gives the horizontal distance in meters. - The left graduation on the instrument is used to obtain percentage readings for the position of the upper diameter and that of the base rod. The alge braic difference is multiplied by the base reading and gives the height above ground level of the upper diameter. Sterba (1976) summarized errors involved in the estimation of stem volume with the aid of the telerelascope. Instrument errors were associated with move ments of the vertex of the sighting angle, which affected the estimation of the operator to tree distance. The distance errors varied between $0.5 \%$ and $1 \%$ and necessitated upper-stem diameter adjustments between $1 \%$ and $2 \%$, whereas height estimates should be adjusted upward by $20-40 \mathrm{~cm}$. The volume esti mates were furthermore associated with errors due to the formulae being used. An upward adjustment of $3-5 \%$ was required to remove this source of bias.

### 2.5.3. Prisms

The prism is a thin wedge made of glass or plastic, which deflects the incom ing rays through an angle that is constant for a given prism. Deflection causes displacement of the tree when viewed through the prism, the amount of dis placement being dependent on the diopter strength of the prism, which in turn is a function of the angle between the two surfaces of the prism. A strength of one diopter is the equivalent of a displacement of 1 unit per 100 units dis tance. The displacement of the image produces a critical angle, similar


Fia 3-10. Decision on counting a tree with a wedge prism. to that established by the relascope. When the left edge of the stem, viewed through the prism, is aligned with the right edge of the stem, viewed over the prism, the corresponding stem diameter is $1 / 100$ of the distance to the tree, for a prism with a diopter strength of one. During a $360^{\circ}$ sweep
with the sampling point as center, the line between the two surfaces of the prism is held vertically above the sampling point and should remain in this position. Trees which are dis placed less than the apparent diameter are counted, those which are displaced fall outside the imaginary plot and are not counted (see Figure 3-10). The borderline trees, i.e., those trees for which the amount of displacement is equal to the apparent diameter, should be checked by measuring the stem diameter and distance from sampling point. Because of the amount of time involved in checking, it is customary to assign a count of one-half to each borderline tree, although this might produce operator-bias. On a sloping terrain, the number of trees counted produces a biased estimate of the basal area, since slope distance instead of horizontal distance is observed. Corrections are necessary for slopes of more than 10 . The basal-area estimate, corrected for slope is calculated as follows:

Gha $=\mathrm{BAF} \cdot \mathrm{n} \cos \mathrm{a}$
where $\mathrm{n}=$ tree count, $\mathrm{a}=$ angle of slope. In order to calibrate the prism, a target of known width, between 30 cm and 1 m is set up. The observer moves towards and away from the target until the image seen through the prism and the right side of the stem viewed over the prism are exactly aligned. The dis tance between the observer and the target in measured with a tape. Because of measurement errors, the calibration exercise should be repeated three or four times. The BAF is calculated from
$B A F=10000$
$1+4(L / w) 2$
where $L=$ distance in meters and $w=$ width of target in meters. The more expensively calibrated prisms, however, eliminate the necessity of field calibrations.

A similar situation may arise during winter, if the ground is covered by snow. In Continuous Forest Inventories, the measuring point is usually permanently marked, partly to establish whether or not a particular tree was measured at the previous occasion. It has the additional advantage of reducing random or systematic errors associated with inaccurately positioning the tape or caliper. When using calipers for re-measuring permanent sample plots, they should be checked regularly for the occurrence of instrument-related errors. In forest inventories, they are to be calibrated annually. Calipers used to measure research plots require a 1 mm graduation. Those with a 1,2 or 4 cm graduation are adequate and
more practicable for management, regional, and national forest inventories. The diameters are marked on the graduated beam, and show the midpoint of the diameter class. When using self-rounding calipers, these midpoints are always printed at the lowest point of the diameter class.

## Unit 3: Tree Measurement II

## Unit Structure

### 3.0 Learning Objectives

### 3.1. Bark thickness

3.2: Bark percentage tables
3.3. Crown measurements
3.4. Tree crown measurement
3.5. Crown Class
3.6. Measurement of cross sectional area, basal area
3.7. Bole Surface Area
3.8. Leaf Area
3.9. Crown Cover

### 3.0 Learning Objectives

After going through this unit you shall be able to:

- Understand about the bark thickness
- Major techniques of crown measurement
- Measurement of cross sectional area and leaf area,


### 3.1. Bark thickness

The thickness of the bark varies with species to species and tree to tree. The bark thickness varies with the age of tree and base to top of the tree. This affects the calculation of thick standing trees. The thickness of the bark is measured by Swedish hark gauge.

## Bark Measurement

- The thickness of the bark and its percentage of volume in the tree or log are importantparameters in mensuration because most measurements on standing trees have to be made on over bark.
- Some species have very thick bark. In general, bark thickness varies
genotype, rate of growth and position in the tree
- The bark thickness of the living tree may be measured with little damage to the treesusing a Swedish Bark Gauge.

This chisel form of instrument is provided with a curved arm fitted with a graduated (cm and tin) moving tube. This is pushed into the bark. Afterwards the instrument is removed and thickness of the bark is read off on the scale. Relationship among girth over bark (gob), girth under bark (gub) and bark thickness is given below:
$g=g^{\prime}-2 \pi t$ (where $g=$ gub, $g^{\prime}=$ gob, $t=$ bark
 thickness)

## Conversion of D.O.B into D. U.B. and G. O.B. into G.U.B.

Girth over bark (G. O. B.) can be converted into girth under bark (G.U.B.) by deducting twice bark thickness. Since the thickness of the bark is seldom uniform it should measured at 2 pairs of opposite points corresponding to the two diameters measured, added up and divided by two to get twice bark thickness.
G.O.B. can be converted into G.U.B. by the formula $g$
$g=g^{\prime}-2 \pi t$ where $g=G . U . B$ and $g^{\prime}=G \cdot O \cdot B \quad$ and $t=$ bark thickness.
The formula has been derived as follows :

$$
\begin{array}{ll}
g=2 \pi r=2 \pi\left(r^{\prime}-t\right) & \text { (where } r \text { is radius U.B. and } r^{\prime} \text { is radius O.B.) } \\
=2 \pi\left(\frac{g}{2 \pi}-t\right) & =g^{\prime}-2 \pi t
\end{array}
$$

## Bark thickness table

On the basis of measurement of thickness of bark on a large number of trees of a species, the bark thickness is plotted against the diameter of trees and a smooth curve drawn. From this curve, the thickness of bark is read off against the middle of diameter class interval and tabulated. Bark thickness tables are available for many species now. One such prepared by Lohani (44) for twisted chir is given as an example :

| Diameter class (cm) | Bark thickness (cm) | Twice bark thickness (cm) |
| :--- | :--- | :--- |
| $30.1-40.0$ | 1.80 | 3.60 |
| $40.1-50.0$ | 1.94 | 3.88 |
| $50.1-600$ | 2.09 | 4.18 |
| $60.1-70.0$ | 2.24 | 4.48 |
| $70.1-80.0$ | 2.38 | 4.76 |

## 3.2: Bark percentage tables

For species in which bark is important or in which the volume or weight of wood is considerably influenced by bark, it is usual to compile bark percentage tables. These tables give the percentage of bark and wood by diameter classes. The table is prepared by measuring a large number of trees and calculating the percentage of bark and wood in each. This data is plotted against diameters of trees and smooth curve drawn. From this curve percentage of bark and wood against the middle of the diameter class interval is read and tabulated. One such table pre-pared by Lohani (44) for twisted chip- is given below as example :

| Diameter class (cm) | Bark percentage of total <br> volume | Wood percentage of total <br> volume |
| :--- | :--- | :--- |
| $20.1-30.0$ | 25.70 | 74.30 |
| $30.1-40.0$ | 24.15 | 75.85 |
| $40.1-50.0$ | 21.80 | 78.20 |
| $50.1-60.0$ | 18.85 | 81.15 |
| $60.1-70.0$ | 16.25 | 83.75 |
| $70.1-80.0$ | 14.20 | 85.80 |
| $80.1-90.0$ | 12.80 | 87.20 |

### 3.3. Crown measurements

Crown measurement is one of the tree measurements taken at the crown of a tree, which consists of the mass of foliage and branches growing outward from the trunk of the tree. The average crown spread is the average horizontal width of the crown, taken from dripline to dripline as one moves around the crown. The dripline being the outer boundary to the area located directly under the outer circumference of the tree branches. When the tree canopy gets wet, any excess water is shed to the ground along this dripline. Some listings will also list the maximum crown spread which represents the greatest width from dripline to dripline across the crown. $\frac{[1|2|] 3]}{}$ Other crown measurements that are commonly taken include limb length, crown volume, and foliage density. Canopy mapping surveys the position and size of all of the limbs down to a certain size in the crown of the tree and is commonly used when measuring the overall wood volume of a tree.

Average crown spread is one of the parameters commonly measured as part of various champion tree programs and documentation efforts. Other commonly used parameters, outlined in tree measurement, include height, girth, and volume. Additional details on the methodology of tree height measurement, tree girth measurement, and tree volume measurement are presented in the links herein. American Forests, for example, uses a formula to calculate Big Tree Points as part of their Big Tree Program ${ }^{[3]}$ that awards a tree 1 point for each foot of height, 1 point for each inch of girth, and $1 / 4$ point for each foot of crown spread. The tree whose point total is the highest for that species is crowned as the champion in their registry. The other parameter commonly measured, in addition to the species and location information, is wood volume. A general outline of tree measurements is provided in the article tree measurement and more detailed instruction in taking these basic measurements is provided in "The Tree Measuring Guidelines of the Eastern Native Tree Society" by Will Blozan.[1][2]

### 3.4. Tree crown measurement

Crown depth (Cdep, $m$ ) and crown radius ( $\mathrm{Cr}, \mathrm{m}$ ) are two crown dimensions that can be measured in the field (Figure 1). Cdep is defined as the depth of the crown (Figure 1A), calculated as the difference between total tree height ( $H$ in m ) and the bole height
defined as the height from the ground to the first living branch or to the height to lowest foliage ( $H f$ in m ). Cdep is measured for most trees using a trigonometric approach with either a manual
clinometer or electronic hypsometer. Crown radii are measured in the four (north, south, east and west) or eight (north, north-east, north-west, south, south-east, south-west, east and west) cardinal directions from ground projections of crown edge to stem centre, with the person viewing the edge of the canopy standing perpendicular to the line to the bole

(Figure 1B).

Figure 1. Measurement of crown depth (A) and crown radius (B) (image adapted fromPretzsch et al. 2015)

These basic crown measurements are used to estimate: crown diameter (Cdia, m) from 2 times the mean of the four or eight crown radii; crown area ( $\mathrm{Ca}, \mathrm{m}^{2}$ ) from the projected area of four ellipse quarters ( $\pi \times C_{r_{-}}$Ns $\times C_{r_{-}}$Ew $)$, where $C_{r}$ NS is the mean of Cr north and Cr south and $\mathrm{Cr}_{2}$ EW is the mean of Cr east and Cr west; and crown volume ( $\mathrm{Cv}, \mathrm{m}^{3}$ ) assuming a half-ellipsoid shape as ${ }^{1} \quad 4$

$$
2 \times 3 \times C_{a} \times C_{\mathrm{dia}}
$$

Crown measurements can be taken during initial plot census or during plot recensus (see

Phillips et al. 2016 for plot establishment and census protocols). The crown measurementscan be paired in the field with estimates of tree height (Banin et al., 2012; Feldpausch et al., 2011), liana infestation (van der Heijden et al, 2010), and crown illumination (see RAINFOR protocols). Crown measurements are generally taken to evaluate the allometry of intact crowns. Therefore, field measurements exclude trees with significant crown damage. When crown measurements are taken to assess crown damage (e.g., due to selective logging), an assessment of the fraction of the crown lost to damage should be noted on the field sheets and when uploading data to forestplots.net. The number of crowns measured per site varies from 30 to 9000 trees across field sites and studies (Loubota Panzou et al. in review). An analysisof the number of tree height measurements needed to reduce uncertainty in biomass estimates suggested a conservative threshold of sampling 50 trees per location, including the ten trees with the largest diameter (Sullivan et al. 2018). A similar threshold could be taken for crown diameter measurements, with individual trees measured at the same time to save time and to evaluate crown size-bole diameter-height allometry. Crown diameter measurements can be uploaded to forestplots.net with the census data.

Damage from branch and tree-fall, wind, lianas, and logging, as well as preferential growth towards light gaps can cause asymmetric crown development. Sometimes, these changes can result in negative radii values relative to the centre point of the base of the bole. Figure 2 shows an example where the crown is shifted to the side away from the bole centre point, causing one radius to be negative. Despite this negative radius, the overall estimated crown diameter is correct.

### 3.5. Crown Class

Crown class is a term used to describe the position of an individual tree in the forest canopy. In the definitions below, "general layer of the canopy" refers to the bulk of the tree crowns in the size class or cohort being examined. Crown classes are most easily determined in evenaged stands, as depicted in Figure 5.5. In an unevenaged stand, a tree's crown would be compared to other trees in the same layer. Kraft's Crown Classes are defined as follows (Smith et al. 1997 and Helms 1998 modified for clarity):

Dominant trees: These crowns extend above the general level of the canopy. They receive full light from above and some light from the sides. Generally, they have the largest, fullest crowns in the stand (Figure 5.5).

Co dominant trees: These crowns make up the general level of the canopy. They receive direct light from above, but little or no light from the sides. Generally they are shorter than the dominant trees.

Intermediate trees: These crowns occupy a subordinate position in the canopy. They receive some direct light from above, but no

### 3.6. Measurement of cross sectional area, basal area

The most important use of area measurement in trees is for calculation of volume of a tree. For thi5i purpose, the tree is divided into sections and the area of the plane passing through the stern of the tree at right angles to its longitudinal axis is referred to as crosssectional area. While in the case of felled trees these sections are cross-cut, in standing trees only diameter is measured and the cross-sectional area calculated. The area of the cross section of a stem at breast-height is called the basal area, of the tree. The total basal area of all or specified classes of trees per unit area, referred to as basal area per hectare, is a very important measure specifying characteristics of a stand.

If the cross-section of the tree is circular, the sectional area can be determined by the formulae :

$$
\begin{aligned}
& \text { Area }=\pi \frac{d 2}{4} \quad \text { where } \mathrm{d} \text { is the diameter of the tree at the section. } \\
& \text { Area }=\frac{d 2}{4 \pi} \quad \text { where } \mathrm{g} \text { is the girth of the tree at the section. }
\end{aligned}
$$

It has already been pointed out in previous chapter, that the section of the tree is seldom a circle; it approaches sometimes to an ellipse and therefore calculation of sectional area requires the measurement of two diameters corresponding to the major and minor axes of the elliptical section. With two diameters, the area of the section could be found out either by the formula $\frac{\pi}{4}\left(\frac{d 1+d 2}{2}\right)^{2}$ or by the formula formula $\frac{\pi}{4}\left(\frac{d 12+d 22}{2}\right)$ where $\mathrm{d}_{1}$ and $\mathrm{d}_{2}$ are the two diameters corresponding to the major and minor axes. The cross-sectional areas of the cut sections of a tree such as logs, whose shapes are irregular, can be calculated
more accurately by measuring several radii from the pith, and calculating the area by the formula $\pi r^{2}$

$$
\text { where } \mathrm{r}^{2}=\frac{r_{1}^{2}+r_{1}^{2}+r_{1}^{2}+r_{1}^{2}+\cdots \ldots .+r_{1}^{2}}{\mathrm{n}}
$$

where $\mathrm{r} 1, \mathrm{r}_{\mathrm{n}} \mathrm{r}$, are the radii in different directions and n is the number of radii.

### 3.7. Bole Surface Area

The bole surface area is of interest since it represents $4.1^{*}$ the surface upon which the growth material is distributed and can be used in the estimation of tree and stand growth. The area of the bole:surface of a tree and log can be calculated by the formula:

$$
\mathrm{S}=\mathrm{gl} \quad \text { or } \frac{g_{1}+g_{2}}{2} \mathrm{XI}
$$

when, $S$ is the bole surface area of a $\log , g 1, g$ and $g 2$ are girths at thicker end, middle and thinner end of the log respectively, and I is the length of the log or tree.

### 3.8. Leaf Area

The surface area of the foliage of forest trees is a useful measure for studying interception of precipitation and light, evapo-transpirational studies, etc. It is also of great importance in case of certain species such as Diospyros melanoxylon whose leaves are of great economic value for making bides. Not much quantitative work has, however, been done so far in this direction in India.

### 3.9. Crown Cover

Crown cover is defined as the horizontal projection on the ground of the tree crown. Besides being a measure of canopy density, it is important in the use of aerial photographs.

## AREA MEASUREMENT FOR SAWN TIMBER

Though conversion and sale of sawn timber is mostly by cubic measures, area calculation is required in certain categories of timber, such as planks, for payment of sawing charges. As conversion of thinner pieces of timber from a beam or a log consumes more power as
well as time, the rate of conversion is by surface area exposed in sawing. Therefore in such cases, the area exposed has to be calculated for each plank by multiplying its length by breadth.

## Unit 4: Height Measurement:

## Unit Structure

### 4.0 Learning objectives

### 4.1. Height Measurement

### 4.2. Basis of Height Measurement

### 4.3. Principles of height measurement

4.3.1. Trigonometric principles
4.3.2. Geometric principle of similar triangle
4.4. Height measurements: Direct and indirect methods
4.4.1. Height by Direct Methods
4.5. Instruments used in height measurements

### 4.6. Sources of error in height measurement

### 4.0 Learning objectives

After studying this unit you will be able to understand about:

- Height measurement and its principles
- Principle and Methods of height measurement
- Instruments used for height measurement


### 4.1. Height Measurement

- Height is the linear distance of an object normal to the surface of the earth.
- Tree height is the vertical distance measured from the ground surface.
- Height of standing tree is measured to find out its volume. Height of selected trees in aforest are also required to read volume tables, form factor tables, yield tables etc.
- Lastly, heights of trees are required to find out productive capacity of site. Height is generally considered as an index of fertility and with the knowledge of age it gives areliable measure of the site quality of a locality.

Total height of a standing tree is the distance along the axis ofthe tree stem between the ground and the tip of the tree.

Bole height is the distance along the axis of tree between ground level and crown point. (crown point is the position of thefirst crown forming branch).

Commercial bole height is the height of bole that is usually fitfor utilization as timber.
Height of standard timber bole is the height of the bole fromthe ground level up to the point where average diameter over bark is 20 cm .

Stump height is the distance between the ground and basalposition on the main stem where a tree is cut.
Crown length-The vertical measurement of the crown of the tree from the tip to the point half way between the lowest green branches forming green crown all round and the lowest green branch on the bole.

Crown height - The height of the crown as a measured vertically from the ground level to the point half way between the lowest green the lowest green branches forming green crownall round.


Fig.1. Tree structure

### 4.2. Basis of Height Measurement

The height of trees is important to forestry particularly because:

- The length of the stem is important as part of the calculation of the total amount of wood contained within it.
- The height of the tallest trees in the forest is the basis of one of the most important measures used in forestry to assess site productive capacity.

In forestry, tree height is defined as the vertical distance from ground level to the highest green point on the tree (which will be referred to here as the tip of the tree).It might seem odd that tree height is not defined in terms of stem length (since it is usually the wood-containing stem of the tree with which forestry is most concerned) or as the height to the top of the stem itself. However, near the tips of treesof many species, it is difficult to define exactly what constitutes the stem, because of the proliferation of small branches there. Even if the main stem can be seen clearly near the tip, it is often very difficult to see exactly where it stops. This is particularly so when viewing, from the ground, a tall tree with a dense crown.

Whith the highest green point of a tree is much easier to identify than its stem length, care must be taken to ensure that the tree is viewed from sufficiently far away so that its tip can be seen clearly. Even then, in dense forest it is often difficultto see the tip amongst the crowns of other trees; care must be taken to ensure the tip one can see is indeed that of the tree being measured.

Even if the tree is leaning, its height is still defined in forestry as the height to the highest green point, rather than by its stem length. Most trees, in most forest circumstances, stand just about vertically; if they do lean a little, perhaps inresponse to strong prevailing winds, the lean is usually no more than a few degrees. For general forestry purposes, it is sufficiently rare to encounter trees leaning sufficiently that special consideration has to be given as to how their height should bemeasured; the lean would have to exceed about $7-8^{\circ}$ before it would be sufficient to affect appreciably the result of a tree height measurement.

### 4.3. Principles of height measurement

- Instruments used for measuring tree heights are collectively referred to as hypsometers.
- All height measuring instruments are based either on geometric principles of similar triangles or on trigonometric principles based on relations between the sides of rightangled triangle.


### 4.3.1. Trigonometric principles

The principles follow the basic rules of trigonometry for deriving heights of trees from distance and angle measurements. Two laws are applicable for this purpose and they are: tangent law and sin law. Instruments based on Trigonometrical principles are Brandis hypsometer, Abney's level, Haga Altimeter, Topographical Abney's level, Relaskop, Tele Relaskop, Barr and Stroud dendrometer, BlumeLeiss hypsometer

1) Tangent method: This method utilizes the tangents of the angles to the top and base of the tree and the distance of the observer from the tree. This method is applicable to :
2) Applicable to right angle triangle
3) For accurate results, trees must not lean more than $5^{\circ}$ from the vertical, and the fixedhorizontal distance must be determined by taped measurement.

$$
\operatorname{Tan} \theta=A B / B C, \mathrm{~h}=\mathrm{d} x \tan \theta
$$

## Condition 1: On Level ground:

Let $A B$ is the tree, $E F$ is the observer, $E F$ is the horizontal distance from the tree and a is the angle to the top of the tree. Now,

$$
A B=A D+B D
$$

$=E D \tan \alpha+B D(\tan \alpha .=A D / E D)=B F \tan$ $\alpha+E F(E D=B F)$
$B F$ is the ground distance and $E F$ is
 the eye height of the observer.

## Condition 2: On Sloping Ground

The top of the tree is above the eye level and base of the tree is below the eye level
Let $A B$ be the tree, $\alpha$ be the angle to the top, $\beta$ is the angle to the base of the tree and $E D$ ground distance from the observer to tree.

(i) $A B=A D+D B$

$$
\begin{aligned}
& =E D \tan \alpha+E D \tan \beta \\
& =E D(\tan \alpha+\tan \beta)
\end{aligned}
$$

## Condition 3:

The top and the base of the tree are at above the eye level
$A B=A D-B D$

$=E D \tan \alpha-E D \tan \beta$
$=E D(\tan \alpha-\tan \beta)$
$=E B \cos \alpha(\tan \alpha-\tan \beta)$

## Condition 4:

The base and top of the tree is below the eye level


## $A B=B D-A D$

$=E D \tan \beta-E D \tan \alpha$
$=E D(\tan \beta-\tan \alpha)$
$=E B \cos \beta(\tan \beta-\tan \alpha)$

## 2. Sine Method:

In this method, the ground distance and angles are used according to sine theory.

$$
\begin{aligned}
& \frac{\operatorname{Sin}<A E B}{\mathrm{AB}}=\quad \frac{\operatorname{Sin}<E}{\mathrm{~EB}} \\
& \text { Hence, } \mathrm{AB}=\frac{\mathrm{EB} \mathrm{Sin}<A E B}{\operatorname{Sin}<E A B} \\
& =\quad \frac{E B \operatorname{Sin}(\alpha+\beta)}{\operatorname{Sin}(90 \alpha)} \\
& =\quad \frac{E B \operatorname{Sin}(\alpha+\beta)}{\cos \alpha}
\end{aligned}
$$



### 4.3.2. Geometric principle of similar triangle

In similar triangles. The corresponding angles are equal and the corresponding sides are proporational. in the following similar triangles viz. $A B C$ and $A^{\prime} B^{\prime} C^{\prime}$
$=A^{\prime} B^{\prime}: A B:: B^{\prime} C B C$
$=A^{\prime} B^{\prime} \times B C=A B \times B^{\prime} C^{\prime}$
Solving this

$$
A^{\prime} B^{\prime}=\frac{A B X B^{\prime} C}{B C}
$$



By using Geometric principles of similar triangles we can find:

- Corresponding angles are equal and the corresponding sides are proportional.
- By knowing the two sides of a triangle and only one side of the other, the correspondingsecond side of the latter can be found.
- Each angle of a triangle is equal to its corresponding angle of the other triangle


### 4.4. Height measurements: Direct and indirect methods

In the case of small trees, height can be directly measured with a pole or a rod. For taller trees, telescoping of sliding poles made of wood, light metal or fiber glass can be used. Measurement of heights of standing tall trees, particularly when their number may be large, requires some special consideration. Height measurement is a more time-consuming operation than diameter measurement because the measurement of 'height of a standing tree with instruments takes about 10 times more time than it takes to measure its diameter. Therefore, 'heights of all trees are measured only in small permanent sample plots. For all other purposes, heights of a few trees are measured with. instruments and for the others, it is estimated by eye or by some other noninstrumental method. Thus; the methods of measurement of height may be classified into ocular, non-instrumental and instrumental methods.

### 4.4.1. Height by Direct Methods

Direct height measurement involves simply holding a vertical measuring pole directly alongside the tree stem. Devices with a telescoping set of pole segments can be purchased readily. These are able to measure tree heights to about 8 m .

## (1) Ocular estimate

In estimating heights of trees by eye, .a height scale has to be fixed in mind. This is easily done by measuring the heights of a few trees with some instrument before the start of the work and that of a few trees again in the middle of the work. With this standard in mind the estimator judges the heights of trees to be measured and records them. To make it more reliable a pole of. 3 rn length may be placed against the tree and then the tree is imagined to be divided in 3 m sections and the height is calculated, If the estimator has no practice in dividing the tree in imagination, he can make use of a pencil. A pencil may be held in a stretched-out hand in such a manner that a portion of the pencil sticking out of hand covers the 3 m pole placed against the tree. With that length of pencil, he marks off sections on the stem in imagination and then the number of sections multiplied by 3 gives the height of tree in metres. Ocular estimate is not very reliable and serious errors may result if the estimator has no previous experience of this work. Constant practice can, however,
make foresters quite proficient in estimating heights correctly.

## (2) Non-instrumental methods

No tree is truly vertical. All lean on one side or the other. If a leaning tree is assumed to be vertical, the calculated height does not give the actual height of tree. All non-instrumental and instrumental methods are based on the assumption that the tree is vertical. But the heights of leaning trees can be calculated by any height measuring instrument if the angle of lean is also measured. Therefore, the methods of measurement of heights of vertical and leaning trees will be described separately. The -heights of vertical trees can be calculated either by instruments or without them. Since height measurement by instruments is slow and therefore expensive and by ocular estimate, not very reliable, several non-instrumental methods have been developed to meet the requirements of routine forest operations, some of which are described below:

## (a) Shadow method:

In this method a pole of convenient length is fixed on the ground and its height above the ground is measured. The shadows of the pole and the tree distance also measured. The height of the tree can then be calculated by simple proportion as follows:

$$
\frac{A B}{a b}=\frac{B D}{b d}
$$

$$
\text { Therefore } \mathrm{AB}=\frac{A B B D X a b}{b d}
$$

where $A B$ is the tree, $a b$ is the portion of the pole above ground level, $B D$ is the length of the shadow of the tree, and bd is the shadow of ab.


Fig. 5 Shadow method of measuring tree height
Obviously, this method can be applied only on clear sunny days. It will give accurate results if it is applied in the early hours-of morning or late in the evening
(avoiding the time between 11 a.m. and 2 p.m ) when the tree will cast long shadow. This method is difficult to apply in well-stocked forests where it may be difficult to get shadow of the tree in question on ground.

## (B) Single pole method



In this method, the observer holds a pole of about 1.5 m length vertically at an arm's length in one hand in such a that the portion of the pole above the hand is equal in length to the distance of the pole from the eye. Without changing the position of hand with reference to the eye, the observer moves slowly forward_and backward till the line of sight to the tip of the tree passes through the tip of the pole is held by hand. This means that the portion of the pole above the hand covers the tree completely. The height of the tree is then equal to the distance of the observer's eye from the base of the tree as shown below : Let $A B$ be the tree and a c a pole about 1.5 m long, held at $b$ vertically so that distance from observers eye E to $b$ is equal to $a b$ then in Fig. 6

## (3) Instrumental methods

The instruments used in height measurement of trees are called hypsometers, altimeters and clinometers. Hypsometer is an instrument used for determining the height of standing trees from observations 'taken at some distance from the tree. Though altimeters are generally altitude measuring instruments, one instrument has been devised to determine heights of trees. Similarly, clinometers are instruments which measure angle of slope. Any instrument which measures angles of slope can be used for determining the heights of trees by trigonometrical methods and as such clinometers are also used for height measurement of trees. But some clinometers arc specially designed for this purpose by addition of a scale which directly gives the value of tangent of angle of inclination as percentage of horizontal distance, i.e., slope percent. These clinometers may, then, be called hypsometers. All these instruments are based either on geometric principle of similar triangles or on trigonometrical principles based on relations between the sides of a right angled triangle. Both these principles are explained earlier.

### 4.5. Instruments used in height measurements

- Height measuring instruments are called hypsometer.
- Those instruments based on trigonometric principles are more accurate than the ones employing geometric principles.
- The Abney's level, Haga Altimeter, Blume-Leiss, Altimeter and Sunto Clinometer are similar in accuracy.


## Christen's Hypsometer

- It is based on the geometric relationships of similar triangles.
- Consists of a strip of metal, thin wood or card board about 2.5 cm wide and 33 cm length.
- It has two flanges or protruding edges one at the top and other at the bottom.
- Each flange has a hole in it, the upper one to suspend the instrument by some thread passing through it at and the lower one to suspend a weight from it to prevent it from swinging.
- To use it, a pole (usually 5 or 10 ft long) is
 held upright against the base of the tree, or a mark is placed on the tree at a height of 5 or 10 ft above the ground.
- The hypsometer is then held vertically at a distance from the eye such that the two inside edges of the flanges are in line with the top and base of the tree.
- It may be necessary for the observer to move closer to or farther from the tree to accomplish this, but except for this, the distance from the tree is immaterial.
- The graduation on the scale that is in line with the top of the pole, or the mark, gives the height of the tree.

The following proportion gives the formula for graduating the instrument

$$
\begin{aligned}
& \frac{A^{\prime} C^{\prime}}{A C}=\frac{A^{\prime} B^{\prime}}{A B}= \\
& \mathrm{AB}=\frac{A C X A^{\prime} B^{\prime}}{A^{\prime} C^{\prime}}
\end{aligned}
$$

For a given length of instrument $\mathrm{A}^{\prime} \mathrm{B}^{\prime}$ and a given pole length or mark height AC , the graduation $\mathrm{A}^{\prime} \mathrm{C}^{\prime}$ can be obtained by substituting different values of height AB in the equation.

- Although the christen hypsometer may be used to measure any type of height, it is practical only for total height measurements.
- A crowding of graduations at the bottom of the scale, makes the instrument unreliable for the determination of the height of tall trees.


## Advantages

- It is light, easily made and easy to transport
- The height of the tree can be read directly.
- It is quicker to use and so it is useful inconditions where speed is required.


## Disadvantage:

- Extra care has to be taken to hold the top and bottom of the tree within the flanges whilereading the heights.
- It should be held in the true vertical plane
- It is not suitable for more than 30 m tree height.
- It requires the use of staff.
- Skill is necessary to use the instrument with consistent accuracy.


## 2. Sunto Clinometers:

Hypsometers based on tangent of angles such as Abney's level, Haga altimeter, the Blume-Leiss altimeter and the Sunto clinometers are used in height measurement oftrees.

- The sunto clinometer is a handheld device
 house in a corrosion-resistant aluminumbody.
- A jewel-bearing assembly supports the scale, and all movingparts are immersed in
a damping liquid inside a hermetically sealed plastic capsule.
- The liquid dampens undue scale


## 3. Abney's level

- It is used to measure tree heights as well as land elevations.
- The instrument consists of a graduated arc mounted on a sighting tube about 6 inches long.
- The arc may have a degree, percentage or topographic scale.
- When the level bubble, which is attached to the instrument, is rotated while a sight is taken, a small mirror inside the tube makes it possible to observe when the bubble is horizontal.
- The angle between the bubble tube and the sighting tube may be read on the arc.
- The abney's level, however, is slower to use, and large vertical angles are difficult to measure because of the effect of refraction on observations of the bubble through the tube beneath.
- This makes the abney level difficult to use in tall timber that is so dense that the tops cannot be seen from a considerable distance.
- When used correctly, the Abney Level has an accuracy of about +/- 0.5 m for a 20 m tall tree (ie about 2.5\%).

Use

- Measure the horizontal distance from the base of a vertical tree (or the position directly beneath the tree tip of a leaning tree) to a location where the required point on the tree (e.g. tree tip) can be seen.
- Sight at the required point and move the index arm over the scale until the bubble tube is level.
- Read the percentage scale (or the degrees and minutes of the angle).
- Calculate the height by multiplying the percentage read by the horizontal distance (or by multiplying the horizontal distance by Tan of the angle).
- Site to the base of the tree and repeat steps 2-4.
- Combine the heights from steps 4 and 5 to determine total tree height:
i. Add the 2 heights together if you
 looked up to the required point in step 2 and down to the base of the tree in step 5.
ii. Subtract the height to the base of the tree from the height to the required point if you are on sloping ground and had to look up to both the required point and the base of the tree.

Check all readings and calculations.

## Advantages

- It gives accurate angles of elevation and depression
- Reading can be taken after sighting the tree without distrubing the index.
- It is small and light and can be used even in hills without difficulty.


## Disadvantages

- Shaking of the hand makes the sighting of the top or bottom of the tree a little difficult and time consuming.
- The spirit level has to be adjusted by moving the head of the screw while simultaneously looking to the top or bottom of the tree.


## 4. Haga altimeter

It consists of a gravity-controlled, damped, pivoted pointer, and a series of scales on a rotatable, hexagonal bar in a metal,

- pistol-shaped case.
- The six regular American scales are $15,20,25,30$, percentage, and topographic scale.
- Sights are taken through a gun-type peep sight; squeezing a trigger locks the indicator needle, and the observed reading is taken on the scale.
- A range finder is available with this instrument.
- When used correctly, the Blume Leiss has an accuracy of about $+/-0.5 \mathrm{~m}$ for a 20 m tall tree (ie about 2.5\%).


## Use

1.Select a location, preferably $15,20,30$ or 40 meters horizontal distance from the base of a vertical tree (or the position directly beneath the tree tip of a leaning tree) where the required point on the tree (e.g. tree tip) can be seen.
2.Select the appropriate distance scale on the rotating rod.
3.Release the pointer by pressing the button on the side of the instrument.
4.Sight at the required point on the tree, wait for a moment for the pointer to settle then pull trigger.
5.Read the height directly from the appropriate scale if you are $15,20,30$, or 40 meters away from the tree. If you were unable to find a position at one of these distances:

- If the horizontal distance is a simple fraction of one of the scale distances (e.g. 10 m is half of 20 m ), read from the scale distance and
 multiply by the appropriate fraction.
- $\quad$ Read from the percent scale and multiply this percentage by the horizontal distance measured in step 1.

6. Site to the base of the tree and repeat steps 3-5.
7.Combine the heights from steps 5 and 6 to determine total tree height:

- $\quad$ Add the 2 heights together if you looked up to the required point in step 2 and down to the base of the tree in step 6.
- $\quad$ Subtract the height to the base of the tree from the height to the required point if you are on sloping ground and had to look up to both the required point and the base of the tree.
8.Check all readings and calculations.


### 4.6. Sources of error in height measurement

1. Instrumental errors: Instrumental error occurs as a result of some limitations in instrument. Christen hypsometer keeps on swinging due to wind and hence taking correct readings is a difficult task. In Smythies hypsometer, the plumb bob is liable to slip and thereby giving wrong reading. In Brandis hypsometer, the continuous swinging of wheel distract the patience of the observer while taking reading.
2. Personal errors: The personal error varies with person to person according to their experience and knowledge. Besides that, the shaking of hand can give very inaccurate results while reading in case of Abney's level and Christen's hypsometer.
3. Errors due to measurement: The horizontal distance calculation is difficult due to bushes and shrubs on the ground level. These obstruct the visibility of tree base and affect the angle measurement. For a tree standing on level ground, the height is given by the formula;

## Sources of the major errors in height measurement

Failure to measure correctly the horizontal distance from the observer to the tree
If the distance from the observer to the tree is not measured horizontally, the observer will stand too near the tree, and the height will be overestimated by the direct reading on the instrument scale.

## Wind sway

Wind causes tree tops to sway and this can be very serious hindrance in tree height measurement and cause serious errors. Accurate readings cannot be made in high winds. The errors may be reduced by averaging readings taken at the extremes of the sway towards and away from the observer.

## Leaning trees

If the tree is leaning away from the observer, height will be under estimated and if the lean is towards the observer, height will be overestimated.

Non linearity of the relationship of tree height and angle of sight.

The smaller the angle of the sight the easier it is to define the highest point in the crown; but the nearer the angle of sight to 450 the smaller is the error caused by an inaccurate reading of that angle. The best compromise between these two conflicting considerations is to select the observation point so that the angle of sight lies between 300 and 450 , i.e the observer should stand between one and one-and-a-half times the tree height away from the tree. Angles greater than 450 must be avoided as the probability of mistaking a side branch for the top of the tree is unacceptably high.

Instrument error. All instruments should be checked periodically against some standard or known height and adjust as necessary.

Operator and recording error. - Personal error is always likely, e.g.
incorrect setting of distance or booking of angles and distances, incorrect reading;
forgetting to add on the section of tree below eye level or forgetting to sight to the tree base;

Measuring to wrong tip - shaking the tree may help!
Difference of opinion amongst observers in nominating the tip of an umbrageous crown.

## Unit-5: Measurement of Form

## Unit Structure

### 5.0. Learning Objective:

### 5.1. Definition

5.1.1. Metzger's Theory or Girder Theory
5.1.2. Methods of studying form
5.2. Form factor and its type
5.2.1. Types of form factor
5.2.2. Uses of form factor
5.2.3. Kinds of form factor

### 5.3. Form quotient and its type

5.3.1: Form Class
5.3.2. Form Point Ratio

### 5.4. Taper table and formulae

### 5.0. Learning Objective:

After studying this unit you are able to understand about:

- Tree stem form and Metzger's theory
- What is form factor and kind of form factor
- What are taper and classification of form factors and form quotient


### 5.1. Definition

Even a superficial observation of trees stems of trees are not cylindrical and therefore calculation of volume of trees or logs requires knowledge of diameter from which cross sectional area can be calculated, height (or length in case of logs) and stem form. Form is defined as the rate of taper of a log or stem. Taper is the decrease in diameter of a stem of a tree or of a log from base upwards. The taper varies not only with species, age, site and crop density but also in the different parts of the same tree.

## Tree Stem Form

- Form is the rate of taper of a log or stem
- It is the decrease in diameter of a stem of a tree or of a log from base upward.
- The taper varies not only with species, age, site and crop density but also in the different parts of the same tree.
- The basal portion of the tree corresponds to the frustum of a neiloid, the middle portion to the frustum of a paraboloid, and the top portion to the cone
- Trees often are combinations of form.



Fig. 1: Tree form
Fig. 2. Curve, revolution forms a cylinder

For instance, as seen in fig. 1 the basal portion of the tree corresponds to the frustum of a neiloid, the middle portion to the frustum of a paraboloid, and the top portion to a cone. These solids are formed by revolution of curves given by the following equations:

1. Neiloid
$Y=k \sqrt{x}{ }^{3}$
or $\quad Y^{2}=k x^{3}$
2. Paraboloid
$Y=k \sqrt{x}$
or $\quad Y^{2}=k x$
3. Cone
$Y=k \sqrt{x}{ }^{2}$
or $\quad Y^{2}=k x$
4. Cylinder $\quad Y=k \sqrt{x} 0$
or $\quad Y=k$

### 5.1.1. Metzger's Theory or Girder Theory

- Several theory put forward to explain variations in taper from tree to tree and in the same tree as well
- Metzer's theory assumes that the tree stem should be considered as a cantilever beam of uniform size against the bending force of the wind.
- The wind pressure acts on the crown and is conveyed to the lower parts of the stem in an increasing measure with the increasing length of the bole.
- Thus, the biggest pressure is exerted at the base and there is a danger of trees snapping at the place, to counteract this tendency, the tree reinforces itself towards the base.
- The pressure of wind crown keeps on changing as the tree is growing in open crowded portion.
- Tapering increase if it is an isolated area, an area where largest density, in the area tapering decreases.
- Though tapering is the natural process which can be controlled by human interference. If competition increases, tapering decreases.
- Trees growing in complete isolation or exposed situation have short but rapidly tapering boles while the trees growing in dense crops, which are therefore subjected to lesser wind pressure, have long and nearly cylindrical boles.

Mathematically,
Let,
$p=a$ force applied to a cantilever beam at its free end
I= the distance of a given cross section from the point of applications of this force
$d=$ the diameter of the beam at the point
$\mathrm{s}=$ the bending stress in $\mathrm{kg} / \mathrm{cm}^{2}$

By the rule of mechanics, $\quad S=\frac{p^{*} l}{d^{3}} * \frac{32}{\pi}$
As the force $p$ in case of trees consists of components
$W=$ wind pressure per unit area, $F=$ crown area, than $p=w^{*} F$
Then,

$$
\begin{aligned}
& s=\frac{w^{*} F * l}{d^{3}} * \frac{32}{\pi} \\
& d^{3}=\frac{32 * w^{*} F * l}{\pi^{*} s}
\end{aligned}
$$

- For a given tree $w, F$, s can be considered as constant, therefore $d^{3}=k l$, where $k$ is a constant
- Thus, the diameter raised to the third power increases proportionately with lengthening of the lever or with the increasing distance from the central point of application of wind force
- According to this logic, the tree stem must have the shape of a cubic paraboloid.


### 5.1.2. Methods of studying form

1. By comparisons of standard form ratios (form factor and form quotient) : For this purpose two form ratios are in vogue :
(i) Form Factor
(ii) Form Quotient
2. By classification of form on the basic of form ratios and
3. By compilation of taper table

### 5.2. Form factor and its type

Form factor is defined as the ratio of the volume of a tree or its part to the volume of a cylinder having the same length and cross of the volume of a tree section as the tree. In other words, form factor is the ratio between the volume of a tree to the product of basal area and height. Depending on the height of measurement of basal area and on the parts of the tree considered, the following classes of form factors are distinguished. It is the ratio between the volume of a tree to the product of basal area and height.

$$
F=\frac{V}{S h}
$$

Where,
$F=$ form factor
$\mathrm{V}=$ tree volume in cubic units
$\mathrm{S}=$ basal area at breast - height in area units
$\mathrm{h}=$ height of the tree in linear units

### 5.2.1. Types of form factor

(A) Artificial form factor: This is also known as Breast-height form Factor. For this form factor, the basal area is measured at breast-height and the volume refers to the whole tree both above and below the point of measurement (Fig. 3).

The artificial form factor is not a reliable guide is of the tree form. The point of diameter measurement is fixed and as this bears no fixed relation to the height of the tree which is that of the whole tree and not of the portion above the breast height, the tree of same form but different heights will have different not of the portion above the breast-height, the trees form factors. Notwithstanding its unreliability as a Fig. 3 measure of tree form, the artificial form factor is


Fig. 3. Artificial Form Factor universally used because its computation involves handy measurement and because with the standardization of diameter measurement at breast-height, it is just the factor needed to convert height and breast height sectional area into the tree volume.
B. Absolute form factor: It is the ratio between the volume of the tree above the point of diameter or basal area measurement with the cylinder which has the same basal area and whose height is equal to the height of the tree above that point. For this form factor basal area is measured at any


Fig. 4. Absolute from factor convenient height and the volume refers only to that part of the tree above the point of measurement. In other words, it is the ratio between the volume of the tree above the point of diameter or basal area measurement with the cylinder which has the same basal area and whose height is equal to the height of the tree above that point as shown in Fig. 4.
C. Normal (or true) form factor: In this form factor basal area is measured at a constant proportion of the total height of the tree, e.g $1 / 10^{\text {th }}, 1 / 20^{\text {th }}$ etc. of the total height and the volume refers to the whole tree above ground level.

## Disadvantages

1. The height of tree to be determined before the point of measurement can be fixed.
2. Point of measurement very inconvenient in case both very tall and short trees.

Absolute form factor and normal factor are no longer used. Unless stated, form factor implies artificial form factor whose basal area calculated at 1.3 m . The natural form factor corresponding to the total volume of a stem is generally between 0.3 and 0.6.

### 5.2.2. Uses of form factor

## 1. To estimate volume of standing trees

- Form factor compiled in tabular form to give average form factor of different dimensions by dbh and height classes.
- Table used to estimate volume by measuring dbh and height.
- Table prepared from measuring large number of trees, so application to individual tree not satisfactory, however used to estimate volume of group trees.
- Uses limited to similar growing conditions


## 2. To study laws of growth

- Gives insight to laws of growth, particularly to stem form of trees.


### 5.2.3. Kinds of form factor

Depending upon volume represented, form factor may be of following kinds

1. Tree form factor
2. Stem timber form factor
3. Stem small wood form factor

## Form Height

It is the product of form factor and total height of the tree.
$F h=\frac{V}{S}$
Where,
$F h=$ form height
$\mathrm{V}=$ volume of the tree
$\mathrm{S}=$ basal area
Volume is calculated from under bark measurements and the basal area is calculated from dbh (ob). Form height is used to determine how far is it reasonable to assume that volume
is proportional to the basal area. If form height remains constant with increasing diameter, then it is clear that the assumption is justified.

### 5.3. Form quotient and its type

## Form Quotient

- It is the ratio between the mid-diameter and the dbh.

$$
F \cdot Q=\frac{\text { mid }- \text { diameter }}{\mathrm{dbh}}
$$

- Taper depends upon form quotient (A. Schiffel)


## Types of form quotient

1. Normal form quotient

- Ratio of mid-diameter or mid-girth of a tree to its diameter or girth at breast height.


## 2. Absolute form quotient

- Ratio of diameter or girth of a stem at one half its heights above the breast height to the diameter or girth at breast height.

Form quotient is the third independent variable of volume table that can be used to predict the volume of a tree stem.

### 5.3.1: Form Class

- Form class is defined as one of the intervals in which the range of form quotients of trees is divided for classification and use.
- It also implies to the class of trees which fall into such an interval.
- Trees may be grouped into form classes expressed by form quotient intervals such as 0.50 to $0.55,0.55$ to 0.60 and so on or by mid-points of these intervals such as $0.525,0.575$ and so on.


### 5.3.2. Form Point Ratio

- It is defined as the point in the crown as which wind pressure is estimated to be cantered.
- Form point ratio is defined as the relationship, usually expressed as a percentage, of the height of the form point above ground level to the total height of the tree.
- Form point ratio bears a consistent relation to the form quotient.
- If form point ratio is known, the form quotient and form class of a tree can be determined.


### 5.4. Taper table and formulae

- It provides the actual form by diameters at fixed points from the base to the tip of a tree.
- Volume tables can thus be prepared from taper tables in desired unit.


## Use of taper table

1. Volume of the average tree for each diameter and height class can be found readily in office without direct measurement. The only measurement that will be needed is the $\mathrm{dbh}(\mathrm{ob})$ and the height of standing tree.
2. Volume table can be prepared from taper tables in desired units.

The ultimate purpose of all taper tables is to show upper stem diameters, which can then be used to calculate the volume of the sections of a tree and the entire tree. Taper tables can assume several forms.

## Types of taper table

1. Ordinary taper table or diameter taper table

- It gives the taper directly for diameter at breast- height without reference to the tree form.


## 2. Form class taper table

- This tables gives for different form classes the diameters at fixed points on the stem expressed as percentage of dbh (u.b)


## General formulae or equations for tree form

- Taper equation represent the expected diameter as a function of height above ground, total tree height and dbh irrespective of tree species and generalized for form class
- Many different forms of taper equations have been developed as no single one can adequately represent all species in all situations. The use of taper equations
allow us to obtain volumes for any desired portion of a tree stem by predicting upper stem diameters.


## Hojer's formulae

- It determines the diameter quotient (i.e the ratio of the diameter of a stem at any given height to its breast-height diameter) for each form class
- It gives the percentage of the length of the tree between breast height and top
$\frac{d}{d b h}=C \log \frac{c+l}{c}$
Where,
$d=$ Diameter at any point on the stem
C \& c = Constant for each form class
$1=$ Distance from the top of the tree to the point at which $d$ is measured


## Behre's formulae

$\frac{d}{d b h}=\frac{1}{a+b}$
Where,
$d=$ Diameter at any point on the stem
$\mathrm{a} \& \mathrm{~b}=$ Constant for each form class, $\mathrm{a}+\mathrm{b}=1$
$1=$ Distance from the top of the tree to the point at which $d$ is measured

- This formulae is more consistent


## Bark Measurement

- The thickness of the bark and its percentage of volume in the tree or log are important parameters in mensuration because most measurements on standing trees have to be made on over bark.
- Some species have very thick bark. In general, bark thickness varies with: species, age, genotype, rate of growth and position in the tree
- The bark thickness of the living tree may be measured with little damage to the trees using a Swedish Bark Gauge

Bark Percent $=\frac{V_{o b}-V_{u b}}{V o b} * 100$
Where,
$V_{o b}=$ Volume over bark
$\mathrm{V}_{\mathrm{ub}}=$ Volume under bark

## Unit 6 Volume estimation

## Unit Structure

### 6.0 Learning Objectives

## 6.1: Introduction

6.2. Tree Volume
6.3. Volume of Felled Trees
6.3.1. Cross-cutting the tree stems in logs
6.3.2. Calculation of volume of logs
6.3.3...................................................... Calculation of volume by quarter girth formula
6.3.4....................................................................................Solid volume of firewood
6.4. Volume of standing trees
6.5. Volume Tables
6.5.1. Variables
6.5.2. Classification of volume tables
6.6. Method of preparation of volume tables
6.6.1. Graphical Method of Preparation of General Volume Tables
6.6.1.1. Checks
6.6.1.2. Principal sources of error in graphical method
6.6.2. Regression Equation Method or Method of Least Squares Fit
6.7. Preparation of Local Volume table

### 6.0 Learning Objectives

After studying this unit you are able to understand about:

- Volume estimation
- Volume of felled and standing trees
- Volume tables and its types
- Method of preparation of volume tables


## 6.1: Introduction

The ultimate object of all mensurational activity in forest is to calculate or estimate quantity of wood contained in trees and Consequently in crops not only for sale but also for research, predicting future yields, estimating increment to assess return on capital, etc. Volume has been the traditional measure of wood, quantity and continues to be the most
important measure inspite of increasing use ). 1 weight or biomass as a measure of forest productivity.

### 6.2. Tree Volume

In case of trees, the volume may be calculated after the tree has been felled or when it is standing. As different methods are used in calculating or estimating volume of trees when felled and standing, they are being discussed separately.

### 6.3. Volume of Felled Trees

Volume of felled trees consist of volume of stem wood, branch wood, and root wood where it may be valuble and its digging is permitted. As stem provides most of the timber in the tree, calculation of its volume will be dealt with relatively in greater detail

### 6.3.1. Cross-cutting the tree stems in logs

As the tree tapers irregularly, it is usual to cut the tree into logs. The lengths of the logs depend upon the rate of taper and market requirements As the diameter at the thin end of the log determines the sawn volume that can be taken out of it, the greater the rate of taper, the lesser is the length of the log. But in certain cases, higher rate for a longer timber is so profitable that longer lengths of logs are cut inspite of decrease in converted volume. Another consideration that affects the length of a log is the mode of transport, Carts usually transport much longer logs than the trucks can do and so when truck transport is to be adopted, logs are generally not longer than 4.5 m or so. But when logs are made for calculating volume of felled trees for research work, all logs including the first are uniformly 3 m in length except the top end $\log$ which may be upto 4.5 m . But if the end section is more than 1.5 m in length, it is left as a separate log.

### 6.3.2. Calculation of volume of logs

It has been mentioned id chapter 4 that the basal portion of a tree corresponds to a frustum of neiloid, the middle portion to a frustum of paraboloid and the top portion to a cone if uncut, and to a frustum of a cone if it is cross-cut at some point. The following table gives formulae for calculation of volume of the three solids of revolution together with the formula for cylinder for comparison:

|  | Form of Solid | Volume of full solid | Volume of a <br> frustum of solid | Remarks |
| :---: | :---: | :---: | :---: | :---: |
|  | (1) Cylinder | sl | SI |  |
|  | (2) Paraboloid | $\frac{s l}{2}$ | $\frac{S 1+S 2}{2} \times I$ <br> (ii) $S_{m} \times I$ | Smalian's formula <br> Huber's formula |
|  | (3) Cone | $\frac{s l}{3}$ | $\frac{(S 1+S 2+\sqrt{(s 1+s 2)}}{3} \mathrm{xl}$ |  |
|  | (4) Neiloid | $\frac{s l}{4}$ | $\frac{(S 1+4 S m+s 2}{6} \mathrm{x}$ | Prismoidal or <br> Newton's formula |
| Where | $s$ is the sectional area at the base in square units, $\mathrm{s}_{1}$ is the sectional area at the thick end in square units, $\mathrm{s}_{\mathrm{m}}$ is the sectional area at the middle in square units, $s_{2}$ is the sectional area at the thin end in square units, and I is the length of the log or height of the solid in linear units. |  |  |  |



Fig. 1: Figure to illustrate formulae used to calculate volumes of various solids or their frustums

Out of the formulae mentioned above for frustums, Newton's formula is the most accurate.. It gives the volume not only of frustum of a neiloid correctly but also those of frustums of cylinder paraboloid and cone and all other formulae can be derived from it. But it is a cumbersome formula to use as it necessitates the measurement of diameter and calculation of areas of three cross-sections. This formula is difficult to apply particularly when the logs are stacked and the mid-diameter cannot be measured. Hence, Newton's formula is not used in practice. It is only used to calculate the error in volume calculated by other formulae.

Smalian's formula, which requires the areas of end cross sections, overcomes the difficulty of measuring mid-diameter of logs when they are stacked. It is easier to measure the diameters of end -cross-sections of logs with a wooden scale than to measure middiameter with a tape or calliper when the log may be lying on ground. Thus Smalian's formula is easier to apply as compared to Huber's or Newton's formula. It gives volumes of frustums of cylinders and paraboloid with absolute accuracy. But as the tree form is not exactly a frustum of a paraboloid and lies between it and the frustum of a cone, having slightly convex sides, Smalian's formula over-estimates the volume. The difference between volumes obtained by Smalian's and Newton's formula is:

$$
\begin{gathered}
\left(\frac{S 1+S 2}{2} \times I\right)-\left(\frac{(S 1+4 S m+s 2}{6} \times I\right) \\
=\frac{1}{6}\left(3 s_{1}+3 s_{1}-S_{1}-4 S_{m}-\mathrm{S}_{2}\right) \\
=\frac{1}{6}\left(2 \mathrm{~s}_{1}+2 \mathrm{~s}_{2}-4 \mathrm{~S}_{\mathrm{m}}\right) \\
=\frac{1}{3}\left(\mathrm{~s}_{1}+\mathrm{s}_{2}-2 \mathrm{~S}_{\mathrm{m}}\right)
\end{gathered}
$$

In case of cylindrical and paraboloidal shapes, $\mathrm{s} 1+\mathrm{s} 2$ is equal to $2 \mathrm{~S}_{\mathrm{m}}$ and so there is no difference. But in case of cone and neiloid $2 \mathrm{~S}_{\mathrm{m}}$ is less than $\mathrm{s} 1+\mathrm{s} 2$ and so there is a positive error. Huber's formula requires the measurement of mid-cross-sectional area of a log. It is therefore impossible to use it when the logs arc 'stacked without incurring extra expenditure in separating them. Though this formula also gives the volumes of frustums of paraboloid and cylinder with absolute accuracy, it under-estimates the volumes of logs because they lie between frustums of paraboloid and frustums of cone. The following
calculation which shows the difference' between Huber's and prismoidal formulae, substantiates it:

In case of a cylinder and a paraboloid $s 1+s 2$ is equal to $2 S_{m}$ and so there is no difference. Huber's formula is more accurate than the Smalian's formula. It is also more accurate than the Smalian's formula because the effect of the root swell does not vitiate the result in the lower-most log.

### 6.3.3. Calculation of volume by quarter girth formula

In India and Britain, volumes of logs are often calculated by the quarter girth formula :
Volume of $\log =\left(\frac{g}{4}\right)^{2} \times$ I
where $g$ is the girth of the log at the middle, and $I$ is the length of the log.
It is also known as Hoppus's rule in Britain, after the name of Hoppus, who first prepared tables for use with it. The formula has been evolved as follows :

Volume $=\pi r^{2}$ I but $g=2 \pi r$ and so $\quad r=\frac{g}{2 \pi}$
Volume $=\pi\left(\frac{g}{2 \pi}\right)^{2} I$, substituting the value of $r$ in terms of

$$
\left.g=\pi \frac{g 2}{4 \pi 2} \right\rvert\,=\frac{g 2}{4 \pi} l
$$

$\pi$ is actually equal to 3.14 but it has been roughly taken to be equal to 4
Therefore volume $=\frac{g 2}{4 \times 4} l=\left(\frac{g}{4}\right)^{2} \times 1$
As an approximation has been made in the value of $\pi$ from 3.14 to 4 . The volume drive by this formula is lesser than the true volume. The following comparison of the volume by quarter girth method and the full circular method shows that the former gives only $78.5 \%$ of the true volume:

Full circular volume $\mathrm{V}_{1}=$ Volume $=\frac{g 2}{4 \pi} \mathrm{XI}$,
Quarter Girth formula $\mathrm{V}==\left(\frac{g}{4}\right)^{2} \times 1$ or $=\left(\frac{g 2}{16}\right) \times 1$
Thus $\frac{V}{V 1}=\quad \frac{4 \pi}{16} \quad=0.785$ or $\quad 78.5 \%$

### 6.3.4. Solid volume of firewood

The stacked volume is not the actual volume of firewood. It is only for the convenience of paying the labour in the forest where there is no arrangement for weighment that the stacked volume is calculated. This can be done by any of the following methods:

## (i) Xylometric method:

The volume of billets can be calculated with the help of a xylometer which consists of a graduated vessel and volume of wood is calculated by the principle of water displacement. Water is poured in the vessel and reading of water level is taken. The pieces of wood are submerged and the reading is again taken. The difference between two readings gives the volume of the pieces of wood submerged. To obviate the necessity of submerging large quantities of wood, the whole stack is first weighed and only a portion is submerged. Let W be the weight of the whole stack of wood an]. w the weight of submerged pieces, V the volume of the former and $v$ that of the latter, then,

$$
\begin{aligned}
& \text { W:w:: V:v } \\
& \mathrm{V}=\frac{W x V}{w}
\end{aligned}
$$

This method is, however cumbersome and seldom used in practice.

## (ii) Specific gravity method

If the specific gravity of wood is known then volume can be calculated from the weight of the billet. Specific gravity (sp. gr.) of a piece of wood
$=\frac{\text { weight of wood }}{\text { Weight of the same volume of water }}$
$=\frac{\text { Density of wood }}{\text { Density of water }}$
As density of pure water is 1 gm per cc, the density of wood in gms per cc is the same as its specific gravity minus the units.

Volume $=\frac{\text { Weight }(\text { in gms })}{\text { Sp.gr. }} \mathrm{Cc}$
In F.P.S. System, Volume $=\frac{\text { Weight (in lbs.) }}{\text { Sp.gr.X } 62.5} \mathrm{cft}$

### 6.4. Volume of standing trees

The processes of management and sales frequently-require estimation of volume of trees without felling them. It is-therefore necessary to know the methods by which volume of standing trees can be estimated. The following are some of the methods used :
(1) Ocular estimate-The first possible method of estimating the volume of standing trees is the ocular estimate) Experience\& persons can make fairly accurate estimate of volume of standing_ trees marked in felling lots by careful inspection on the basis of-records of past fallings. But this method is too subjective forreliable results. Not only do different workers produce different results for the same tree but even the same worker may estimate differently under the influence of fatigue, hunger, etc. All the same every forester must train his eye to make a fair estimate of the; cubical contents of standing trees. To be approximately correct the estimator requires great practice and occasional opportunity to-compare his estimate with the actual measurement after the trees have been felled. Even then the results are subject to considerable errors.
(2) Partly ocular and partly by measurement-The uncertainty of purely ocular estimate can be overcome by measuring the diameter and height of the tree and then estimating the volume keeping--the tree taper in view. In this method, the estimator estimates the-diameters of subsequent logs after the basal log and from them the volume till whole length is covered. This method also requires lot-of experience and practice without which estimates May not be-correct.
(3) Direct measurements-The unreliability of the ocular estimate is completely removed if the diameters of tree at different heights are measured by a man climbing the tree with the help of ladder to some height and thereafter by his own effort. This method then becomes similar to the .method of calculating volume of trees. After they have been felled, with the difference that in this case the-' tree is not felled but a man climbs it and measures the diameters at different heights on the tree. This method, is however, very time consuming and tiring and cannot be used, when large number of trees are involved.
(4) Indirect measurements - With the development of optical dendrometers, it is no longer necessary to climb trees for measurement of volume. These instruments are Spiegel Relaskop, Tele Relaskop, Wheeler Pentaprism calliper and Barr and

Stroud dendromete. The principle on which these instru-ments, are based and the methods of measuring upper stern diameters.

### 6.5. Volume Tables

Volume table is defined as a table showing for a given species the average contents of trees,: logs or sawn timber for one or more given dimensions. The given dimensions may be (i) d.b.h. alone, (ii) d.b.h. and height or (iii) d.b.h., height and some measure of form or taper. The main object of these tables is to estimate the volume of an average standing tree of known dimensions and thus to estimate the volume of a given crop or of marked trees in a. given coupe. These tables are based on the actual measurement of sufficiently large number of trees and have been prepared on the assumption that the trees of the same species with the same dimensions will have the same volume. The scope of applicability therefore depends on the sources of measure-17nents on which the averages given in the volume table are based: Consequently before using a table, it is necessary to see if the table is applicable to the locality where it is being used. Even then the volume table does not give exact volume of an individual tree because the volume of the individual tree may be different from the average based on several individuals. Therefore the volume table approach can be truly applied only to a group of trees, e.g., coupe, but not to individual trees.

### 6.5.1. Variables

The volume of a tree depends mainly upon -three variables, viz., (i) diameter, (ii) height and (iii) form. The choice of variable or variables on which to base the tables depends on the extent of their intended application, simplicity, speed with which they have to be applied and desired accuracy, For a restricted area only one variable, e g., diameter, may serve the purpose. In a small area trees of the_ same, diameter can be expected to have the same height and form and therefore the same value. But this does not hold good over larger areas where wide differences in heights and form may be present. in such cases, at least two variables should form the basis of compilation of volume tables. Volume tables based on a single variable are the easiest to use. More than one variable not only complicates the use of the table but also requires more time for measurement of the two variables. But volume tables based on a single
variable are not accurate if these are applied to larger areas because the assumption that the trees of the same diameter will have the same height and form and consequently the same volume does not hold good. Therefore volume table applicable to larger areas should be based on at least two variables of height and diameter. Volume tables based on three variables are more accurate but they need more measurements for applying them in the field. As the advantage expected of a volume table is the ease and rapidity with which volume of trees can be determined, there seems no advantage in the use of a table which requires measurement of diameter, height and form. Diameter at breast-height is the most important of the three variables. -Next to this comes the height and last of all the form.

### 6.5.2. Classification of volume tables

Volume tables can be classified in three ways, viz., (i) according to the number of variables on which they are based, (ii) according to the scope of their application and (iii) according to the kind of outturn given by them.
(I) Classification on the basis of the number of variables-The volume tables can be classified into following kinds according to the number of independent variables on which these are based :
(a) Volume tables based on one variable, viz., diameter, alone - In this kind of volume tables, the trees are classified by d.b.h. (o.b.) only. Since it is cumbersome and at the same time not very useful to show average volumes against individual diameters, diameter classes are used 'Thus these volume tables show average volumes of trees by diameter classes. Since height variation in the same diameter may be frequent in different localities, such volume tables cannot be used for extensive areas. They can be used only locally and hence these are called local volume labels) These are easy and quick to use as they require the measurement of only(d.b.h4 (o.b.).
(b) Volume tables based on two variables, i.e., diameter and height-As trees of the same diameter may have different heights and different volumes in different localities, volume tables based on these two variables, are applicable to larger areas. Thus, these volume tables give volumes of trees by diameter classes as well as by height classes pertaining to the total height of the trees. Sometimes instead of giving height classes for
the total height, these give volumes for different height classes of the merchantable bole. As total height is taken to be an index of site quality of a locality, these, sometimes, give volumes by diameter classes and site qualities provided the quality classes for that species have been differentiated.
(c) Volume tables based on three variables-These tables are based on diameter, height and form quotient and are called the form class volume tables. Though these are more accurate, these are expensive and difficult to prepare and inconvenient and timeconsuming in their application. Such tables have not been prepared in India. They have, however, been prepared in Sweden and England.
(II) Classification on the basis of scope of application-On the basis of the scope of their application, volume tables are classified as under:
(a) General volume tables: These volume tables are based on the average volume of trees growing over a large geographical area. These are therefore applicable to a wider range of distribution of the species. These tables are usually based on two variables, viz., diameter at breast-height and total tree height. These tables show volumes of trees by diameter classes and in each diameter class by height classes.. As these are based on measurement of trees growing in wider range of distribution of the species and as they require measurement of diameters and heights for their application these tables have a limited direct application. These tables are used for deriving local volume tables. Several kinds of general volume tables are met with depending on the kind of volume they show, eg. Standard volume table, commercial volume table, sawn outturn table, assortment tables, etc. These will be described in detail later.
(b) Regional volume tables-These are compiled from measurement of trees growing in a region and therefore have a limited application when compared to general volume table.
(c) Local volume table-these volume tables are compiled from the measurements ot trees growing in restricted locality. These are generally based on one independent variable, viz., d.b.h. (o.b.). These are, therefore, applicable to such restricted localities as a coupe or a compartment because in such small areas, the assumption that the trees of the same diameter will have the same height, will hold good. The local volume tables are used for (i) estimating the volume of standing trees before felling with the object of regulating yield in accordance with working plan prescriptions, or (ii) for Making confidential estimate of the volume of a coupe. These are either prepared directly from field data or derived from general volume table, either by graphical method or by regression equation method.
(III) Classification on the basis of the kind of outturn-Volume tables are classified on the basis of the kind of outturn shown by them as under:
(i) Standard volume tables-These volume tables give separately the estimated outturn in the form of standard timber, i.e., from „round level to the limit of the portion of tree stem or branch where diameter is 20 cm measured overbark) and small wood, i.e., the volume between the diameter limits of 20 cm and 5 cm both measured overbark. The volume is-given in terms of round timber and includes volume of stump. It is calculated on full basal area basis from overbark diameters but standard stem timber excludes the volume of bark while standard small wood includes it.
(ii) Commercial volume tables-These are volume tables in which the contents of round timber' given as volume measured clown to a thin end diameter to which conversion is done, the stump -volume being omitted. As the diameter level to which conversion is done may vary with time and place, the scope of applicability of these tables is limited to areas having the same closeness of con-version and to certain periods of time when standard .of conversion is the same. As market requirements decide the minimum limit of exploitation, violent fluctuations in market render the table unfit for use. The chief use of this table is to serve as a basis for preparation of local and usually temporary tables for individual coupes.
(iii) Sawn outturn tables-These are volume tables in which contents of sawn timber are given as volume measured down to a thin end diameter to which conversion is done, the stump volume being omitted. Thus these are, similar to commercial volume tables except for the fact that these give volume of sawn timber instead or volume in round.
(iv) Assortment tables--These arc volume tables which give volume in round down to various stated thin end diameters. For example, it will be possible to find out volume of a tree of given. Linear dimensions when the conversion was done up to $25 \mathrm{~cm}, 20 \mathrm{~cm}$ or 15 cm diameter limit. Thus the standard volume table and commercial volume table are special cases of assortment table.
(v) Sawn outturn assortment tables-These tables are similar to assortment tables except that they give sawn outturn in the number of standardized pieces instead of volume in round.

### 6.6. Method of preparation of volume tables

The volume tables are prepared by the following methods :
(i) Graphical method;
(ii) Regression equations method or the method of least squares fit; and
(iii) Alignment chart method.

The first two of these methods, which are commonly used in This country are described below:

### 6.6.1. Graphical Method of Preparation of General Volume Tables

(A)Selection of trees-For preparation of volume tables the first important thing to do is the selection of trees. Trees of typical height and development are so selected throughout the entire range and distribution of the species that they may be evenly distributed over the entire area. Trees with defects other than those which are common in the species, e.g., fork, broken top, etc., are not selected: Selection of trees is very important and is done very carefully. A smaller number of trees suitably selected gives better result than a much larger number with improper selection, such as conscious or unconscious selection of trees of one type. Care is taken not to select trees above the average.

The number of trees required depends upon (i) the grouping adopted, (ii) the precision required and (iii) the deviation of the individual tree volumes from the mean in each group. In India, the Silviculture Research Code regards 1000 trees as a satisfactory number provided that the whole range of diameters and heights is fairly evenly covered, though some of the volume tables prepared in Germany have been based on 40,000 trees. In actual practice, how-ever, the Indian volume tables have been based on a much smaller number. For instance, the Volume Table and Diameter Growth Curve for Picea smithiana by Seth and Lohani (53) is based only on 211 trees.
(B) Measurements-Measurements vary with the kind of general volume table to be compiled. For example, standard volume tables require measurements of standard timber and small wood while. the commercial volume tables require measurements of commercial timber. In the case of standard timber volume, standard timber and standard small wood volumes of a felled trees are measured by dividing the total length into sections of 3 m length. The data of individual trees is then summarized and finally grouped into height and diameter classes in a table called table of basic average of which a specimen is given below:

| Diamete <br> r classes <br> (cm) | Height class in meters |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5.1-10.0 |  |  |  | 10.1-15.0 |  |  |  | 15.1-20.0 |  |  |  |
|  | Dia. <br> (cm ) | $\begin{aligned} & \hline \text { Heigh } \\ & \mathrm{t}(\mathrm{~m}) \end{aligned}$ | $\begin{aligned} & \hline \text { Vol } \\ & \left(\mathrm{m}^{3}\right. \\ & \text { ) } \end{aligned}$ | No <br> of <br> tree <br> s | Dia. <br> (cm <br> ) | $\begin{aligned} & \text { Heigh } \\ & \mathrm{t}(\mathrm{~m}) \end{aligned}$ | $\begin{aligned} & \hline \text { Vol } \\ & \left(\mathrm{m}^{3}\right. \\ & \text { ) } \end{aligned}$ | No <br> of <br> tree <br> s | $\begin{aligned} & \hline \text { Dia. } \\ & \text { (cm } \\ & \text { ) } \end{aligned}$ | $\begin{aligned} & \text { Heigh } \\ & \mathrm{t}(\mathrm{~m}) \end{aligned}$ | $\begin{aligned} & \hline \text { Vol } \\ & \left(\mathrm{m}^{3}\right. \\ & \text { ) } \end{aligned}$ | No <br> of <br> tree <br> s |
|  |  |  |  |  |  |  |  |  |  |  |  |  |

Curves-The values recorded in the table of basic averages by diameter and heights are then plotted in the following three successive steps and smooth curves are drawn.

Step 1—Average diameter is plotted against corresponding average volume separately for each height class using distinctive height class marks such as $+,{ }^{*}, 0$, etc. The number of measure-ments forming the basis of each average is also noted against the point.

Smooth curves are then drawn for each height class and then harmonized with reference to one another. Volumes against the middle of each diameter class are then read from the curves and tabulated

Step I a—Average heights given in the table of basic averages ale plotted against corresponding average diameter separately for each height class using distinctive marks for each. The number of measurements forming the basis of each average is noted against each point. Smooth curves are then drawn for each height class and harmonized with reference to each other. Average heights against middle of diameter classes are then read from the curves and tabulated.

Step 2-The average volumes tabulated in Step 1 are then plotted against corresponding average heights tabulated in step 1 a separately for each diameter class using distinctive marks for each class. Smooth curves are then drawn and harmonized. Volumes are then read from the curves against the middle of each height class and tabulated.

Step 3- The volumes read from curves in Step 2 are again plotted against the middle of diameter classes separately for each height class. Smooth harmonized curves are drawn through these points. Final volumes are read from these curves against middle of diameter classes and tabulated.

### 6.6.1.1. Checks

In order to check the accuracy of the volume table so compiled, the following checks are applied : (a) Aggregate check—The actual volume of trees measured should be checked against the total volume read from the final curve for the actual average diameter and interpolated actual average height both available from the table of basic averages. The difference should not exceed one percent.
(b) Height/diameter class check-This check is merely an aggregate check applied to each diameter and height class. The difference between the total volume obtained from field data and corresponding volumes read from the curves against actual average diameter and interpolated actual average height, multiplied by the number of the trees in the class should not differ by more than $5 \%$. There should be at least 20 trees in the class for the check to be reliable.
(c) Relative check-When two or more tables are derived independently from the same data, they should be checked against each other, e.g., local volume table prepared directly from the given data and the local volume table derived from the general volume table with the help of the same data. The difference should not exceed $3 \%$.
(d) Average deviation check-The average deviation of actual individual tree volumes from those read from the curve is computed and should be as low as possible. This is a very laborious check and is not applied except in cases of exceptional importance.

### 6.6.1.2. Principal sources of error in graphical method

The following are the principal sources of error in preparing general volume table by graphical method
(I) Failure to, select proper trees in sufficient number
(II) Only two variables can Inc; dealt with at, a time. Where more than two variables are involved data has to be separated in groups formed on some artificial definition: This introduces a bias. Any change in the formation of groups changes the final curves. If the number of independent variables exceed three, even the method of grouping cannot give satisfactory results,
(III) Drawing of curves always introduces a personal factor and consequent bias.

### 6.6.2. Regression Equation Method or Method of Least Squares Fit

In this method, while the basic data essentially remains the same as in the previous method, relationships between volume as dependent variable and d.b.h., height and form, etc., as independent variables arc given mathematical expressions by a regression equation. Various workers have developed various equations or models. Some of these are given below:

## (1) Arithmetic standard or non-form class equations

| Naslund | $\mathrm{V}=\mathrm{a}+\mathrm{b}+\mathrm{D}^{2}+c \mathrm{D}^{2} \mathrm{H}+\mathrm{dH}^{2}+e \mathrm{DH}^{2}$ |
| :--- | :--- |
| Comprehensive | $\mathrm{V}=\mathrm{a}+\mathrm{b}+\mathrm{D}+\mathrm{c} \mathrm{DH}+\mathrm{dD}^{2}+e \mathrm{eH}+\mathrm{f}^{2} \mathrm{H}$ |
| Meyer modified | $\mathrm{V}=\mathrm{a}+\mathrm{b} \mathrm{D}+\mathrm{c} \mathrm{DH}+\mathrm{dH}^{2}+e \mathrm{D}^{2} \mathrm{H}$ |
| Austrian | $\mathrm{V}=\mathrm{a}+\mathrm{b} \mathrm{D}+\mathrm{c} \mathrm{DH}+\mathrm{dH}^{2}+e \mathrm{D}^{2} \mathrm{H}$ |
| Combined variable | $\mathrm{V}=\mathrm{a}+\mathrm{bD} \mathrm{D}^{2} \mathrm{H}$ |
| Constant form factor | $\mathrm{V}=\mathrm{aD} \mathrm{D}^{2} \mathrm{H}$ |

(2) Arithmetic form class equations

Combined variable from class

$$
\mathrm{V}=\mathrm{a}+\mathrm{b} F+c \mathrm{D}^{2} \mathrm{H}+\mathrm{dFD} \mathrm{D}^{2} \mathrm{H}
$$

Shortcut form class

$$
V=a+b D^{2} H
$$

## (3) Logarithmic standard or non form class equations

Schumacher

$$
\log V=\log a+b \log D+c \log H
$$

Dwight
$\log V=\log a+b \log D+(3-b) \log H$
Logarithmic
$\log \mathrm{V}=\log \mathrm{a}+\mathrm{b} \log \mathrm{D}^{2} \mathrm{H}$

### 6.7. Preparation of Local Volume table

The local volume tables which give tree volume in terms of c. b.h. only, may be prepared by one of the following methods: (i) Graphical methods, or (ii) Regression equation methods.

In both of these above methods local volume tables may be prepared either directly from basic data of diameter and volume of trees or derived from the general volume tables of that species.

## (a)Graphical method based on basic data collected from field:

In this method, the diameters at breast-height (o.b.) and volumes of sufficiently large number of tree are measured and recorded by diameter classes. The average volume for each diameter class is worked out and it is then plotted against the mean diameter of the class and smooth curve is drawn through these points. As this method requires calculation of volume of a large number of trees it is seldom followed because it is easier to derive local volume tables from a general volume tables which are now available for most of the important tree species.

## (b)Derivation of local volume table from general volume table by graphical method

The general volume table gives volume of a tree by diameter and height classes. These are generally based on data of trees collected from a wide range of distribution. For using general volume table it is necessary to measure d.b.h. and height of individual trees. Local volume table is derived from a general volume table with the help of measurement of d.b.h. and height of some standing trees in the locality. The procedure of preparation of a local volume table from general volume table is as follows The figures of a general volume table are plotted on a graph paper showing volumes against the middle of diameter classes for each height class separately. Thus there will be the same number of curves as the number of height classes and in order to distinguish them, they should be given a number equivalent to the middle of the height class to which it pertains.
(C) Regression equation from the basic field data directly-In this method some regression equations or models are first prepared From the basic field data and out of these models, the model which fives results of desired accuracy is selected for preparation of local volume table. For example, the senior author developed following models for preparation of local volume tables:

$$
V=a+b D+c D^{2}
$$

$$
\text { And } \sqrt{V=} \mathrm{a}+\mathrm{bD}
$$

where V is the volume in $\mathrm{m}^{3}, \mathrm{D}$ is the overbark diameter of tree at breast-height in $\mathrm{m}, \mathrm{a}$ is the regression constant, and b and c are regression coefficients. The first equation is a parabola. This form has one maxima or a minima and at either side of this point we may find this equation to give lower volume for higher diameter class trees.

## Unit-7 Stand Growth and Yield tables

## Unit Structure

### 7.0 Learning objectives

### 7.1 Introduction

7.1.1 Stand Growth
7.1.2 Stand Structure
7.1.3 Growth of Stand
7.2 Methods of Determining Past Growth of Stand
7.3 The Method of Control
7.3.1 Continuous Forest Inventory Method
7.4 Method of Predicting Future Growth of Stands
7.4.1 Stand Density
7.4.2 Canopy Density
7.4.3 Crown Competition Factor
7.4.4 Site Quality
7.4.5 Fractional Quality
7.5 Yield Table
7.6 Kinds of Yield Table
7.7 Preparation of Yield Table
7.8 Application and Use of Yields Table
7.9 Money Yield Table

### 7.10 Stand Table

7.10.1 Application and Uses of Stand Table
7.11 Yield Table for Mixed Forest

### 7.12 Biomass Measurement

### 7.0 Learning objectives

After studying this unit you will be able to understand about:

- Stand growth, site quality, site index and stand structure
- Yield tables and preparation of yield tables
- Biomass measurement


### 7.1 Introduction

### 7.1.1 Stand Growth

The growth of the stand depends upon the volume of trees growing in it. As it is not possible to calculate the volume of each and every tree in a stand, different methods of calculate the volume of each and every tree in a stand, different methods of calculating volumes of sample plots carefully laid out throughout the geographical distribution of species. As the estimate of the forest population was
obtained by sampling, it was necessary to have some knowledge of statistics. Thesemethods are sufficient for estimation of growing stock but are not enough for forecasting future yields. For this purpose we require tables which may give yield of stand on unit area basis for the greater part of its useful life. Yield of a forest depends on several factors such as its structure, growth, density, the productive capacity of the site, etc. It is, therefore, necessary to have some knowledge of these aspects of forest crops.

### 7.1.2 Stand Structure

Stand structure defines the distribution and representation of age and / or size classes of trees in a stand. It keeps on changing with passage of time or age and at particular period in its life, it is the result of growth habits of the species constituting stand, the factors of locality and above all, the management practices under which it has originated and developed. The stand structure of the properly managed forests can be broadly classified into two groups, viz., even-aged and uneven-aged.

The even-aged stand is a stand consisting of trees of approximately the sameage. The period required to regenerate a stand under different methods of regeneration varies from 1 or 2 years to about 20 to 30 years, depending on the system of management and the species concerned; differences in age upto $25 \%$ of the rotationage are allowed in cases where the stand is not harvested for 100 years or more. Though the trees are nearly of the same age-class, competition for light and moistureresults in crown differentiation with the vigorous trees occupying dominant positions, the less vigorous they dominated position and those left behind in the struggle occupying suppressed position. This results in wide range of diameters found in a stand at a particular age. The pattern of diameter variation naturally varies with age and is not the sameat the rotation age as it is in the early part of its life. Studies made by various workers indicate that the diameter distribution in an evenaged stand follows definite laws and the relationship between number of trees and diameter can be described by computed data. These studies have also shown that from the point of view by statistical analysis of estimated parameters of even-aged stand are more closely related to averagediameter of the stand than to its age or a measure of the site quality though both these have a profound effect on the diameter distribution.

The uneven-aged stand is a stand in which individual stems vary widely in age, the range of difference being usually more than 20 years and in case of long rotation crops, more than $25 \%$ of the rotation. When nearly all age gradations or age classesare present in a particular area it is called a selection forest. Proper studies in structure of uneven-aged forests in India are wanting but it is clear that such forests do not followthe same pattern as that of even-aged forests. The proportion of trees of different diameter classes varies unsystematically from place to place. Generally, there is a preponderance of trees of higher diameter classes at the cost of trees of lower diameter classes. The shortage in the lower diameter class as a result of lack of regeneration and establishment gradually makes the structure more and more unbalanced. A French Forester De Liocourt during a study of diameter distribution in selection forests foundthat stem numbers in successive diameter classes had a fixed ratio within a stand asshown below:

$$
\mathrm{N}_{1} / \mathrm{N}_{2}=\mathrm{N}_{2} / \mathrm{N}_{3}=\mathrm{N}_{3} / \mathrm{N}_{4}=\mathrm{q}
$$

Where $N_{1}, N_{2}, N_{3}, \ldots .$. are numbers of stems in the successive diameter classes 1 to 4.

The ratio is called the De Liocourt-quotient. In other words, the number of trees in successive diameter classes are in a geometric series. If the number of trees are plotted against diameter classes, the resultant curve is of a laterally inverted J shape.

Meyer (1953) termed stands where the number of stems by diameter classes decrease in constant geometric progression as a balanced forest. Balance can apply to an uneven-aged forest as well as to the management unit of an even-aged forest where age classes are partially separated, i.e., they occupy separate areas. Balance exists if the decrease in the number of stems follows a geometric progression. In such a forestit is at least theoretically conceivable to remove the current growth annually or periodically while maintaining at the same time the structure and initial volume at leastas long as no major catastrophy befalls the forest.

### 7.1.3 Growth of Stand

As already stated, stand structure changes from year to year. These changes take place as a result of growth, death and cutting of trees. The growth of stand makesthem the biggest renewable natural energy resource and therefore numerous studies have been made to predict it, treating the stand as a unit as well as a population of trees.

Treating the stand as a unit, growth function models have been suggested by many workers.

### 7.2 Methods of Determining Past Growth of Stand

The usual method of determining past growth of stands is by repeated measurement of sample plots, as measurement of the whole stands is not a practical proposition. Even this could be done either by laying out temporary sample plots each time an estimate of growth is to be made or by laying permanent sample plots. The methods of laying out a temporary sample plot each time growth is to be determined results in measuring different sampling units each time and therefore the precision willbe poorer and the accuracy generally lesser. The method of repeated measurement in permanent sample plots is better and even where such sample plot is not representative of the stand, the precision will be high, though the growth estimate may be biased. The following two methods are commonly used for the purpose.
(i) The method of control
(ii) Continuous forest inventory method

### 7.3 The Method of Control

This method was originally devised by Gurnaud in France and later adopted in various European countries. As devised, the method requires a complete enumeration of the total stand. But this is possible only when the stand is small. Therefore in largerforests, representative sample plots are used. Thus, at the initial inventory, the entirearea, whether of stand or sample plot, is enumerated and trees are classified by diameter classes. Then the record of all trees removed till the next inventory is kept by diameter classes as well as the volume of trees removed. At the next inventory, the entire area is again enumerated. The volume of trees in different diameter classes iscalculated by the use of a local volume table and added up to obtain the volume at each inventory. Then the growth of stand in the period intervening between the two inventories is calculated by the following formula:

$$
I=V_{2}+V_{c}-V_{1}
$$

Where I is the periodic growth of stand
$V_{2}$ is volume of the sample plot or stand at the second inventory
$V_{c}$ is the volume of trees removed during the period intervening betweenthe first and second inventory
$V_{1}$ is the volume of the sample plot or stand at the first inventory
Local volume table used at the initial inventory is not changed at subsequent inventories. However, if the changes in diameter/height or form relationship in the crop over the years make it necessary to modify the volume table to reflect these changes, following correction factor may be applied:

## Actual volume of felled trees

C.F. $=$ $\qquad$
Volume of felled trees according to local volume table
The diameter increment by diameter classes under the method of control is calculated by an elaborate computation method which takes into account the number of trees rising into a diameter class, trees remaining stationery in the class, and the treesgoing out of the class in successive inventories. Increment in each diameter class is calculated by the formula:

$$
\begin{aligned}
& \qquad \qquad \frac{D R}{D E} \times C \\
& \text { And } \\
& \text { Average periodic increment for all trees }=\frac{\sum D R}{\sum D E} \times C \\
& \text { Where, } \\
& D R \text { stands for double rising which means the sum of trees rising out } \\
& \text { and risinginto a class } \\
& D E \text { stands for double effective which means the sum of trees in the } \\
& \text { first and second inventories after correction for trees removedC is width } \\
& \text { of diameter class } \\
& \Sigma \text { is signs, i.e., sum of all classes }
\end{aligned}
$$

For a specific diameter class, number of trees rising into the class is calculated as follows:

Tree rising $=$ No. of trees of $2^{\text {nd }}$ inventory -No . of trees of $1^{\text {st }}$ inventory +No . of trees rising out of the class

### 7.3.1 Continuous Forest Inventory Method

Continuous forest inventory (abb. CFI) method is an American modification ofthe European method of control to serve two specific needs:
(i) Use of sample plots and
(ii) Use of modern computing machines

Thus, the continuous forest inventory differs from the method of control essentially in having systematically distributed representative sample plots, periodically surveyed instead of having total area enumeration by diameter classes. While the plotshould be representative of the whole forest, they should also be representative of cutting, mortality and growth, etc., of the entire forest. In other words, these plots should undergo the same silvicultural operations as the rest of the forest. In this respect, the linear increment plots laid in certain forests in India, are an example of CFI. Generally sampling intensities range from 0.03 to 0.1 per cent using circular plot of about 0.50 ha to 0.08 ha. The centre of the plot is marked on the ground by an aluminum or wooden stake. The trees that are measured are numbered with paint or metal tags and place of diameter measurement is marked on them by paint cross mark. Diameter, height and volume of each tree so marked is measured at each inventory carefully and recorded in field on field-tally sheets and then transferred to punch cards. The measurements and re-measurement at each subsequent inventory are carefully checked by a checking crew to ensure error free card and nowadays machine checks arealso used in addition to field checks. From the study of growth in the sample plots, growth of the stand between the two inventories is estimated. CFI method can give more reliable growth information with respect to mixed forests. They provide basic datafor simulation studies also.

### 7.4 Method of Predicting Future Growth of Stands

The future growth of stands can be predicted with the help of yield tables prepared from the studies of growth in sample plots, systematically distributed over the entire forest. Growth of a stand is a function of stand density and site quality. Therefore, before describing yield table, method of its preparation and its uses, it is necessary to have knowledge of stand density and site qualities and their effect on growth prediction.

### 7.4.1 Stand Density

Stand density is a measure of the relative completeness of the tree stocking expressed as a decimal coefficient, taking normal number of trees, basal area or volume as unity. In India it is considered synonymous with terms crop density, density of stocking and stocking, though in America distinction is often made between stand density and stocking. According to Gingrich (29), stand is a quantitative measurementof a stand in terms of square feet of basal area, number of trees or volume per acre. Itreflects the crowding of stems within the area. Stocking, on the other hand, is a relativeterm used to describe the adequacy of a given stand density in meeting the management objective. Thus, a stand with a density of 70 square feet of basal area per acre may be classified as over stocked or under stocked are used to describe stand orcrop density if it exceeds, equals or is less than 1.0.

### 7.4.2 Canopy Density

Canopy density is a measure of relative completeness of canopy and is expressedas a decimal coefficient taking closed canopy as unity. Thus, canopy density describes only the relative completeness of the canopy and has no bearing with the crop volume, basal area or even number of trees or consequently with stand density. Yet for forests for which yield tables do not exist, it gives an indication, however rough, of the density. The following classification of canopy density is in vogue:
(i) Closed - when the density is 1.0
(ii) Dense - when the density is between 0.75 and 1.0
(iii) Thin - when the density is between 0.5 and 0.75
(iv) Open - when the density is under 0.5

### 7.4.3 Crown Competition Factor

Crown competition factor (abb. CCF) is another measure of density and is claimed to be independent of site quality and stand age. For determining CCF, relationship between crown width and dbh is obtained for a sample of open grown trees.
For example, Chaturvedi (13) found the following relationship between crown width and diameter in case of Shorea robusta.

$$
C W=-0.6117+20.812 \mathrm{D}
$$

Where CW is crown width D is dbh, both in m

Maximum crown area expressed as a percentage of one hectare is called maximum crown area (MCA) since it indicates the maximum proportion of one hectarethat the crown of trees of a given dbh can occupy. Thus,


$$
=0.007854(C W)^{2}
$$

Substituting the value of CW in the regression for Shorea robusta given above

$$
\begin{aligned}
\text { MCA } & =0.007854(-0.6117+20.812 D)^{2} \\
& =0.002938-0.2000 D+3.4019 D^{2}
\end{aligned}
$$

By adding MCAs for all trees on the average hectare of forest land, an expression of stand density called CCF is obtained. Thus, for the above example,

$$
C C F=1 / A 0.002938 \Sigma N_{i}-0.2000 \Sigma D_{i} N_{i}+3.4019 \Sigma D_{i} N_{i}
$$

Where $\mathrm{N}_{\mathrm{i}}=$ Number of trees in the $\mathrm{i}^{\text {th }} \mathrm{dbh}$ class
$D_{i}=$ Mid-point of the $i^{\text {ith }}$ dbh class
A = Stand area in hectares
$\Sigma=$ Sum
CCF estimates the area available to the average tree in the stand in relation tothe maximum area it could use, if it were open grown.

### 7.4.4 Site Quality

Site is the complex of physical and biological factors of an area that determinewhat forest or other vegetation it may carry. Site quality is a measure of the relative productive capacity of a site for a particular species. Every site gives a different growth response to different species. Knowledge of this inter-relationship can help a forester in utilizing this growth potential of a site to its optimum level. Site quality can be evaluated by measurement of either site factors or vegetative characteristics.
(i) Site factors - Site factors are the effective climatic, edaphic and biotic factors which influence the growth and development of forest or other vegetation in a locality. While it is obviously difficult to quantify the effect of all these factors together, the measurement of productivity based on one or a few of these factors makes the estimate too general and so not very reliable. This index is too broad-based as it does not take into account the
characteristics of soil and other environmental factors. An ideal index must take into account not only all the climatic factors but also the topographic, edaphic and biotic factors, if any. But, as already stated, it is extremely difficult to quantify variables relating to the various sub-factors of these major factors to enable development of a regression or any other equation to describe the site quality and this has been a major obstacle in these factors forming the basis of any useful site quality index.
(ii) Vegetative characteristics - In view of the difficulties described above in using site factors to form the basis of any efficient site quality index, the only alternative is to look for some characteristic of the vegetation, which may be suitably used as a basis to describe and differentiate site qualities. The characteristics of the vegetationwhich could serve this purpose could either be the types of plants occurring naturally in the area or the volume, basal area, diameter or heights of trees which form the dominant part of the vegetation in the area.
(iii) Plant indicators - One possible way to evaluate site quality is to use the composition of the vegetation and search for some characteristic plants which may becorrelated with site qualities. This approach is based on the theory that certain species of lower vegetation, i.e., herbs and shrubs are clear indicators of the suitability or otherwise of the site for a particular tree species or forest type. As such they can be used to describe and differentiate site qualities. This method has been used for site quality classification in certain undisturbed forests of Finland and Canada. Though this way suggest a meaningful correlation between plant indicators and site quality, yet the method cannot be adopted universally because of the following reasons:
(a) It can be applied only to forests of simple composition as exist in northern latitudes
(b) It requires considerable ecological knowledge
(c) The lower vegetation which may provide plant indicators is easily affected by forest composition, density and above all the biotic factors and
(d) Finally, the lower vegetation remains in the top few centimeters of the soil and so cannot indicate the conditions of the deeper layers of soil which also affect the tree growth considerably
(iv) Tree characteristics - The most important characteristics of the tree whichreflect the productivity of the site are its volume, basal area, diameter and height. Asthe primary object of site quality differentiation is to assess its productivity, the quantity of wood produced per unit area should therefore be the ideal measure of site quality butas this method requires prior calculation of volume, it is difficult to apply in practise.

The site classes are delimited by one of the following methods:
(a) Strip height method based on Baur's method
(b) British forestry commission (BFC) method
(c) FRI method

### 7.4.5 Fractional Quality

Fractional quality is defined as the site quality expressed as a decimal subdivision of the height range of half a quality class, the figures running 0.0 to 2.0 within the whole quality class.
(i) Importance of knowledge of site quality or fractional site qualityknowledge of the quality of the site is a pre-requisite for the prediction of growth and yield of a stand with the help of yield tables because they give the information by quality classes. Therefore it is not only necessary for the prediction of growth and yields of stand at any future time but also to determine the quality class or fractional quality classof a stand from the yield tables before using them.

### 7.5 Yield Table

Yield table is a tabular statement which summarizes on per unit area basis all the essential data relating to the development of a fully stocked and regularly thinned even- aged crop at periodic intervals covering the greater part of its useful life. It differs from the volume table in the sense that while the volume table gives the volume of an average tree by diameter and/or height classes, yield table gives different parameters ofa crop such as number of trees, crop height, crop diameter, crop basal
area, volume of standing crop, volume removed in thinnings, m.a.i., c.a.i., etc. In short, it gives all thequantitative information regarding development of a crop.
As stated in the above definition, yield tables is not applicable to uneven-agedforest because in its present form it has been compiled from even-aged pure crops and is therefore applicable to them alone. Yield tables have not been prepared so far for uneven-aged crops because of the difficulties involved and the main difficulty is that there is no one representative average age. Some tables have been prepared for suchcrops but they show the ratio of increment (current or mean annual) to help in deciding the policy of management.

### 7.5.1 CONTENTS OF YIELD TABLE

Though the contents of yield table can vary depending upon the circumstances of data collection or the objects with which it has been prepared, most of the yield tables in India give the following information by site qualities per unit area basis at interval of 5 or 10 years for the greater part of the useful life of the crop:

## Main Crop

(i) Average diameter
(ii) Average height
(iii) Total basal area
(iv) Number of trees
(v) Stem timber form factor
(vi) Total (stem and branch) small wood form factor
(vii) Standing volume stem timber
(viii) Standing volume total small wood
(ix) Total standing volume, i.e., stem and small wood

## Thinning

(i) Volume stem timber
(ii) Volume total small wood
(iii) Total volume

## Final Yield

(i) Volume stem timber
(ii) Volume total small wood
(iii) Total volume

## Accumulated Yield of Thinning

(i) Volume stem timber
(ii) Volume total small wood
(iii) Total volume

## Total Yield

(i) Volume stem timber
(ii) Volume total small wood
(iii) Total volume

## Mean Annual Increment

(i) Volume stem timber
(ii) Total volume

## Current Annual Increment

(i) Volume stem timber
(ii) Total volume

The above contents of the yield table can be divided into two classes:
A. Primary data comprises of volume of main crop (i.e., the crop left after thinning at that age), thinning at that age, final yield (i.e., the main crop and thinning at that age), accumulated yield of thinnings upto that age (i.e., thinning at that age andthe accumulated yield of thinning upto the previous age), total yield (i.e., final yield at that age and accumulated yield of thinning upto the previous age or main crop and accumulated yield of thinning upto that age).
B. Secondary data includes crop averages for diameter and height, number of stems for unit area, crop basal area, form factor, m.a.i. and c.a.i
In addition to the above information, yield tables give the following additionalinformation:
(i) Top height by site quality and age
(ii) Spacement table: (a) by age and site qualities
(b) by crop diameter and site qualities
(iii) Number of trees per unit area by crop diameter and site qualities
(iv) Normal growing stock of stem timber volume for 40 hectares (100 acres) under different rotations, in case more than one rotation is in vogue
(v) Stand table - It is table showing the distribution of stems by diameter classes for each of a series of crop diameters. It is a common practice in India to give stand table along with yield table.

### 7.6 Kinds of Yield Table

Yield table are further classified on the basis of the grades of thinning and whether the outturn is expressed in volume or value:
(a) On the basis of the number of grades of thinning used - On this basis, yieldtables are classified into:
(i) Single yield table - It is an yield table in which parameters have been given only for one grade of thinning which is usually c grade
(ii) Multiple yield table - These are yield tables in which data are given fordifferent grades of thinning

Thus these tables, area series of single yield tables. In India multiple yield tables have been prepared for Cedrus deodara only.
(i) On the basis of volume/value given - On this basis, yield tables are classified into:
(ii) Volume yield table - It is an yield table which expresses outturn in terms of volumes
(iii) Money yield table - It is an yield table constructed from volume yield table in which outturn is expressed in terms of money instead of volume

### 7.7 Preparation of Yield Table

The yield tables are prepared in a series of steps as described below:

1. After constructing top height/top age curve by site qualities by any of the methods described under site quality, all plots are assigned to quality classes with reference to the position of top height/top age points in the scatter diagram with limiting curves for site qualities. If the points of successive measurements of one and the same plot do not lie within the limiting curves of the same quality, the plot is assigned to one or the other quality on the basis of
the majority of measurements or on the basis of latest measurement. As a rule, the latter course is generally preferred.
2. Crop age and mean crop basal area per hectare figures of the plot measurements are grouped separately for each quality by decade or half decade age classes and their averages are computed. The average values of basal area are then plotted against the corresponding average values of crop age and a smooth freehand curve is drawn for each quality. Reading against the crop age values are taken and compared with the actual measurements. The standard deviation and coefficient of variation are/then computed and curved against crop ages. They are mutually checked and adjusted. The standard deviations are then read against crop ages. The differences between the curved and actual values of basal area of individual measurements are compared with the corresponding curved values of the standard deviation. Measurements in which the difference exceeds twice the standard deviation are rejected. Similarly, rejections on the basis of number of trees per hectare, average crop diameter, stem timber and stem small wood volume per ha are also done.
3. For each quality class, the main crop data of acceptable measurements are grouped by decade or half decade age classes and the following averages are computed.
(i) Basal area per hectare
(ii) Number of trees per hectare
(iii) Average crop diameter
(iv) Average crop height
(v) Stem timber and stem small wood volume per hectare
(vi) Stem timber and stem small wood form factor and
(vii)Top age and top height
4. The averages for each age class are plotted against average crop age and smooth curves are drawn separately for each quality. Necessary adjustments are then made between mutually related curves so that
(i) Number of trees per hectare x average basal area $=$ basal area/ha
(ii) Basal area/ha x crop height x form factor - Volume/ha
5. The values against the decade or half decade of age are read from these curves and tabulated. The top age and top height values read from top age/crop age and top height/crop height curves are interpolated on the top height/top age curves to determine fractional qualities at each decade or half decade. The values tabulated from the remaining curves are plotted against their respective fractional qualities and smooth curves are drawn for each decade or half decade of crop age. The mutually related curves are crosschecked and adjusted. The values are read against each mean sitequality and also against each limiting site quality and tabulated. These values are plotted against crop age and smooth curves are drawn for mean and limiting site qualities and the mutually related curves are cross-checked and adjusted again. Thevalues are read from these finally adjusted curves and tabulated.
6. The data for subsidiary crop is similarly curved and tabulated. This method of yield table preparation suffers from the following defects:
(i) The sample plot data are grouped in different site qualities. Many plots grouped in one site quality in early years are found to belong to another site quality in later years. Allotment of such plot to neither group is correct.
(ii) In each site quality there are varying number of points and the curves drawn for different site qualities do not show consistent trends
(iii) No sample plots are available for certain decades in some site qualities
(iv) A good deal of the sample plot data gets rejected

### 7.8 Application and Use of Yields Table

When yield tables are used to predict growth and yield or to supply any other information, care should be taken that the correct site quality table is sued and factor of density is applied. Thus before the yield table can be used knowledge of age of crop and its density is required. Age of the crop may be determined either from records
orby any of the methods suitable for finding age of stand. The density stand should be based on actual basal area which is quickly determined by instruments like wedge prism and comparing it with the basal area given in the yield table for the age and quality of the stand. The yield table, because of the lot of information contained in it, can be put to many uses in management or silvicultural practices. Some important uses of yield table are described below:
(i) Determination of site quality or fractional site quality
(ii) Estimation of total yield or growing stock
(iii) Determination of increment of stand
(iv) Determination of rotation
(v) Preparation of stock-map by site qualities
(vi) As a guide to silvicultural thinnings

### 7.9 Money Yield Table

Money yield table is a table constructed from a volume yield table in which theyields are expressed in terms of money instead of volume. Its main object is to determine best financial rotation. As the selling rate of timber varies with size, the volume yield table cannot be directly converted into money yield table by simple multiplication. The various steps by which money yield table is prepared are brieflygiven below:
(i) The volume per hectare for main crop and thinnings at different ages is obtained from the volume yield table
(ii) The number of trees of different sizes is found out for various ages from the stand table
(iii) A price-size gradient relationship is established between the crop age and its value in terms of money. Curves giving valuelage or value/dbh on per hectare basis are known as price-size gradients
(iv) With the help of this gradient, stand table data are converted in terms of money.
(v) Total value per hectare is found out by adding the values of all trees of different classes. These values are found out for main crop and thinnings
(vi) The value of the crop per cubic metre at different ages is worked out for main crop and thinnings
(vii) Finally money yield table data are tabulated in appropriate columns

### 7.10 Stand Table

Stand table is a table showing the distribution of stems by diameter classes for each of the series of crop diameters. In India, it is usually appended at the end of yield table of the species and the distribution is given in the following forms:
(i) As percentage of trees over given diameter limits in crops of various diameters
(ii) As number of trees per hectare (or acre) over given diameter limits, in crops of different qualities for successive 5 years age interval
(iii) As percentage of the total number of trees by 10 cm ( 4 inches) diameter classes in crops of various diameters
(iv) As percentage of the total number of trees by 10 cm (4 inches) diameter classes for various partial crop diameters excluding trees below a certain limit
(v) As number of trees by 10 cm (4 inches) diameter classes for 40 ha ( 100 acres) of normal forest under different rotations before the annual fellings.

Stand table is very useful for forest management and it is today generally accepted that yield table is not complete without it. The average diameter of the main crop stems and the stems removed in successive thinnings can be ascertained from the yield but the range of diameters comprised in these averages cannot be known unless stand table is also prepared. The calculation of the final values of forests, preparation ofmoney yield table and application of commercial volume tables made for local standards of conversion, to yield table crop data require a knowledge of trees per unit area underdifferent diameter classes and this is given by stand table.

### 7.10.1 Application and Uses of Stand Table

Being statistical averages, stand table may not be applicable correctly to individual cases unless they are normally stocked. In normally stocked crops, however, they give a clear idea of the distribution of different diameter classes and can be put toseveral uses, some of which are given below:
(i) Determination of the financial value of crops and
(ii) Preparation of money yield table

### 7.11 Yield Table for Mixed Forest

Mixed forests constitute the maximum percentage of the forests in India. No yield tables have been prepared for such forests. The value of regression coefficient $b$ in
the stand volume equations shows that it does not show wide variations. A precise relationship can be found between the volume and the product of basal area and crop height for mixed forests. Some work was done in this direction for plantation crops and promising results were obtained C.F.I. plots are required to be laid in mixed forests invarious forests types of enable studies being carried out on their growth. Chaturvediobtained the following regression equation for plantation crop of mixed species in Ramnagar Forest Division (U.P.).

$$
\begin{gathered}
\text { Volume }=2.6018+0.2417 \text { BA } x \text { crop } \\
\text { Height } r=0.8026 \text { d.f. }=21
\end{gathered}
$$

### 7.12 Biomass Measurement

Forest ecosystem plays very important role in the global carbon cycle. It stores about $80 \%$ of all above-ground and $40 \%$ of all below-ground terrestrial organic carbon (IPCC, 2001). During productive season, $\mathrm{CO}_{2}$ from the atmosphere is taken up by vegetation and stored as plant biomass (Losi et al., 2003; Phat et al., 2004). For this reason, the UNFCC and its Kyoto Protocol recognized the role of forests incarbon sequestration. Specifically, Article 3.3 and 3.4 of the Kyoto Protocol pointed out forest as potential carbon storage (Brown, 2002; United Nations, 1998).

When the vegetation decomposes, they release carbon back to the atmosphere. Disturbances in the forest due to natural and human influences lead to more carbon released into the atmosphere than the amount used by vegetation during photosynthesis (Brown, 2002). Sustainable management strategies are, therefore, necessary to make the forest a carbon storage rather than source. However, the state of tropical forests continues to deteriorate. Land conversion is the main reason for $93.4 \%$ of the annual net forest loss, while conversion to plantation forest explains the remaining $6.6 \%$. Land conversion resulted from forest mismanagement, such as, illegal forest practices and lack of sound policies and regulations for sustainable forestry (FAO, 2001).
FAO (2004a) defined biomass as "organic material both above-ground and belowground, and both living and dead, e.g., trees, crops, grasses, tree litter, roots etc.". Above-ground biomass consists of all living biomass above the soil including stem, stump, branches, bark, seeds, and foliage. Below-ground biomass consists of all living roots excluding fine roots (less than 2 mm in diameter). In forest biomass studies, two biomass units are used, fresh weight (Araujo, et al., 1999) and dry weight (Aboal et al.,

2005; Ketterings et al., 2001; Montagu et al, 2005; Saint-Andre et al., 2005). For carbon sequestration application, the dry weight is more relevant because $50 \%$ of it is carbon (Losi et al., 2003; Montagnini and Porras, 1998; Montagu et al., 2005). Many biomass assessment studies conducted are focused on above-ground forest biomass (Aboal et al., 2005; Brown, 1997; Kraenzel et al., 2003; Laclau, 2003; Losi et al., 2003; Segura and Kanninen, 2005) because it accounts for the majority of the total accumulated biomass in the forest ecosystem. This study also focuses on the assessment of above-ground forest dry biomass.

Biomass assessment is important for many purposes (Parresol, 1999; Zheng et al., 2004). It is aimed at two major objectives: (1) for resource use and (2) for environmental management. It is important to determine how much fuel wood or timber is available for use. Thus, one needs to know how much biomass is available at one given time. In environmental management, biomass quantification is important to assess the productivity and sustainability of the forest. Biomass is also an important indicator in carbon sequestration. For this purpose, one needs to know how much biomass is lost or accumulated over time. Consequently, the amount of carbon sequestered can be inferred from the biomass change since $50 \%$ of the forest dry biomass is carbon (Losi et al., 2003). The Kyoto protocol requires transparent reporting of forest removal and accumulation (biomass change). This implies the use of precise procedure to quantify forest biomass and its uncertainty.

Lu (2006) mentioned three approaches to biomass assessment. These are field measurement, remote sensing, and GIS-based approach. The field measurement is considered to be accurate (Lu, 2006) but proves to be very costly and time consuming (de Gier, 2003). In any of these approaches, ground data is important for validation. In the case of remote sensing, ground data is needed to develop the biomass predictive model. This means, it is always necessary to a field measurement of biomass for predictive modelling or validation purposes. Typically, the procedure is to randomly select sample trees, measure the tree variables (such as DBH or tree height) and the tree biomass, then develop biomass equation using these measurements. The developed biomass equation is used to estimate the tree-based biomass. While measuring the sample tree variables is easy and straightforward, measuring the sample tree biomass is difficult because the trees are large and heavy.

Two methods of measuring sample tree biomass are available: (1) destructive and (2) non-destructive. The conventional destructive method is done by felling the sample tree and then weighing it. Direct weighing can only be done for small trees, but for larger trees, partitioning is necessary so that the partitions can fit into the weighing scale. In cases where the tree is large, volume of the stem is measured. Sub-samples are collected, and its fresh weight, dry weight, and volume are measured. The dry weight of the tree (biomass) is calculated based from the ratio of fresh weight (or volume) to the dry weight. This procedure requires considerable amount of labour and cost, and the use of ratio is biased (Cochran, 1963).

A new destructive method proposed by Valentine et al (1983) and later adapted by de Gier (2003) uses the principle of randomized branch sampling and importance sampling. In the randomized branch sampling, a "path" is determined starting from the butt and ending at the terminal bud. The segments (nodes) comprising the "path" is selected with probability proportional to size (pps). Unconditional probability of selection for each section is calculated. Along the path, points, where a change of taper occurs, are located. The inflated area of points measured along the path is calculated by dividing the diameter squared by its unconditional probability. The calculated inflated area is used to calculate the volume of the segment, say by Smalian's formula. The unbiased woody tree volume is the sum of these segment volumes (de Gier, 2003).

After the path is selected, importance sampling comes in to randomly locate the sample disk. The whole path is viewed as consisting of infinitely many thin disks, of which one is selected with probability proportional to its diameter squared. To determine the location of sample disk, the tree woody volume is multiplied with a random number and the segment where this volume is reached is identified. The exact location of the sample disk within the identified segment is determined by interpolation. The weight per unit thickness of the disk is determined and divided by the unconditional probability assigned to the segment from where it is removed. Multiplying this value with the estimated tree woody volume, and dividing it by the square of the disk diameter gives the woody fresh weight. The woody dry weight is calculated in the same manner as the fresh weight. (de Gier, 2003). The determination of path reduces much of the work as those tree segments not included in the path are not measured. Furthermore, there is no need to weigh the whole tree; hence, it is efficient in terms of
time and cost. However, the procedure uses considerable amount of computation that decent computing equipment (e.g. HP LX200 palmtop computer or iPaq equipment) is necessary.

The non-destructive method does not require the trees to be felled. Measurement can be done by climbing the tree and measuring its various parts and computing the total volume. Tree density which can be found from literature is used to convert the measured volume into biomass estimate (see Aboal et al., 2005). This procedure takes even more time and cost to perform. Another procedure is to taking two photographs of the tree at orthogonal angles. Then the scale of the photograph is calculated so that the volume of each tree components (stem, branch, foliage) can be calculated. Density of the different tree components is calculated and used to convert the volumes into biomass (Montes et al, 2000). However, the calculated biomass from these procedures cannot be validated unless the sample tree is felled and weighed.
Once sample tree variables and biomass data are obtained, and the biomass equation is developed, it is then applied to each tree in the sample plots to obtain the plot biomass. The forest biomass is then estimated by the corresponding sampling design formula for the mean and total estimator or by predictive modeling using remotely sensed spectral data.

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## UNIT 8: Determination of age of trees

## Unit Structure

### 8.0 Learning Objectives

8.1. Introduction
8.2. Increment
8.3. Determination of increment
8.3.1. Classification of Increment
8.3.2 Relation between C.A.I. and M.A.I.
8.3.3. Increment Percent
8.4. Stump analysis
8.5. Stem analysis
8.5. Increment Boring

### 8.0 Learning Objectives

After going through this unit you shall be able to:

- Understand about the tree growth measurements.
- What is stump analysis and stem analysis?
- About the increament and increament boring.


### 8.1. Introduction

Tree growth shows in elongation and thickening of roots, stems and branches. Linear growth takes place as the result of the direct activity of the original tissue, or primary meristem. Subsequent radial growth is brought about by division and enlargement of secondary meristem, or cambium, which produces new wood and bark between the old wood and bark. Root growth is not given enough importance in forest mensuration though in certain tree species, like sandal, the root portion is very valuable. The radial growth and height growth of stem together with the resulting changes in stem form must be measured to determine growth in volume. Fluctuations in growth occur both daily and seasonally. The daily changes cannot generally be measured in trees. The seasonal variations specially in temperate forests are of particular importance ire forestry since they are responsible for the formation of growth rings. The spring wood is lighter and more porous while summer wood is. darker and denser. Thus the annual
growth of a tree as observed on a cross section of the stem forms a pattern of concentric rings. By measuring the width of these rings the annual radial growth of the tree may be determined. This radial growth is not the same along the entire length of the stem. The height and radial growth do not start and end at the same time. In most of the tree species height growth starts and culminates earlier than the radial growth $\bullet$ ' Tropical species as well as broad-leaved trees in temperate climates, barring few exceptions, do not generally form annual rings though they may have pronounced seasons of high and low growth activity. Acacia catechu and Tectona grandis are examples of broad-leaved tropical species which show annual growth rings. Several other broad-leaved species show growth rings but these may not be annual viz., Boswellia serrata, Machilus duthiei. In certain cases several rings may be produced in a year while in other cases one ring may be formed after several years.

### 8.2. Increment

In other words, during every season, a tree increases by elongation of the top shoot, side branches, roots and by laying layers of wood and


Fig. 1. Diameter and height growth curves of a spruce tree height, as
well as the spread of the crown increase consistently upto a certain age, producing an increase in volume or biomass. This is called increment which is defined as the increase in girth diameter, basal area, height, volume, quality, price or value of trees or crops during a given period. Trees grow slowly in the beginning, then more rapidly
upto a certain age reaching an optimum level after which the rate of growth slows down.

### 8.3. Determination of increment

The age at which highest rate of growth is reached is different for different growth parameters, such as diameter, height, form, volume, etc. The growth actually never stops until a tree dies. However, with the increase in age the forces of decay become more active in course of time and the rate of decay more than


Fig. 2. Growth pattern the increment. When such a stage is reached, the tree is said to be putting on negative increment. A. study of this growth pattern is essential for planning management of any forest crop (Fig 2).

### 8.3.1. Classification of Increment

The increment, whether in diameter, height or volume is classified as follows depending upon the period to which it relates:
(i) Current annual increment (c. a. i.)-It is the increment which a tree or a crop puts on in a single year. Because of the-very slow rate of growth in certain tree species, in practice, c. a. i. often refers to average annual rate of increase over a short period or the periodic annual increment.
(ii) Periodic annual increment (p. a. i.)-It is average annual increment for any short period.
(iii) Total increment-It is the increment that a tree or a crop puts on from origin upto the age at which the tree or crop is cut. it is the sum of the c. a. s and therefore represents the volume of the tree or the crop.
(iv) Mean annual increment (m.a.i.)-It is the mean N of a tree or crop at the desired age, i.e., the total increment up to given an age divided by that age. if the mean annual increment is calculated for a portion of the total age, it is called periodic mean annual increment. If it is calculated at the rotation age, it is caned the final mean annual increment.

### 8.3.2. Relation between C.A.I. and M.A.I.

The m. a. i. whether in a tree or in a stand, is the expression of the average yearly response in the past to the growth factors of the site and the c. a. i. is the actual response in any one year. As the m. a. i. is the average and the c.a.i. is the individual item from several of which the average is drawn, the two do not coincide with each other throughout the life of the tree or stand except twice viz., one at the end of first year and the second in the year of the culmination of the m. a. i.

The c. a. i. is not uniform throughout the life of a tree or a stand. It rises to a maximum and then gradually falls off. In the case of some species, it is small to start with, then it rises steeply to a climax and then falls off. In either case, falling off is not so steep as the rise. The m. a. i. also increases to a maximum but at a much lower rate because the effect of increase of a single year is distributed over all the years that the tree or stand has existed. The m. a. i. continues to rise towards a. maximum even after the c. a. i. has started falling because the amount of growth added to the tree or stand volume during the year, although lerser than before, .is still greater than the average or the mean. When C. a. i. finally falls to such an extent that it is equal to the average or the entire life of the tree or the stand, the m. a. Y. reaches its highest point. This is the year of the culmination of the mean annual increment. During the following and subsequent years, the c.a.i. is less than the mean and so the mean also begins to drop. But this drop, too, is not as rapid as that of the c. a. i. because the effect of the decrease of c.a.i. is spread over the whole life of the tree or stand. The m. a. i. therefore drops only to the extent it is being pulled down by the effect of the total volume lesser c. a i. for single years upon the fraction

$$
\frac{\text { total volume }}{\text { age in years }}
$$

A time may come when there may be no growth at all and so the c.a.i. will be zero. Unless the tree or the stand is cut, loss in volume may start taking place due to rot or other damages and then the c. a. i. will become negative. But the value of m. a. i. Is never zero and in no circumstance, it is negative like the c. a. i. in other words, until the tree or the the stand is cut or destroyed, the $\mathrm{m} . \mathrm{a}$. i. • always
positive. The curves of c.a.i. and cross each other at the maxima of m.a.i. curve. The age at which they cut, is the age of maximum production.

### 8.3.3. Increment Percent

Increment percent is defined as the average annual growth in diameter, basal area or, volume over a specified period expressed as a percentage of diameter, basal area or volume either at the beginning or more usually halfway between the beginning and end of the periods.

This is an expression of increment from the business man's $p$ Ant of view. The statement that the tree or a crop has been growing in diameter; basal area or volume at a certain rate during a period omits reference to the capital on which the increase is taking place. . 1 f the increase per year is expressed as a percentage of the diameter, basal area or volume of the tree or crop to which it relates, it indicates the rate of growth clearly and makes it comparable to other forms of investments, on which interest accrues.

### 8.4. Stump analysis DEFINITION

It is the analysis of a stump cross section by measuring annual rings in order to estimate the age of the tree and its past rate of diameter and basal area growth.

## OBJECT

Stump analysis determines the progress of diameter increment on stump throughout the life of the tree analyzed or over any desired period but its most important object is to find out age/ diameter relationship to determine the rotation age corresponding to a particular exploitable diameter in the absence of yield tables: It also shows the influence of external factors affecting single trees or the stand, on the rate of diameter increment of trees or stand by with effects correlating rate of diameter increment with effects of the factors involved.

## FIELD WORK

If old stumps are available, sound stumps of trees of rotation size and over are selected taking care to see that they were the suppressed at any time in the past
life as evident from the extreme closeness of rings on any part of the stump. The stumps should be of the average trees so that they can serve as representatives of the crop.

If old stumps are not available, trees are specially selected and felled for this purpose. The trees should be of the rotation size and over and should be representative of the site quality to which the results of investigation are to be applied. They should be of the typical shape and development and the tendency to select trees above the average quality must be avoided. Suppressed, markedly dominated trees and those with any abnormality or malformation should not be selected.

After selection, the breast-height diameter is measured and re-corded and then the tree is felled. As falling cut is seldom smooth and horizontal, the stump is cross-cut by a saw to remove a disc leaving smooth and horizontal cross-section, Cross-cutting to be done in case of old stump has also selected for analysis. Stump height and underbark girth of each stump are then measured and recorded. From the girth, average radius underbark is calculated and recorded for each stemp in the form


Fig. 3. Stump analysis number one given on next page.

Four radii, angularly as far apart as possible, with length not appreciably longer than the calculated averages are marked on fire, stump with pencil. If rings are not clearly visible, the surface is moistened with water or chiseled along the marked radius. On each radius, decades ( 10 rings) are counted from pith outwards and a pin inserted at each decade, leaving incomplete decade (odd rings) on the outer periphery. Incomplete or false rings are not counted. To ensure this, pins, are fixed at each decade independently on all the four radii and then checked to see that all
the 4 pins of the same decade lie on the saw continuous ring. The radius of each decade on each marked radii is then measured from pith outwards in centimeters and millimeters in one setting of the scale on each radius to avoid error and recorded in the following form in which all the other measurement so far taken have been recorded.


Fig. 4. Stump analysis Forms
The radii at successive decades are then totalled for each stump/tree and halved to get average diameter at successive decades. These are then consolidated in a separate form No. 1 a, similar to the one above except for the difference in heading as diameter at successive decades. Besides this study of stump, data are collected to determine age to grow to stump height and for drawing taper curves. For the first purpose some freely growing seedlings equal in height to the height of stumps are cut flush to the ground and their ages estimated from rings. Alternatively, a few, say 10, saplings of about 2 to 3 m height growing in the open are cut not only at ground level but also at 10, 20, 30, 50 and 80 cm for ring
counting and calculating ages at different heights. This information is tabulated, totalled and averaged in the following tabular form (Fig 4). For the second purpose sufficient number of trees of different diameter classes are selected and their under bark diameters at $15 \mathrm{~cm}, 45 \mathrm{~cm}$ and 75 cm from ground level and the d. b. h. (o.b.) are measured and tabulated in the following form to be totalled and averaged.

### 8.4.1. Stump Analysis Curves

First of all seedling height curve is drawn. The data collected in form 2 are plotted showing heights on abscissa and a ordinate. The average ages at various heights are plotted and a smooth curve is drawn. From this curve, the age corresponding to the average stump height is read. (Fig. 5)

The second curve drawn is for conversion of stump diameter u.b. to (o.b.). For this, the average stump diameter (u.b.) for each height (i.e., 15 $\mathrm{cm}, 45 \mathrm{~cm}, 75 \mathrm{~cm}$ ) is plotted separately


Figure 5.


Figure 6 against correspond-. gd.b.h. (o.b.), the data being available from form 3 . This gives a set of curves-one for each height class, which should be harmonized. (Fig. 6)

If the dispersion of these curves is appreciable, an additional curve is interpolated corresponding to the average stump height. Then the d.b.h. (o.b.) corresponding to convenient stump diameters u.b. say $10 \mathrm{~cm}, 15 \mathrm{~cm}, 20 \mathrm{~cm}$ and S so on, are read from this new curve and tabulated.

Finally, a curve showing d.b.h. (o.b.) against total age is drawn in the following steps:

1. As a first step, stump diameter/stump age curve is drawn. The average diameters at successive decades given in form 1 a are plotted against the
corresponding decades taking stump diameter u.b. on abscissa and decades at stump height on ordinate and a smooth curve is drawn.
2. As a second step, the axis of $Y$ which shows age at stump is corrected to show total age. This is done by shifting down the scale by the number of units equal to the number of years a tree takes to grow to average stump height, read from the first curve.
3. As the final step, the axis of $X$ which gives stump diameter u.b. is corrected to read d.b.h. (o.b.). this can be done with the help of the interpolated curve in the curve No. 2 (fig. 81) which gave d.b.h, (o b.) corresponding to the various stump diameters (u.b.). From this curve the d.b.h.


Fig. 7. Age diameter curve (o.b.) corresponding to various stump diameters (u.b.) were read. Now the points of the stump diameters (u.b.) for which corresponding d.b.h. (o.b.) figures are available, are located on curve 3 (fig. 82) and shifted backward on the same age axis to a point corresponding to the corresponding d,b.h. (o.b.). Thus a new set of points are marked through which a smooth curve is drawn. This curve gives $d$ b.h. (o.b.) corresponding to total age of the tree and the age which a tree takes to reach a particular diameter can be read from this curve (Fig 7)

## ADVANTAGES

As compared to other methods, stump analysis offers the following advantages: (i) Data can be collected from stumps of felled trees as long as the wood remains sound. Thus it can he carried out with or without special fellings.
(ii) Data can be collected at any time with minimum of manual labour.
(iii) The data can be multiplied to any desired extent with no objection other than the time taken in measuring.
(iv) Field work is simple and easily learnt.
(v) Each stump provides data for the whole life of the tree.

### 8.5. Stem analysis

## DEFINITION

Stem analysis is defined as the analysis of a complete stem by measuring annual rings on a number of cross-sections at different heights in order to determine its past rates of growth.

## OBJECT

The most important object of this investigation is to 'determine -age-diameter, age-height and age-volume relations throughout the life of the tree analyzed and thus assess the average rate of diameter, height and volume increment. The effect of external factors, e.g., lopping, defoliation, etc., on the rates of growth of trees, can also be determined. The data of the stem analysis can also be used for preparing local volume tables by correlating diameter volume relations. The accuracy with which the past increment of an individual tree can be obtained by this method often encourages the use of the results to stands but caution is, however, necessary in doing so because the study pertains to a few trees. The number of trees cannot be increased beyond a certain limit as the study is slow and 'expensive. The difficulty can be overcome by selecting trees of average form but there is no guarantee that the average trees selected today were average trees in the past or will remain as representative for future crop.

## FIELD WORK

As in the case of stump analysis, 'the trees selected for the purpose should be of typical shape and development and of rotation size or over. Suppressed or markedly-dominated trees or trees with abnormal growth or, malformation are not selected

After selecting the tree, its diameter at breast height, is measured and recorded. A horizontal mark is made all round the bole at this level. The height of the tree and its crown width is also calculated and recorded) (After this the tree is felled and the following measurements. are taken :
(1) Height of the first green branch and the lowest point of the full crown to calculate clear bole and crown length.
(2) Total height of the tree.
(3) Height of the points with d.o.b. 20 cm and 5 cm and d.u.b. at these pointes.

The felled tree is then considered as divided into sections from base including stumi2 the lowest section being 2'74 m long so that breast-height corresponds to its mid-point and the following ones, as far as possible, all 3m long except the upper most. The odd length at the top is treated as a separate section if it exceeds 1 '5 m in length but included in the previous section if 1.5 m or less.

The mid-points of these sections are marked on the tree at 1'37 m (already marked), 4'24 m [2'74 (i.e., height of the previous section+1'5 (mid-point of this section)], 7.24 m $(5.74+1.5)$, and so on. The mean diameter over bark is callipered at each mark and recorded. 1 A ring of bark is' then peeled off at these points and the underbark diameters callipered and recorded. If the mid-points of sections fall on points unsuitable for ring counting on account of knots, rot or breakage, they are shifted up or down for a distance not exceeding 30 cm , the sections being treated as though they were actually at the midpoint. H-illis range is not sufficient, then the length of the section is altered to avoid the difficult portion and mid-point marked off accordingly.

The tree is then cut at these mid-points by a cross-cut saw for ring counting. If the measurements cannot be done in the field, transverse sectional discs 5 to 8 cm thick are sawn but in such eases. The section height is noted in the field on the surface to be counted to avoid likely confusion later. The length of successive sections and the height of their mid points above ground level are recorded. For each section, average radius is calculated on the basis of two diameters (u.b.) callipered at right angles, totalled, averaged and halved. This is also recorded. Two radii angularly as far apart as possible and in length as nearly equal to the calculated average radius are located on each section and marked on them with a soft pencil and analysis rule from pith to circumference. If the annual rings are not clear on these radii, they are brightened by moistening or chiseling. Rings are first counted on the section at 1 '37 m. Counting is done by decades from the pith outwards along the marked lines. A pin is inserted on completing each decade, leaving incomplete decade at the circumference. It should be checked that the decade pins are on the same annual ring and that the number of rings on each radius is the same. On each of the remaining sections, number of rings corresponding to the outermost incomplete decade on the 1.37 m section should be marked off first, counting from the cambium. The remaining rings are marked off in decades counting inwards and the total number noted.

The radii are measured from the pith outwards to each pin beginning with the outermost. The measurements are taken to the nearest millimeter. This work is first done on the 1.37 m section on which the radial measurement for the outermost full decade along, the selected radii are first entered under the appropriate decade heading in columns of form No. 1 (Specimen given on the next page) and then the successive radial measurements to other decades are entered in continuation across the form to the right. The measurements of incomplete decade on the outside are not recorded. It is preferable to take all readings along one radius in one setting of the scale. Then the two are totalled in the red ink. This will be $d \cdot u \cdot b \cdot$ 'Measurements of remaining sections are successively recorded in\order, the first entry for each being made under the first entry of first section.

In the form, there is a column for age to section. This is filled by considering age at breastheight as 0 (as correction for age to breast-height is done later). Thus the difference between the ring3 on breast-height and any other section gives the age to that section; the data for entry in column for twice bark thickness is obtained by subtracting u.b. diameter

from o.b. diameter both measured before cutting the section. Besides this ring counting work, seedling height data is also collected in the field and recorded in form No. 2 in the same way as it is done in case of stump analysis.

## COMPUTATION

Before drawing curves, some computation work has to be done. For this work, the data collected in the field are tabulated in the following forms:
(1) Age computation-The ages to different sections of the tree are entered in form 3 a (specimen given on next page) under appropriate column for the height of section. Thus all the vertical entries of column 3 of form No. 1 will appear horizontally against the respective tree number. As the height of the last or uppermost section is usually different from the standard height due to the section being longer or shorter than 3 m , deviation will occur. For example, if the height of the last section is 35 m instead of being 34.24 m , there is a deviation of +0.76 m . On the other hand, if the height of the last section is 34 m instead of 34.24 m , there is a deviation of $0-0.24$. In order to find out the average of deviation, they are algebraically added and divided by the total number of times arid of the trees in which deviation actually occurs, the average age and average deviation is that tree was of height equal to height of section + average deviation at 6.2 For instance, if under the height class 34.24 the ay years and average deviation - 0.14 , it means that the tree was $34.24-0.14$ 34.10 m in height.


## (ii) Mean diameter computation for decade

The diameter under bark at different heights is collected from I they are given in red ink in columns of radius at for all trees separately for such decade and (specimen given on next page). Diameters of one I and entered in the form against the particular tree under appropriate sections. The lower decades will have lesser number of entries to record and
that too under fewer height sections while the highest decade will have the maximum number of height sections covered. The second column under each height section is of deviation as in the ease or age computation. The diameter u.b. and the deviation are totalled for each decade and averaged by dividing the total by the total number or trees appearing in that decade, irrespective of whether they reach the height of the section or not.
(iii) Mean tree volume computation for decades-In order to calculate volume of the mean tree of each decade the diameter is obtained from curve IV which gives d .0 b./height curves separately for each decade. Taking the curve for the decade in question, diameters are read at $1.37 \mathrm{~m}, 4.24 \mathrm{~m}, 7.24 \mathrm{~m}$ and so on. The basal areas for these dianieters are calculated by referring to the table giving areas of circles of given diameters. The basal area in then multiplied by the length of the section to get the volume. This procedure is adopted for all sections except the last which is regarded as a cone.


The diameter of cone is the smaller end diameter of the previous section and can be read against height of previous section from curve for the decade. The height of the cone can be read on the height axis from end of the previous section to the apex of the cone which is the point where the curve for the decade touches the height axis and the volume is calculate by the formula $\frac{\mathrm{BH}}{3}$. The height of the cone is mentioned on top of the diameter of this column. By adding the volumes of all the sections, volume of the mean tree for the decade is obtained and it is recorded at the end. The record is kept in the following form:

(IV) Computation for bark thickness -The d.u.b. of all section and of all the trees being analysed are arranged in 10 cm diameter classes and twice bark thickness is recorded against d.u.b. in the following form. The d.u.b. is obtained from Col 6 and twice bark thickness from Col 7 of form No. 1. The figures of d.u.b. and twice, bark thickness of each diameter class are then totalled and averaged.


## Curves:

Several curves are drawn in stem analysis to obtain age diameter, age height and age volume relationships of trees as shown below:

## (I) Age Height curve (Curve I)

From the age computation form 3 a, the age is plotted against


Fig.1. Age Height curve
corresponding height of section with age on X axis and height on Y axis. While plotting the the height, the standard height is corrected by average deviation. The number of observations on which the average are based are noted down near the point. Then smooth curve is drawn, giving greater weightage to points based on averages of larger number of observations (Fig. 1)

The average ages on axis denote ages above the breast height. In order to show total age on that axis, the zero of the axis is shifted to the left by the necessary units which represent the period that the tree takes to grow from ground level to breast-height obtainable from curve VI based on seedling height (tail. From this corrected X axis for age, height corresponding to the decades of total age may be read and tabulated on the graph for ready reference.
II-a Age-diameter (u.b.) curve-
The final age-diameter curve is drawn in several stages. For this purpose first of 'all, diameter (u.b)'height curves by decades are drawn. The average values of d.u.b. obtained from a series of form 3 b (mean diameter computation for decades) are plotted against corresponding heights of sections separately for each decade and the curves smoothened and harmonized in the following three steps.

Step 1—Average diameters are plotted against height, of section for each decade taking heights on $\mathrm{A}^{\circ}$ axis and diameters on Y axis. A distinctive symbol is given for each decade so that the 50 points of that- decade are not con. 45 fused with the point of other decades Smooth and harmonized curves are drawn through the points of each decade separately. Thus there will be as many curves as the decades. From these smoothened curves, values are read against heights of sections (1.e. $1.37 \mathrm{~m}, 4.24 \mathrm{~m}, 7.24 \mathrm{~m}$ and so on) and tabulated on the right hand top corner of this curve, called curve II (Fig 2).

Step 2-The values read from curve II are plotted against decades separately for each section taking age on X axis and diameter on Y axis. Distinctive marks are used for points of one sectional height to so avoid confusion. Thus separate d.u.b./age curves for each height class are obtained. The curves are smoothened and harmonized and values are again tabulated for $1.37 \mathrm{~m}, 4.24 \mathrm{~m}, 7.24 \mathrm{~m}$.........heights (Curve III) (Fig 3).

Step 3-The values read from curve III are replotted taking height on $X$ axis and diameter on $Y$ axis using as usual, distinctive marks for each decade. Thus there will be as many curves as the decades. These curves are smoothened and harmonized. In harmonizing the higher decades are given more weight age. Values of diameters as read against sectional heights are used for volume computation. (Curve IV Fig. 86 on next page).

Curve IV gives d.u.b. at different heights at each decade. These data can be used for drawing a curve showing the relationship between age and d.o.b. at breast-height. But before this can be done he following two curves has to be drawn.

## Advantages:

As compared to other methods for collection of data for increment of trees, stem analysis offers the following advantages:
(i) The data are collected from standing trees carefully--selected and so it is more reliable. This is more so because the .entire tree is analysed and data is not collected only from the lower part of the stem as in stump analysis or in increment borings.
(ii) It gives complete information about the growth of trees in .respect of diameter, height' and volume and so it is self-contained.

### 8.5. Increment Boring

Increment boring is defined as the boring of a tree stem with Pressler's increment or any other borer to determine increment of trees with annual rings.

OBJECT:
The main object of increment boring is to determine age and diameter increment of trees and with this knowledge to determine the average time required by the trees to reach a particular exploitable diameter or to pass through successive diameter classes. Besides, it helps to determine the effect of adverse or favourable factors on diameter increment and the response made to any improvement of these factors

## FIELD WORK

After selecting a few typical trees of each diameter class, two d.b.h. (o.b.) are measured with callipers and recorded together with their averages. Then the bark of the places where the callipers arms touched the tree is removed and under bark diameters are measured and recorded along with their averages. Then the tree is bored at the two ends of the diameter at right angles to the axis of the tree to a depth of 4 to 5 cms so that at least 15 to 20 rings are obtained on each radial core. The length of the outermost 5 rings of one end of each of the two diameters on a particular tree are recorded and totaled. The data are collected diameter classwise in the following form :


## COMPUTATION AND CURVES

Age d.b.h.(o.b.) curve-In order to draw this curve, first a diameter increment curve is drawn, For this, the data com-piled in column 4 and 11 of the form given before is totalled and averaged for each diameter class. The average width of 5 rings is plotted against the average diameter of that diameter class. This increment curve is then transformed into a following steps:

| Initial Diameter | Increment | Final Diameter |
| :--- | :--- | :--- |
| 1 | 2 | 3 |
|  |  |  |

(ii) The diameter values so obtained are plotted against a succession of equidistant points spaced at intervals corresponding to the number of years in the period (i.e., five years) with diameters on y axis and years on x axis and the growth curve is drawn through the plotted points.
(iii) The time axis of the curve is corrected to read age by shifting the zero point to the left by the necessary number of units corresponding to the estimated time required to reach the lowest diameter plotted. This will be age/d.b.h. (u.b.) curve. Twice bark thickness is added either from bark thickness table or twice bark thickness is separately measured on the diameters on, which borings have been done, curved on the diameters and a smooth curve drawn to read values of twice bark thickness. When the twice bark thickness is added to each of the d.b.h. (u.b.) points, series of new points are obtained and a smooth curve is drawn\ through them. 'This is the curve for d.b.h. (o.b). Thus the curve becomes age/d.b.h. (o.b.) curve.

## Unit 9: Measuring Forest Crops

## Unit Structure

### 9.0 Learning Objectives

### 9.1. Introduction

9.2. Determination of Diameter Of Crop
9.3. Determination of Height Of Crop
9.4. Determination of Age of Crop
9.5. Determination of Volume of Crops
9.6. Stand tables
9.6.2. Application and Uses of Stand Table
9.6.3. Stand Volume Table
9.6.4. Yield Table for Mixed Forest

### 9.0 Learning Objectives

After going through this unit you shall be able to:

- Understand about the crop diameter.
- Determination of Age, Height and Volume of a crop
- What are stand tables:.


### 9.1. Introduction

Forest crops consist of trees, but methods of measurements of individual trees, do not necessarily apply to their measurements due to certain characteristics described below
(i) Gradual diminution of number-Most of the forest crops start with a very large number of plants and as they grow, there is a competition between the individuals constituting them and the number of trees goes on reducing by the death and decay of some individuals. With the removal of smaller trees by death, the aver-age of the diameters and heights of the remaining trees increase. Similarly, if the old mature trees die there is a reduction in average diameter and height. Thus, in a forest crop, the average diameter, height, etc.,
change not only by the increments of the individual-- trees constituting it but also by the gradual diminution of the num-ber of trees.
(ii) Stand structure-The measurement of crop is further complicated by its constitution or stand structure. All crops are neither even-aged nor pure crops of one species. Some are regular, others are irregular, some are pure, and others mixed. Even in regular crops, the distribution and representation of age and/or size classes may vary from place to place. As the growth of trees depends not only on their growth potential but also on the size and form of the neighboring trees, the growth in regular and irregular forests is different. Therefore the methods of measurement of different dimensions of the crop may have to be varied with the type of forest in order to get a clear idea of its growth.
(iii) Object of measurement -In case of individual trees, the object of measurement of different dimensions of the tree is to find out the rate of growth of the individual tree. But in case of a crop, the object is to find out the behaviour of the crop as a whole. In other words, while in case of individual trees, the object or measurement is to find out its growth in terms of its diameter, height, volume, etc., in case of crops the object is to find out the number of trees, crop diameter, crop height, distribution of trees by Various sizes, volume, etc. In view of these causes, it is necessary to study the methods of calculation of various dimensions of a crop.

### 9.2. Determination of Diameter Of Crop

As the main object of determination of diameter of a crop is to find out its volume and as volume is dependent on basal area, the crop diameter is based on the mean basal area and not the mean of diameters alone. The following two terms are used for the purpose :
(i) Crop diameter- It is diameter corresponding to the mean basal area of a uniform, generally pure crop.
(ii) Mean diameter-It is diameter corresponding to the mean basal area of a group of trees or a stand; sometimes used for the arithmetic mean of the summated diameters.

There is slight difference between the two terms. While the former is used for even aged pure crops, the latter is used to any group of trees or any forest. IThus, mean diameter is a general term while crop diameter is a specialized term applied to only even-aged pure crops. inspite of this difference in scope, both are obtained by finding out the mean basal area. For this the diameters of the trees are measured and they are classified by diameter classes. The number of trees in each diameter class is multiplied by the basal area corresponding to the middle of that diameter class and then basal areas of all diameter classes are totalled and divided by the number of the trees to get mean basal area. Thus, mean basal area is obtained by the following formula:

$$
\text { Mean basal area= } \frac{\mathrm{n} 1 \mathbf{s} 1+\mathrm{n} 2 \mathbf{s} 2+\mathrm{n} 3 \mathrm{~s} 3+\cdots \ldots}{\mathrm{n} 1+\mathrm{n} 2+\mathrm{n} 3 \ldots \ldots}
$$

Where $n_{1}, n_{2}, n_{3}$ are the number of trees in each diameter class and $s_{1}, s_{2}, s_{3}$ are the basal areas of mean trees of different diameter classes.

Crop diameter is the diameter of the trees corresponding to mean basal area.
Mean diameter could also be calculated as an age of the diameter of trees, i.e.,

$$
\text { Mean diameter } \mathrm{D}=\frac{\mathrm{n} 1 \mathrm{~d} 1+\mathrm{n} 2 \mathrm{~d} 2+\mathrm{n} 3 \mathrm{~d} 3+\cdots \ldots . .}{\mathrm{n} 1+\mathrm{n} 2+\mathrm{n} 3 \ldots \ldots . .}
$$

Where $n_{1}, n_{2}, n_{3}$ are the number of trees in each diameter class and $d_{1}, d_{2}, d_{3}$ are the average diameters of these diameter classes.

But this average diameter would not be the crop diameter. As volume of a tree is proportional to the square of its diameter mean or crop diameter has been so defined that it is based on mean basal area, i.e., squares of diameters and not the mean of the diameters. The mean diameter obtained through the mean basal area is always smaller than that obtained by the direct average of diameters.

Yet another term top diameter is often used. It is the diameter corresponding to the mean basal area of the biggest trees in a uniform, generally pure crop, taking into
consideration 250 biggest diameters per hectare. It is used for determining top height of a crop, which, in turn, is used for assessing the 'site quality. In this tease two diameters of each tree are measured at breast height at right angles to each other. Each diameter is then considered separately.

### 9.3. Determination of Height Of Crop

Like diameter, the height of the crop is described by the following two terms : (i) Crop height-It is the average height of a regular crop as determined by Lorey's formula given below :

$$
\text { Crop height }=\frac{s 1 h 1+s 2 h 2+s 3 h 3+\cdots \ldots}{s 1+s 2+s 3 \ldots \ldots}
$$

Where $s_{1}, s_{2}$ and $s_{3} \ldots$...are are the total basal areas of each of the diameter classes, and $h_{1}, h_{2}, h_{3} \ldots$ are the mean heights of trees of these diameter classes.
(iii) Mean height-It is the height corresponding to the mean diameter of a group of trees or the crop diameter of a stand. Thus, for determining mean height, a height diameter curve has to be drawn from the data collected on a sufficiently large number of trees classified into diameter classes. The height and diameters of individual trees are plotted and a smooth curve is drawn. Then the mean crop diameter is calculated by the formula given earlier. Against this mean diameter height is read from the curve and this height is known as the mean height. Though there are several other formulae for calculating meant height, in India only the above two are commonly used. Mean or crop height is used for determining the volume of a crop but when the object is to assess the quality of the locality, the height used is top height which is defined as the height corresponding to the mean diameter (calculated from basal area) of 250 biggest diameters per hectare as read from height diameter curve. Thus, while the crop or mean height relates to all the trees of the crop, the top height relates to only 250 biggest diameters (or about 125 trees) of the crop.

### 9.4. Determination of Age of Crop

Depending on the variation in age of trees constituting a crop, forest crops are classified into even-aged and uneven-aged crops. The term even-aged is applied to crops consisting of trees of approximately the same age but differences upto $25 \%$ of the rotation age may be allowed in case where a crop is not harvested for 100 years or more. On the other hand, the term uneven-aged is applied to crops in which the individual stems vary widely in age, the range of difference being usually more than 20 years and in the case of long rotation crops, more than $25 \%$ of the rotation. In view of these differences different methods are used for determination of age of these two classes of crops.

## EVEN-AGED CROP

The age of an even-aged crop is described by the term crop age which is defined as the age of a regular crop corresponding to its crop diameter. Thus when the difference in ages of trees is cry small as in case of plantation crop or the crops raised under uniform system of forest management, crop age is the age corresponding to age of the tree of mean diameter. The age of such a tree can be found out either by felling it, if it belongs to a species which produces clear annual rings or by reading age corresponding, to crop diameter from the age diameter curve.

If however, the difference in ages of trees is appreciable, they
(ii) As volume depends, to a large extent on basal area, the -ages may be correlated with it and thus the mean age can be determined by the following formula :

$$
\text { Mean Age }=\frac{s 1 a 1+s 2 a 2+s 3 a 3+\cdots \ldots . .}{s}
$$

Where s1, s2 and s3....are are the total basal areas of even aged groups,
a1 a2, a3 $\cdots$ are the average ages of these groups, and $S$ is the total basal area of the whole forest.

This formula is simple and easy to the apply and it works well if form-heights of the diameter classes are equal.
(iv) A less dependable method is to substitute the number of trees for the basal area in each age or diameter class. Then,

Where $n_{1}, n_{2}$ and $n_{3} \ldots$...are number of trees,
$a_{1} a_{2}, a_{3} \cdots$ are the average ages of even aged groups,
This formula stresses the effect of numbers and disregards the size of trees within the age or diameter class. If there is a -preponderance of smaller-sized trees which do not contribute much to thelvolume and which are relatively younger than those in larger diameter classes, the mean age calculated by this formula would show the stand to have reached the present volume in too short a time.
(iv) If the even-aged groups occupy distinct areas: the area occupied by each age class may be used to determine mean age. Thus,

Mean age $=\frac{\mathrm{m} 1 \mathrm{a} 1+\mathrm{m} 2 \mathrm{a} 2+\mathrm{m} 3 \mathrm{a} 3+\cdots \ldots . . . . . . . . . . . . . . . . . . ~ G u m p e l ' s ~ f o r m u l a ~}{\mathrm{~m} 1+\mathrm{m} 2+\mathrm{m} 3 \ldots \ldots .}$
Where $m_{1}, m_{2}$ and $m_{3} \ldots$...are the areas occupied by different age classes,
And $a_{1} a_{2}, a_{3} \cdots$ are the average ages of even aged groups,
(v) The easiest method, though not very accurate, is to take the mean age as equal to the average age of sample trees representing different age classes. Then,

Where $a_{1} a_{2}, a_{3} \cdots$ are the average ages of even aged groups,
Il where al, at, a3 are the ages of sample, trees representing different_ age classes, and n is the total number of sample trees. Theoretically, this method can be improved by taking a, larger number of sample trees in the higher diameter classes but while this increases personal bias, it does not remove the main. source of error arising out of the fact of sample trees being not truly representative of the crop. The main drawback of the last three methods is that they are-based on purely arithmetical averages but although the age increa-ses in arithmetical progression, the volume which is a function of" age, follows a different trend and increases in geometric progression. As volume and its increment are the ultimate objective when. determining age, the mean age should alwayS be weighted by vQiume or basal area and not by the number of trees or area of the-- age class. Another term top age is often used in connection with age of crops. It is defined as the age
corresponding to the top diameter of a regular crop. It is read from age diameter curve against the-top diameter calculated from basal area of 250 biggest diameters per ha.

### 9.5. Determination of Volume of Crops

Determination of volume of crops by direct measurement extending over the whole area is neither practicable nor economic, Therefore the volume of crops over small areas is eitherestimated with the help of volume tables or with the help of studies made in small plots representing the crop. When volume table is. to be used, the crop in the small area is enumerated and classified:, by diameter classes. If the local volume table is available, then the number of trees in each diameter class is multiplied by the-volume corresponding to the diameter class given in the table and the sum of the volumes of all diameter classes is the volume of the crop in that area. If local volume table is not available, then the diameters and the heights of a few trees are measured to prepare local volume tables from general volume tables. As measurement of largee areas is time consuming and expensive, study of growth of crops is carried out by laying small plots. Such plots are called the sample plots.

### 9.6. Stand tables

The stand table gives the expected number of stems per unit area in each diameter class within a given stand. It may also reflect the average distribution for all age classes together, independently of site index, in which case the expected stem numbers are summarized per height class. A stock table gives similar information but expressed in terms of volume. A stand table, based on the sampled diameter distribution, may be constructed for an individual stand. When applied to the metric measurement system, 2 cm wide diameter classes are usually adequate, but the stand tables for conifers of the British Forestry Commission are based on 5 cm wide classes, a class width of 4 cm is customary for working plans in German forestry. It is desirable that the stand table contains at least 10 diameter classes. In young and medium age stands, information about the diameter distribution is sacrificed when selecting a class width of more than 3 cm .

## Parameter prediction and parameter recovery

In order to be useful in the practice of forest management, stand tables should reflect the effect of all influential variables on the parameters of the diameter distribution.

## DEFINITION

Stand table is a table showing the distribution of stems by diameter classes for each of the series of crop diameters. In India, it is usually appended at the end of yield table of the species and the distribution is given in the following forms:
i) As percentage of trees over given diameter limits in crops of various diameters
ii) As number of trees per hectare (or acre) over given diameter limits, in crops ofdifferent qualities for successive 5 years age interval
iii) As percentage of the total number of trees by 10 cm (4 inches) diameter classesin crops of various diameters
iv) As percentage of the total number of trees by 10 cm (4 inches) diameter classesfor various partial crop diameters excluding trees below a certain limit
v) As number of trees by 10 cm (4 inches) diameter classes for 40 ha (100 acres) of normal forest under different rotations before the annual fellings.

Stand table is very useful for forest management and it is today generally accepted that yield table is not complete without it. The average diameter of the maincrop stems and the stems removed in successive thinnings can be ascertained from the yield but the range of diameters comprised in these averages cannot be known unless stand table is also prepared. The calculation of the final values of forests, preparation ofmoney yield table and application of commercial volume tables made for local standards of conversion, to yield table crop data require knowledge of trees per unit area underdifferent diameter classes and this is given by stand table.

- The parameter prediction method fits distributions (Weibull, beta, gamma) and relates the parameter estimates to stand characteristics, possibly with the addition of influential variables, for example, to account for the effect of thinnings.
- The parameter recovery method recovers the parameters of the distribution from the moments of a given distribution from actual or estimated stand attributes.
A considerable amount of research has been done to develop algorithms for a parameter recovery method for the three-parameter Weibull distribution. Burk et al. (1984) presented the following procedure:
- Estimate the first, second, and third noncentral moment of the Weibull distribution, for example, from equations with stand attributes (age, site index, stand density) as predictor variables
- The estimation requires a series of iterations since

$$
\Gamma_{k}=\Gamma\left(1+\frac{k}{c}\right)
$$

- Use this relationship to solve the following equation for $c$ :

$$
\mu_{3}^{\prime}=b^{3}\left(\Gamma_{3}-3 \Gamma_{1} \Gamma_{2}+2 \Gamma_{1}^{3}\right)+3 \mu_{1}^{\prime} \mu_{2}^{\prime}-2\left(\mu_{1}^{\prime}\right)^{3}
$$

- Calculate the parameter $b$ of the Weibull distribution from

$$
b=\sqrt{\frac{\mu_{2}^{\prime}-\left(\mu_{1}^{\prime}\right)^{2}}{\Gamma_{2}-\Gamma_{1}^{2}}}
$$

- The parameter $a$ is calculated as follows:

$$
a=\mu_{1}^{\prime}-b \Gamma_{1}
$$

Pienaar et al. (1988) developed a stand table projection method, based on the relationship between age and the relative size of the $i$ th surviving tree. The latter was defined as the basal area of this tree divided by the basal area of the mean tree. The initial hypothesis of this ratio being constant over time was rejected and required the estimation of the parameter $b$ of the function

$$
\frac{g_{2 i}}{g_{1 i}}=\left\{\frac{g_{1 i}}{\bar{g}_{1}}\right\}^{\left(A_{2} / A_{1}\right)^{b}}
$$

Estimates for $b$ were obtained from permanent sample plots. The projected basal area of a tree in the $i$ th diameter class $(i=1, \ldots, k)$ was then obtained
from

$$
g_{2 i}=G_{2} \cdot\left(\frac{g_{1 i}}{\bar{g}_{1}}\right)^{a} \cdot \frac{n_{i}}{\sum_{i=1}^{k}\left[\left(\frac{g_{1 i}}{\bar{g}_{1}}\right)^{a} \cdot n_{i}\right]}
$$

with

$$
a=\left(\frac{A_{2}}{A_{1}}\right)^{b}
$$

In order to project the stand table, the Clutter and Jones (1980) survival function, as well as a projection equation for basal area

$$
\begin{aligned}
\ln g_{2}= & \ln g_{1}+b_{1} \cdot\left(\frac{1}{A_{2}}-\frac{1}{A_{1}}\right)+b_{2} \cdot\left(\ln h_{2}-\ln h_{1}\right)+b_{3} \cdot\left(\ln N_{2}-\ln N_{1}\right) \\
& +b_{4} \cdot\left(\ln \frac{N_{2}}{A_{2}}-\ln \frac{N_{1}}{A_{1}}\right)+b_{5} \cdot\left(\ln \frac{h_{2}}{A_{2}}-\ln \frac{h_{1}}{A_{1}}\right)
\end{aligned}
$$

and height

$$
\ln h_{2}=\ln h_{1}-b_{1} \cdot\left(\frac{1}{A_{2}}-\frac{1}{A_{1}}\right) \quad \text { were fitted. }
$$

### 9.6.2. Application and Uses of Stand Table

Being statistical averages, stand table may not be applicable correctly to individual cases unless they are normally stocked. In normally stocked crops, however, they give a clear idea of the distribution of different diameter classes and can be put toseveral uses, some of which are given below:
i) Determination of the financial value of crops and
ii) Preparation of money yield table

### 9.6.3. Stand Volume Table

Every forester is confronted with the problem of quick assessment of stand volume. This can be easily done if stand volume table is prepared. Chaturvedi has found out that a linear relationship exists between volume/ha of a stand and the productof crop height and basal area/ha as shown below:

$$
V=a+B A \times H C \text { Where }
$$

V - Volume in $\mathrm{m}^{3} / \mathrm{ha}$
BA - Basal area in $\mathrm{m}^{2} / \mathrm{ha}$
HC - Crop height in $m$
$a$ and $b$ are regression constant and coefficient respectively.

### 9.6.4. Yield Table for Mixed Forest

Mixed forests constitute the maximum percentage of the forests in India. No yield tables have been prepared for such forests. The value of regression coefficient $b$ in the stand volume equations shows that it does not show wide variations. A precise relationship can be found between the volume and the product of basal area and crop height for mixed forests. Some work was done in this direction for plantation crops and promising results were obtained C.F.I. plots are required to be laid in mixed forests invarious forests types of enable studies being carried out on their growth. Chaturvediobtained the following regression equation for plantation crop of mixed species in Ramnagar Forest Division (U.P.).

Volume $=2.6018+0.2417$ BA x crop height

$$
r=0.8026 \text { d.f. }=21
$$

## Unit 10: Forest Inventory and Sampling

## Unit Structure

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10.13.6. Difference between Correlation and Regression
10.14. Introduction to remote sensing and its application in forestry.
10.14.1. Role of Forests

### 10.1. Learning Objectives

After studying this unit you are able to understand about:
i. Enumeration and its types
ii. What is sampling?
iii. Kinds of sampling and sampling designs
iv. Point sampling and central tendency measures
v. Introduction to remote sensing and its application in forestry

### 10.2. Introduction and Definition of Enumeration

Questionnaire of Forest Inventory published by F.A.O. described forests as all lands bearing a vegetative association dominated by trees of any size, exploited or not, capable of producing wood or other products, of exerting an influence on the climate oron the water regime, or providing shelter for live stock and wild life. Forest, therefore, means an area which fulfills the required criteria. 'Inventory' is originally, a commercialterm meaning the record showing the quantity and value of articles in a store. Theforest inventory therefore has to necessarily provide information about the size andshape of the area and the qualitative and/or quantitative information of the growingstock. Loetsch and Haller (46) define the term forest inventory as follows: "Forest Inventory is the tabulated, reliable and satisfactory tree information, related to the required unit, respectively units, of assessment in hierarchic order". In a sense, forestinventory is an attempt to describe quantity, quality, diameter distribution of forest treesand many of the characteristics of land upon which the trees are growing.

It issynonymous with the term 'Cruse' used in North America or the term 'Enumeration' used in India. Forests, define enumeration as the counting, singly or together, of individualsof one or more species in a forest crop and their classification by species, size, condition,etc.

With the increasing importance of forest areas for non-wood values such as recreation, watershed management, wild life, etc., the concept of forest inventory has widened. Regular inventories are now made about the number of tigers, elephants,rhinoceros and other threatened species of wild life. Recently, an inventory was maderegarding the rare plant species growing in silent valley in Kerala to decide if the forestcould be cleared for a river valley project. In the cases where non-timber information isto be compiled it is essential that specialists in these allied fields, work in cooperationwith the inventory specialists, in planning and executing the inventory.

### 10.3. Object

As far as the timber inventories (or enumerations) are concerned, the main object is to determine volume of timber growing in the forest with a view to determinethe yield. Less frequently, they are required for assessing the value for purposes of thesale of exchange or for estimating the return to be expected from clear-felled coupes.

They are also required for determining the current periodic annual increment and lastly they may be carried out to prepare map of the area showing regions of high or low volume production per unit of area to help decision making in setting up of industries. In short, the object of forest inventories is to supply information for forest management and planning and for pre-investment decision on forest industry establishment or expansion. These are also required to assess the feasibility of a project from economic, social or any other considerations.

### 10.4. Kinds of Enumeration

Enumerations are of the following main kinds:
(i)
Total or Complete;
(ii) Partial or Sample
(i) Total or complete enumeration: It means that enumeration of the desired species above the specified diameter limit is carried out over the entire area of the forest unit under consideration. Being expensive and time consuming, it is done in comparatively smaller areas of valuable and intensively managed forests where maximum possible accuracy is desired in the estimate, viz., P.B.I. areas of uniform orother shelterwood systems. Total enumeration is also done in case of miscellaneousforests where the number of species is very large or the species of economic importanceform a small portion and are confined only to favourable localities. Total enumeration is also carried out for statistical studies as check on results of partial enumerations. Having regard to the vast area involved and the time, labour and money that is required, complete enumeration in forestry is forestry is generally avoided.

Partial (or sample) enumeration: On the other hand, means
that the enumeration is to be done only in a representative portion of the whole forest. For example, a forest may be of 1000 ha out of which only 100 ha have been selected forenumeration and estimate of the whole population of 1000 ha is made, it is called partial or sample enumeration. Sample is defined as a part of the population consisting of one or more sampling units, selected and examined as a representative of the whole. Therefore sample enumeration is the enumeration of a representative part of the whole population. The ratio of the sample to the whole population is called sampling fractionor intensity of sampling which is expressed as a percentage e.g., $5 \%, 10 \%$ or $20 \%$. In the example given above a sample of 100 ha was selected out of the whole population of 1000 ha and therefore the intensity of sampling was $10 \%$. Actual enumeration is carried out in a number of sampling units which are subdivisions of the sample area. For example, if sampling units of 1 ha are made, then there would be 100 sampling units in a sample of 100 ha. A sampling unit need not be a compact area but may consist of several areas. The record of each sampling unit is kept separately for workingout sampling errors and other statistical parameters.

### 10.4.1. Choice of The Kind of Enumeration

The choice between the two kinds of enumeration depends upon
(i) extent of thearea to be covered,
(ii) variation in composition and density,
(iii) the intensity of management and consequent degree of accuracy required and
(iv) the resources of labour, time and funds available.

When extensive areas are to be covered, partial or sample enumeration has tobe done to save time and money. In areas where the crop composition and density varies greatly as in some miscellaneous forests, total enumerations have to be carried out. Similarly, where the intensity of management is high and greater accuracy is required as in the P.B.I. areas of the shelterwood systems, total enumeration is carried out. Above all, resources of labour, time and funds
available dictate which method ofenumeration can be carried out.

### 10.5. Sampling - Definition and Advantages

Sampling is the process in which enumeration is to be done only in a representative portion of the whole.

- In Sampling, the information is obtained only from a part of the population assuming that it is the representative of the whole. A part is studied and on that basis, the conclusion is drawn for the entire population.
- For example, a forest area may be of 1000 ha out of which only 100 ha have been selected for enumeration and estimate of the whole population of 1000 ha is made, it is called sampling.

Sampling unit: The population is divided into suitable units for the purpose of sampling. Types of sampling units in forest surveys are:

- Compartments,
- Topographical sections,
- Strips of a fixed width,
- Plots of definite shape and size etc.

Sampling frame: The list of sampling units from which the sample units are to be selected is called sampling frame.

Sampling Intensity (SI): The ratio of sample to the whole population which is expressed on a percentage

$$
\text { SI = sample area/Total area x } 100
$$

## Size of the sample

The number of sample units in the sample is known as sample size. Factors affecting the size of sample

- Nature of population
- Number of classes
- Nature of Study
- Types of sampling
- Standard of accuracy/precision required, and
- Other considerations


## Scope of sampling

- Less time
- Reduced cost
- Administrative convenience
- Better supervision
- Check result of census method
- Suitable for infinite/hypothetical population
- Suitable for destructing sampling


## Limitations of sampling

Sampling is better over complete census only if

- The sampling units are drawn in a scientific manner.
- Appropriate sampling technique is used, and
- the sample size is adequate.

Sampling theory has its own limitations and problems, which are:

- Proper care should be taken in the planning and execution of the sample survey; otherwise the results obtained might be inaccurate and misleading
- Sampling theory requires the services of trained and qualified personnel and sophisticated equipment for its planning, execution and analysis. In the absence of these, the results of the sample survey are not reliable.
- If the information is required about each and every unit of the universe, there is no way but to resort to complete enumeration.


### 10.5.1. Advantages of Sampling

Partial enumeration or sampling offers following advantages over total enumeration:
(i) Reduced cost and saving of time - As data are collected from only a fraction of the total area, expenditure is usually small and the information is obtained quickly. There is no doubt that a lot of time is spent in laying out sampling units for partial enumeration, locating on ground, units marked on map, demarcating them, etc., but unless the size of the sample or the percentage of sampling is high, he partial enumeration is cheaper and less time-consuming than total enumeration.
(ii) Relative accuracy - Though total enumeration is expected to yield most
accurate results, it is, nevertheless, subject to errors which are often considerable. On the other hand, when well-planned and carried out to an appropriate intensity, partial enumeration or sampling yields sufficiently accurate estimates of the growing stock. This is more so because personnel of higher quality can be employed.
(iii) Knowledge of error - Total enumeration gives a false sense of complacency as regards accuracy of result. This is more so because the errors cannot be calculated unless the whole work is repeated. But in sampling, the checking is easy and the errors can be calculated and kept within desired limits $b$ statistical methods.
(iv) Greater Scope - Where highly trained persons or specialized equipment is needed, as in the measurement of volume of standing trees in forest, it is not possible to carry out total enumerations. Thus, partial enumeration or sampling widens the scope of work to include those fields of activity where sophisticated instruments and highly skilled techniques are required.

### 10.6. Kinds of sampling

There are two main kinds of sampling used in forest inventories:

## (A) Random sampling and (B) Non-random-sampling

10.6.1. Random sampling: (simple, stratified, multistage and multiphase sampling).
Random sampling is that method of sampling in which sampling units composing a sample are selected in such a manner that all possible units of the same size have equal chance of being chosen. In this method, -e selected by using a sampling units arm table of random numbers, by unbiased drawing of cards from a well-shuffled pack or by drawing lots. The following variations of this sampling method are used in forest inventories.
(i) Unrestricted or simple random sampling-The unrestricted or simple random sampling is one in which sampling units com-posing the samples are selected by some strictly random process from the whole population or area without dividing it into homogenous blocks. For example, if sampling unit is 041 ha, and the population area is 1000 ha, the population is divided into 10,000 sampling units and each of them given a number 0000 to 9999 . Now for a $5 \%$ unrestricted random sampling, 500 different four digit reference numbers are selected by using a table of random
numbers. In this way, every possible set of 500 units has equal chances of being chosen for the sample.
(ii) Stratified random sampling--The stratified random sampling is that method of sampling in which the population is first divided into sub-populations of different strata and then sampling units are selected from each of them in proportion to their size. This modification in the simple random sampling is necessary because the method of unrestricted random sampling gives good results only if the forest crop is uniform but generally forest populations are characterized by considerable heterogeneity. Under these circumstances, precision or an estimate of sampled mean is appreciably enhanced by recognizing stratifications an] modifying the sampling design accordingly. This is accomplished by sub-division, real or imaginary, of the area to be sampled into homogenous groups which are called strata (single stratum) from each of which sampling units are selected. This method of division of area into homogenous groups is called stratification. In forest inventories this is often done by forming strata by species, site qualities, crop density, etc. For example, if the forest population of 1000 ha is sub-divided on the basis of site quality into 4 strata with area 300 ha in I quality, 350 ha in II quality, 200 ha in III quality and 150 ha in IV quality and sampling units are of 0.1 ha, then, for $5 \%$ sampling, 150, 175, 100 and 75 sampling units will be selected with the help of the table of random numbers or by drawing lots from strata of quality I, H, III and IV respectively.
(iii) Multistage sampling--In the random sampling described earlier; sampling units are drawn from a population at one time or stage and it is therefore called single-stage sampling. But sometimes, it is cheaper to take some samples at that stage and then divide them to take some more sampling units at the next stage and so on. As the sampling units are not taken out at One stage hut arc taken out in two or three stages, the method of sampling is called multi-stage sampling. In this sampling, the sampling units get smaller at each successive stage of sampling but the principle of random sampling 11 WV be preserved at each stage of selection. For example, a forest population of 100 ha may be divided into 200 sampling units of 0.5 ha. With $5 \%$ intensity of sampling, 10 sampling units will be selected in the primary stage. Then each of the selected sampling units, may further be broken up into 0.1 ha sampling units. Thus each of them will have 5 sampling units. Now if one or more of are again selected at random in the second stage, it will two-stag` sampling. "Fh. 2 process can
be repeated further, if necessary, till the desired stage. Two-stage sampling is quite commonly used in forest survey. For example, a forest to be inventoried might consist of numerous compartments that could be considered primarry units in design. Some compartments may be selected in the primary stage. They may be broken into plots of which some would he selected at the secondary stage. Trees selected in these plots for volume measurement could be the third stage of sampling.
(iv) Multiphase sampling - This method of sampling is different from the one described above. In this method some of the same sampling units are used at the different phases of sampling to collect different information or same information by different methods. Two-phase sampling is commonly used in forest inventories and because of the two phases, it is often referred to as double sampling. For example, if the number of bamboo culms in the forest is to be estimated, then in the first phase the number of clumps per hectare is determined by large systematic surveys and in the second phase, number of culms per clump is determined from a small enumeration survey of much smaller area than taken in the first phase. Another example of double sampling in forest inventories is the procedure using a combination of aerial photographic interpretation and field plots. The first phase of sampling consists of estimating the volume on aerial photographs of a large number of relatively inexpensive sampling units employing photo-interpretation and measurement techniques. In the second phase, a subsample of these plots is selected and visited in the field for direct determination of their volumes. This sub-sample is much smaller since field-plots are more expensive to lay than photo-plots. A regression is then obtained between filed plot volumes and photo-plot volumes permitting a corrected volume estimate to be made for the large, less expensive sample of the first phase.
(v) Sampling with varying probability - In the random sampling described

ng units are the same at all times, But in certain populations, the chances may vary as the sampling proceeds and therefore a modified method of sampling known as sampling with varying probability is used. This method of sampling has become increasingly important in forest inventories in recent years. The 'Bitterlich' or 'variable plot' or 'point sampling' or 'p.p.s.' (probability proportional to size) and 'three p' or ‘sampling with probability proportional to prediction' are applications of this aspect of sampling theory.
(vi) List sampling - This is another form of sampling with varying probability. Te method consists of making a list of sampling units along with their measure of size in any order. For example, suppose, 15 compartments are sampling units. They are listed as shown below be giving a number to each hectare of their area:

If it is decided to select a sample of size of 6 items, then 6 random numbers are drawn from the range of integers 1 to 312 which is the total area of the compartments. A compartment is chosen as a sample if the random number falls within the interval indicated in the column of associated numbers. The following may be chosen in this case.

| Random number chosen | Compartment chosen |
| :--- | :--- |
| 299 | 14 |
| 123 | 6 |
| 247 | 12 |
| 305 | 15 |
| 269 | 13 |
| 068 | 4 |

These compartments will be enumerated for the variable, e g, number of trees or basal area. The estimate of total population will be carried out through ratio estimates.

### 10.6.2. Non random sampling (selective, systematic and sequential sampling)

Non-random sampling is that method of sampling in which samples are selected
according to the subjective judgment of the observer on the basis of certain rules or guidelines indicating what sample should be chosen. As the selection is based on judgment of the observer, it is likely to be biased. The following variations of this method are used in forest inventories:
(1) Selective sampling-Selective sampling consists of choosing samples according to the subjective judgment of the observer. Selective sampling may give good approximations of population parameters if it is properly used by a person with intensive knowledge of the population. 4 PEA sampling described by Gyde Land and La Bou (picking plots by personal prejudice and/or ease of access) is one of the forms of selective sampling.For quick and rough estimates, this sampling is often employed.
(2) Systematic sampling: Systematic sampling is that method of non-random sampling in which sampling units are selected according to a predetermined pat-tern without recourse to random selection. Most commonly, the pattern envisages regular spacing of units. For example, sampling units may be located 80 meters apart in rows which may be 200 meters apart. The selection of the first unit may be either random or according to a fixed arbitrary rule and this will result in distinguishing two" varieties of systematic sampling. When the first unit is selected at random and other according to a fixed pattern, the sampling is referred to as systematic sampling with a random start. This method of sampling offers certain advantages as com-pared to random sampling and is therefore, frequently used in fore it inventories in spite of its shortcoming in estimating the variances of the result. The advantages arc even greater in tropical forests where environmental conditions hamper field work. Among these advantages are the simplicity and case in following it, ease of location of sampling units and consequent reduction in cost in reaching it, greater certainty of objectivity in the selection of sample and more uniform distribution of the samples.
(3) Sequential sampling -Sequential sampling is a method of sampling whose characteristic feature is that the number of observations in the sample is not determined in advance but sampling units are taken successively from a population. Each sample includes all the sampling units of the former sample. The con1-dence interval at each stage will go on decreasing at each stage of sampling. Sampling is stopped when the desired precision is reached. This type of sampling is usually adapted to test a hypo-thesis. The hypothesis is tested each time a new observation
is made to determine whether the hypothesis should be accepted or rejected. When the decision is made and the hypothesis is accepted or rejected, then the sampling procedure is completed and no further sampling unit is selected beyond the number necessary to make a decision.

### 10.7. Sampling design

In order that the sampling may give the most accurate estimate of the population, it is necessary that the sampling design should be statistically sound. Choice of the design depends on certain factors, some of the important ones of which are mentioned below

- The objective of the inventory,
- The desired precision,
- Time and money available for the work,
- Topography and accessibility to and within the forest,
- Availability of personnel and equipment,
- Availability of satellite imageries, aerial photographs and maps,
- Availability of automatic data processing equipment, and
- Results of previous surveys carried out in the area.

The objective of the inventory is the first and the most important thing to be considered before deciding upon the design. The design required for certain research investigation is different from the design required for obtaining an inventory of growing stock for routine forest management. Therefore, in order to draw up an efficient design a detailed knowledge of the purpose of inventory and the specific, uses to which the data obtained will be put, is very essential. Besides this the choice of the design also depends upon the precision required and the time and money available for the work.
Theoretically, an optimum design is one which provides estimate within prescribed sampling error at minimum cost and time. Therefore, information about the precision level desired and the time and money available for the work, provides the foundation on which the sampling design can be built and imposes restraints within which the work has to be done. If the amount of money or time for an inventory is fixed, the number of sampling units is determined within that restriction. If the total amount fixed for an inventory is $p$, then the numb.er of sampling units $(n)$ is given by the
following formula :

$$
n=(p-c) / r
$$

Where c is the overhead cost including planning, organization analysis and compilation, and $r$ is the cost per sampling unit. As the success of sampling surveys depends on the reliability of field work, the design should be such as can be worked in the desired forest and with the available human resources in the form of personnel and labour. It is, therefore, necessary to examine in detail its operational efficiency in the field keeping the topography, accessibility of the area, the quality of personnel and labour available, in mind.
The availability of satellite imageries, aerial photographs and maps, automatic data processing equipment and results of previous surveys carried out in the area also affect the design. The design of a statistically sound scheme includes decision on the kind of sampling units, the. size and shape, if on area basis, $s$ the number of sampling units or in other words, the intensity of sampling, distribution of the units over the forest area and the method of analysis of data.

## 10.8: Size and shape of the sampling units

Sampling units arc of two kinds, viz., those having a Fixed area and those having only points. When the sampling units are only points, it is referred to as point sampling which is described in the next chapter. In this chapter sampling units with fixed area are being described.

## SIZE OF FLVED-AREA SAMPLING UNITS

When the sampling units arc of fixed area, it is necessary to describe their size. From general statistical considerations, small sampling units arc more efficient than larger ones because the larger the number of independent sampling units observed, the better the precisions. For a given intensity of sampling, the smaller the sin of the sampling units, the larger is their number and vice-versa. Therefore smaller size of the sampling unit is better. On the other hand, the size of the most efficient unit depends on the Variability and density of stand. Forests of low density are generally more heterogeneous in stocking. Selection of small sampling units in such forests may result in a large portion of the sampling units being empty. In that case, frequency distribution of desired parameters from units become seriously skewed
and does not permit assessment of error using normal distribution theory. Another factor that influences the size is the relative cost. The larger the size for a given intensity, the fewer are- the number of sampling units and therefore lesser time will be required in travelling to and locating the units. Though the smaller the units require relatively lesser cost in enumeration, the over all cost is higher because of higher cost of travelling to and fro and clearing the brush wood etc. therefore the cost of travel and locating the plot is an important consideration. In short, the ultimate choice of the size of sampling units is a compromise between maximum efficiency, cost and convenience.

## Shape of the sampling units:

The usual shapes of the sampling units in vogue in India are plots, strips, topographical units and clusters.

Plots - The term plot is applied to sampling units of small area but of diverse shapes such a square, rectangular, circular or polygonal. Square and rectangular plots are most commonly used in forest surveys. They are easy to lay and can be demarcated by four corners. Majority of sample plots in India are rectangular. They are particularly suited to plantation crops raised in lines. In such plantations two boundaries of the plots are laid in between the plantation lines, thus reducing chances of trees falling on boundary of plots. Usual size of the plots is 0.1 hectare but bigger plots of 1.0 hectare are often laid in mature crops.

Circular plot is very easy to lay on level ground once its center and radius are fixed. It has the minimum perimeter for a given area compared to any other geometric shape and thus has the minimum number of border line trees. In several surveys circular plots of 0.05 hectare (radius 12.62 metres) and 0.1 hectare (radius 17.84 metres) have been used in India. In hilly terrain radius is determined either by stepping or by correcting for slope. The desired radius is obtained by multiplying the prescribed radius ( 12.62 m for 0.05 ha plots) by the slope factor. Slope factors for some angles and percentage slopes are given below:

| Slope angle in <br> degrees | Slope factor | Slope percent | Slope factor |
| :---: | :---: | :---: | :---: |
| 5 | 1.004 | 20 | 1.020 |
| 10 | 1.015 | 30 | 1.044 |
| 15 | 1.035 | 40 | 1.077 |
| 20 | 1.064 | 50 | 1.118 |


| FOREST MENSURATION AND BIOSTATISTICS |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| FR1    <br> 25 1.103 60 1.166 <br> 30 1.155 70 1.221 <br> 35 1.221 80 1.281 <br> 40 1.305 90 1.345 <br> 45 1.414 100 1.414 |  |  |  |  |

The same correction is required for the side of rectangle along slope. In spite of their theoretical advantages, circular plots have not been popular in India in practice. In difficult terrain or in areas with dense weed growth, laying out plots expensive and time-consuming. In such cases, a plot size should be such that the layout and measurements can be completed in a day to avoid to and fro journey to the plot. Efficiency of the plot size is determined by the factor $(S E)^{2} \times p$, where $S E$ is the standard error of a particular plot size and $p$ is the cost of inventory of the plot. Thelower the value, the more efficient is the plot size.

Strips - In certain areas, instead of laying out plots, strips 20 to 40 m wide are laid across the forest from one end to the other at a particular bearing at regular intervals and inventory of these strips is made to serve as a sample. For this the survey party decides a base line at one end of the forest and starts cutting a line through the forest at the fixed bearing from a fixed point on the base line. This line is the centralline of the strip on both sides of which enumeration is done till the other end of the forest is reached. The party, then, moves tot the center line of the next strip and cutting that line, enumerates the strip back to the base line. Though the entire strip is the sampling unit, it is often divided into 100 m lengths to serve as recording units. Ifthe volume per unit area has a tendency to increase in one direction due to fertility gradient or some other cause, strips are laid parallel to this direction. The width of thestrip and the distance between the two strips determine the intensity of sampling, which can be calculated by the following formula:

$$
I=------------------100
$$

Where I is the intensity of sampling,
$W$ is the width of the strip in metres, and
$D$ is the distance in metres between the central lines of the two adjacentstrips.

For instance, if strips are laid 100 m apart and the width of the strip is 20 m , thenthe intensity of sampling is $20 \%$.
Strip method of sampling suffers from certain drawbacks. While it is not possible to use strips as sampling units in hill, it is difficult to maintain constant bearing and width of the strip even in plains. On the basis of comparison of relative efficiency ofplots and strips used as sampling units in 'Mount Stuart' (Tamil Nadu), Chacko (11) concluded that a sample of strips should be three times as large as sample of individual plots in order to give an estimate of equal precision. 5 to $10 \%$ strip enumerations carried out in sal and chir forests in U.P., however, led Gupta (33) to conclude that (i)they are sufficient for estimation of total stock in terms of values and good enough forindividual size classes for a particular species, (ii) systematic in comparison to randomsampling gave more accurate an precise results provided fertility gradient was taken intoaccount, (iii) irrespective of the sampling intensity, the optimum number of samplingunits was found to be 200 and (iv) results could not be relied upon for mixed crops.
Topographical units - Topographical sampling unit is defined as a samplingunit whose boundaries are predominantly topographical or natural features such as nalas, streams, ridges, etc. These are shown on survey maps and can be fixed with exactitude on ground. Even artificial features such as roads, inspection paths, block, compartment or sub-compartment boundaries, can also be conveniently used as they are accurately marked on the maps and are clearly located on ground. The use of footpaths for the purpose is avoided unless they are verified to be accurately marked on the map. In cases where natural or artificial features may not provide the whole of theboundary of a sampling unit, lines may be run at fixed compass bearings for short distances to join such features. They are, then, accurately marked on the map to enable correct plan metering of the area of the unit.

Topographical units are mostly used as sampling units in hill forests where other survey methods cannot be conveniently and accurately carried out. After the sampling intensity has been decided compact portions of the total area to be enumerated (after deducting inaccessible or unworkable areas which are excluded from enumerations) are divided into topographical units of convenient size varying from 12 to 24 ha first on the map and then on the ground. The units are then
stratified into homogeneous blockssuch that the enumeration of at least two units per block gives the desired intensity ofsampling. In case a compact area is so small that the required number of sampling units cannot be accommodated there, two or more compact areas lying reasonably close may be treated as one unit. Broadly speaking, vegetation and productivity zones in hills are the function of altitude, aspect, slope and geology. Though it is not difficult to provide the same aspect throughout a stratification block, it is difficult to arrange unitsin such a way that they may be comparable with respect to relative altitude and slope.However, the best should be attempted.

Any difference in the location of the sampling unit between the map and ground can vitiate results. As the size of the sampling unit in this method is quite large, the number of sampling units is necessarily small and the standard error is higher than that in the method of plots or strips.
Cluster - The term cluster is used to define a sampling unit which is, in fact, agroup of smaller units. The cluster is the statistical unit where as the smaller ones are only record units. Information is collected separately in each record unit and is then merged with information from the other record units to constitute the information related to the sampling unit (cluster). The record units are not the statistical units. In no case should cluster sampling be understood as being synonymous with two stage sampling.

### 10.9. Point sampling - Horizontal and Vertical point sampling,

- Point sampling is a method of selecting trees to be tallied on the basis of their sizes ratherthan by their frequency of occurrence.
- It has been found that counting from a random point, the no. of trees whose breast-heightcross-section exceeds a certain critical angle when multiplied by a factor gives an unbiased estimate of basal area/ha.
- This technique is called: angle count cruising, Point Sampling, variable plot cruising, PPS (Probability proportional to size) sampling.
- Sample points are located within a forested tract, and a simple prism or angle gauge that subtends a fixed angle of view is used to sight in each tree diameter at breast height.
- Tree boles close enough to the observation point to completely fill the fixed
sighting angle are tallied; stem to small or too far away are ignored. The resulting tree tally may be used to compute basal area, volumes, or numbers of trees per unit area.
- Point sampling can be either horizontal (For basal area estimation) or vertical (For ht. estimation)
- horizontal sampling has been widely used
- The probability of tallying a given tree depends on its cross-sectional area and the sighting angle used. The smaller the angle, the more stems will be included in thesample.
- PS does not require direct measurements of either plot areas or tree diameters. A predetermined basal area factor (BAF) is established in advance of sampling and resulting tree tallies can be easily converted to basal area per unit area.
- BA conversion factors are dependent or the sighting angle (or critical angle) arbitrarily selected.


## 1. HORIZONTAL POINT SAMPLING

- In horizontal point sampling, a series of sampling points are selected randomly orsystematically distributed over the entire area to be inventoried.
- Trees around this point are viewed through any angle -gauge at breast height and all trees forming an angle bigger than the critical angle of instruments are counted.
- Even though all trees are of same basal area, some are counted while others are not because of being far away from sampling points and they do not form an angle bigger than critical angle.

- Inclusion of trees in tally for a givenangle depends upon (i) sizes of trees (ii) their distance from the sampling point.
- Basal area per ha = BAF x number of tally trees/ numbers of points
- Number of trees/ha $=(\mathrm{no}$. of trees tallied) $\times$ (per-ha conversion factor)/ total numbers of points (per-ha conversion factor= BAF/BA per tree)
- Volume/h = basal area x stand form
height
From fig, considering the tree section A
$\sin \theta / 2=\left(D_{a} / 2\right) / R_{a}=D_{a} / 2 R_{a}$
Similarly, considering the tree section B
$\sin \theta / 2=\left(D_{b} / 2\right) / R_{b}=D_{b} / 2 R_{b}$
$2 \operatorname{Sin} \theta / 2=D_{a} / R_{a}=D_{b} / R_{b}$
$K=D / R=2 \operatorname{Sin} \theta / 2$


## INSTRUMENTS USED IN HORIZONTAL POINT SAMPLING

- Angle gauge
- Wedge prism
- Spiegel Relaskop
- Tele Relaskop


## CHOICE OF INSTRUMENTS

- When steep slopes are regularly encountered, the Spiegel Relaskop is preferred.
- For relatively flat topography, either the wedge prism or the stick gauge may be used.
- The prism is primarily desirable for persons who wear eyeglasses.
- The simple stick gauge is preferred in dense stand.


## INTENSITY OF POINT SAMPLING

- There is no fixed plot size when using point sampling.
- Each tree has its own imaginary plot radius (depending on the BAF used), and the exact plot size cannot be easily determined, even after the tally has been made.
- However, approximations can be made on the basis of the average stem diameter encountered at a given point.
- From the statistical point view, however, the selection of trees according to size rather than frequency may more than offset this reduction of sample size and with an additional saving in time.
- Conversely, it must be remembered that smaller samples of any kind require larger expansion factors.
- Thus, when point sampling is adopted, the so-called borderline trees must always
be closely checked.
- The only accurate method of determining how many sample points should be measured is to determine the standard deviation (or coefficient of variation) of BA or volume per ha from a preliminary field sample.


## If the statistical approach is not feasible, the following rules of thumb will often provide acceptable results:

- If the BAF is selected according to tree size so that an average of 5 or 12 trees are counted as each point, use the same number of points.
- With a BAF 10 angle gauge and timber that averages 12 or 15 in. in diameter, use the same number of points.
- For reliable estimates, never use fewer than 30 points in natural timber stands or less than 20 points in even-aged plantations.


## ADVANTAGES

-It is not necessary to establish a fixed plot boundary; thus greater cruising speed is possible.

- Large high value trees are sampled in greater proportions than smaller stems.
-BA and volume per ha may be derived without direct measurement of stems.
-When volume per ha conversions are developed in advance of fieldwork, efficient volume determinations can be made in a minimum of time. Thus the method is particularly suited to quick, reconnaissance type cruises.


## DISADVANTAGES

-Heavy underbrush reduces sighting visibility and cruising efficiency.
-Because of the relatively small sized of sampling units, carelessness and errors in the tally (when expanded to tract totals) are likely to more serious than in plot cruising.

- Slope compensation causes difficulties that may result in large errors unless special care is exercised.
- Unless taken into account, problems can arise in edge-effect bias when sampling very small tracts or long, narrow tracts.


## 2. VERTICAL POINT SAMPLING

- A method for deriving the mean stand height
- Within a full 360 degree sweep around the sample point all trees appearing taller than acritical angle are counted.
- Instrument used - Conimeter
- This instrument subtends a critical angle of 45 degree.
- By using Conimeter, if the number of trees per hais N , then number of tree counted ( $n$ ) in the area can be counted from the Conimeter.
- The average height of the trees $(\mathrm{h})$ in meter (when the critical angle is 45 degree) is


For crop height, the height at which the ismetis kept will require to be added.

### 10.10. Central Tendency Measures

Central tendency: Generally it is found that in any distribution, values of the variable tend to cluster around a central value or centrally located observation of the distribution. This characteristic is known as central tendency. This centrally located value which represents the group of values is termed as the measure of central tendency e.g. an average is called measure of central tendency.

In real life, we collect data from population that has same characteristics for a particular objective. The data that are collected contain elements may have different information. Now how we can say anything about the nature of the data. In this scenario, we use a measure that provides an idea about the data. This measure is called 'central tendency measure'. Central tendency is a measure that provides a single value that represents a group of values. However, it should satisfy some certain conditions.

There are some properties that are expected from a central tendency measure:
(i) It should be defined in a rigid manner such that its meaning should be unambiguous.
(ii) It should be calculated on the entire observations in the data.
(iii) It should be calculated in a little time frame and in an easy manner.
(iv) It should possess mathematical properties so that we can further use it.
(v) It should not be influenced by the extreme values of the data.
(vi) It must be sensitive to the small changes in the data values.

Although while applying the measure on the data, one should keep in mind that the data should be collected from homogeneous group. If the data is not from the homogenous group, then the value that we get may lead to incorrect decision. For example, while calculating the average height of the students in a class one should first categorize the gender at first place after that calculate its mean value on category basis. If students are not categorized according to the gender then the average height of students lead to unrepresentative form. Hence one can solve this type of problem by categorizing heterogeneous data into homogenous one.

In Statistics, three measures are considered generally that is mean, median and mode.
Mean measure is further divided into three categories (i) Arithmetic Mean (ii) Geometric Mean and (iii) Harmonic Mean.

### 10.10.1. Types of Measure of Central Tendency

1. Arithmetic Mean
2. Geometric Mean
3. Harmonic Mean
4. Mode
5. Median

10.10.1.1. Arithmetic Mean or Simply Mean:
"A value obtained by dividing the sum of all the observations by the number of observation is called arithmetic Mean"

Mean= $\frac{\text { Sum of tho obsorvations }}{\text { Total number of observations }}$

| Methods | Ungrouped data | Grouped data |
| :---: | :---: | :---: |
| Direct Method | $\bar{x}=\frac{\sum x_{i}}{n}$ | $\bar{x}=\frac{\sum f x}{n} ;$ Here $n=\Sigma f$ |
| Short cut <br> Method | $\bar{x}=A+\frac{\sum D}{n}$ | $\bar{x}=A+\frac{\sum f D}{n} ;$ Here $n=\sum f$ |
|  | Where $D=X_{i}-A$ and A is the provisional or assumed mean. |  |
|  | $\bar{x}=A+\frac{\sum u}{n} \times h$ | $\bar{x}=A+\frac{\sum f u}{n} \times h$; Here $n=\sum f$ |
|  | Where $u=\frac{X_{i}-A}{h}$ and h is the common width of the class intervals |  |

Arithmetic Mean is among the most widely used measure of central tendency for a single representation of observations. The formula for the evaluation of the arithmetic mean (A.M) is given as


For example, let $23,24,25,35,32$ be 5 class size of different sections of 9 th class in a private school. We want to know about the average class size in 9th class. To calculate this we first calculate the sum of all class sizes and divide it by no of sections. The arithmetic mean is $27.8 \approx 28$. It means that there is approximately 28 students on average basis in each class.

This method cannot be applied if the no. of the elements in the data set are very large.

For the convenience purpose frequency tabulation method is used. One can recall the tabulation methods from the previous modules. Let $x_{1}, x_{2}, \ldots, x_{n}$ have frequencies $f_{1}$, $f_{2}, \ldots, f_{n}$ respectively that means that in the complete data set $x_{1}$ appears $f_{1}$ times and $x_{2}$ appears $f_{2}$ times and so on. Hence the formula for A.M. in this case will be:

$$
\bar{x}=\frac{\left\{f_{1} x_{1}+f_{2} x_{2}+\cdots+f_{n} x_{n}\right\}}{f_{1}+f_{2}+\cdots+f_{n} \mid}
$$

This method of calculating Arithmetic Mean in term of frequencies is known as Weighted Anithmetic Mean. As we are multiplying the $x_{i}$ 's with their corresponding frequencies $\left(f_{i}\right.$ ),
An example consists of $n o$. of child in 50 families in a locality is presented below will give you a better understanding of the weighted arithmetic mean.

| No. of child | 0 | 1 | 2 | 3 or above |
| :--- | :--- | :--- | :--- | :--- |
| Families | 10 | 15 | 20 | 5 |

Table 1
Now te calculate the average number of child in a family, One can use weighted arithmetic mean and the value is $1.4 \approx 1$. Hence on average there is a single child in each family.
Also in real life, we have large amount of data so as we know from the module "Diagrammatic and Graphical Representation of the Data" that in such type of cases, one can construct frequency table. For example, we are interested in the marks of the students spending hours on daily basis to learn through online program. A artificial data was available of 1000 students and information is given in the following table as:

| Time Spent (in hours) | No. of students |
| :--- | :--- |
| $0-4$ | 690 |
| $4-8$ | 285 |
| $8-12$ | 20 |
| $12-16$ | 5 |

Table 2
Now if one is interested to find the average no of hours spend on average basis using A.M. then the formula is

$$
\bar{x}=\frac{\left\{f_{1} x_{1}+f_{2} x_{2}+\cdots+f_{n} x_{n}\right\}}{f_{1}+f_{2}+\cdots+f_{n}}
$$

but here $x_{1}, x_{2}, \ldots, x_{n}$ are the midpoint of class interval. So the arithmetic mean of the data can be found as


Hence from the above example, we can conclude that 3.36 hours or 3 hours and 6 minutes on averageare spent by the students on daily basis.

## Merits:

(a) A. M. can be easily understood by a person and does not require any particular need to explainit.
(b) The calculation of the A.M. is very simple and does not require any calculations.
(c) It is derived without ordering the data like other measure of central tendency.
(d) It is considered as comparative very stable and good method of central tendency in comparisonto other measures.
(e) If one has large amount of data then it is considered as a good method of central tendency as the unusual observations in one direction can be offset by the unusual observations in otherdirection.

## Demerits:

There are also certain shortcoming of any measure and $A$. M. is not an exception. There are some important demerits of A.M given below:
(a) A.M. cannot be obtained through guesses like other measure mode etc. whether it is not considered as a major demerit but still it is. Also median can be located graphically but A.M. cannot.
(b) A.M. cannot be found for the data when one has missing observation in the data. Also for a small set of data it is highly sensitive the extreme values. In such cases, A.M. will lead to a wrong conclusions.
(c) A.M. cannot be evaluated for the qualitative characteristics that cannot be measured, like sex, level of illness etc. In such cases it is suitable to use other measure of central tendency like median.
(d) It may be possible that a value calculated through A.M. is difficult to interpret. For example, one can recall the value of A.M. evaluated from Table 1 which is 1.4 that has no interpretation as child is considered in unit not in fraction. So the 1.4 child is meaningless.
(e) It is possible that it may lead to false conclusions.

At the last one can say that A.M. is considered as a simplest measure of central tendency but it will mislead you when data are of heterogeneous form i.e. nonhomogeneous.

### 10.10.1.2. Geometric Mean

### 3.2 Geometric Mean

Geometric Mean is another method to find out the mean of $n$ observations. It is evaluated through the following method.
Let us suppose $x_{1}, x_{2}, \ldots, x_{n}$ are $n$ observations, then

$$
\text { G.M. }=\left(x_{1}, x_{2}, w_{1}, x_{n}\right)^{\frac{1}{2}}
$$

If $x_{1}, x_{2} \ldots x_{n}$ have frequencies $f_{1}, f_{2} \ldots, f_{n}$ respectively, for evaluating the G.M. the method is
where, $N=\sum f_{t}$ is the sum of all observation's frequency and is called the total frequency.
Hence, one can use the above method to find the G.M. of the observations. It is difficult to find out the G.M. by using this formula or one can use the above formula when the no of observations is very small. In this case, one can use log on both side of the formula. Then the modified form the above formula are given below respectively.

$$
\begin{align*}
& \log G . M .=\frac{1}{n} \sum \cdot \log \left(x_{i}\right)  \tag{1}\\
& \log G . M .=\frac{1}{N} \sum f_{l} \log \left(x_{i}\right) \tag{2}
\end{align*}
$$

Now, if one is using the formula of equation (1) and (2), then one has to take antilog at the end to find out the G.M. value.

$$
\begin{align*}
& \text { G.M. } \left.=\operatorname{Antuog}\left(\frac{1}{n}\right) ; \log \left(x_{1}\right)\right)  \tag{3}\\
& \text { G.M. }=\operatorname{Antilog}\left(\frac{1}{N} \Sigma f_{1} \log \left(x_{1}\right)\right) \tag{4}
\end{align*}
$$

Steps for the calculation of the G.M. for simple data using equation (3)
(i) Calculate $\log$ (base 10 ) of the observations
(ii) Take the sum of all $\log$ of observations i.e. $\sum \log \left(x_{1}\right)$.
(iii) To find the $\log G . M$. , divide $\sum \log \left(x_{1}\right)$ by total no. of observations.
(iv) Take antilog (base 10) of ${ }_{n}^{1}$. $\log (x)$ and this is G.M. of the required data.

Ques 1: Find out the G.M. of the following data given as $34,57,37,43.52,57,54,56$.
Ans In Table 3 , the $\log$ values are shown in second column.

| Ohservations | $\log \left(x_{1}\right)$ |
| ---: | :--- |
| 34 | 1.5314789 |
| 57 | 1.7558749 |

As we are using the G.M. as a measure of central tendency one must know where to use this measureto get more appropriate information. Hence it is essential to discuss about the merits and demerits of this measure.

## Merits of G.M.

(a) It is defined in a rigid manner.
(b) It is based on all the values of the data and cannot be calculated for data with missing values.
(c) G.M. can be calculated in those cases where only the total product of observations and number of observations are known without having knowledge of individual values.
(d) G.M. can be used for further mathematical treatment like rations and percentages. Therefore it can be used for further analysis like index number etc.
(e) The main advantage of G.M. is that it is not affected much by extreme large and small values and it gives more weight to small values in comparative to A.M. which gives more weight to large values.

## Demerits

There are few demerits of G.M.. These are
(a) Due to the complexity involved in the calculation of G.M. . It is difficult for a non-mathematicalperson to understand it.
(b) The calculation of G.M. requires the knowledge of logarithm that is difficult for a non-mathematical background person.
(c) G.M. cannot be calculated for the data which contain a value zero.
(d) G.M. is not useful for the data that have different signs that is some of the observations arepositive and other are negative. In such case, G.M. value has no meaning.
(e) G.M. value may not be the actual value of the observations like other measure of centraltendency.
(f) G.M. is not advisable to use in those cases where small observations must be given more weightsand higher observation must be given less weights.
Another measure of central tendency is Harmonic Mean (H.M.). This measure of centraltendency has a great importance in science and mathematical field.

### 10.10.1.3. Harmonic Mean

Harmonic mean is calculated by first taking the reciprocal of the observations and then taking the reciprocal of arithmetic of these reciprocal observations. Let us suppose $x_{1}$, $x_{2}, \ldots, x_{n}$ are $n$ observations in the dataset. Then

$$
H_{1} M_{1}=\frac{n}{\frac{1}{x_{1}}+\frac{1}{x_{2}}+\frac{1}{x_{n}}+\frac{n}{x_{1}} \frac{n}{x_{i}}}
$$

When observations are given with frequencies i.e. $x_{1}, x_{2}, \ldots, x_{n}$ with $f_{1}, f_{2}, \ldots, f_{n}$ are the corresponding frequency values thenthe H.M. is evaluated by using the following formula.

$$
H_{1} M=\frac{f_{1}+f_{2}+\cdots+f_{n}}{{\frac{L}{t_{1}}}_{x_{1}}^{t_{2}}+\ldots+t_{x_{3}}^{L_{2}}} F \frac{N}{\Gamma_{f_{i}} / x_{i}}
$$

where $N=\sum f_{i}$ is the sum of all observation's frequency andis called the total frequency.
Hamonic mean is basically used in the case where rates and ratio are involved like perhour, permorith, per litre, per min etc. It is used for to find out the average of differerit types of rates. These ates are basically used to explain the relationship between two units of opposite nature like as you increase the speed of a vehicle then this shortens the travelling time. Hence HM . is used to find out the average speedinkilometerper hour for a given distance.

In the following example, H.M. is evaluated to find out the average speed of 6 cars gुven in kilometer perhour.
Ques 4: Calculate the Harmonic Mean(H.M.) forthe series $56,45,67,58,53,42$.
Ans

| KMPH | Reciprocal of <br> observations |  |
| ---: | ---: | :---: |
| 56 | 0.017857143 |  |
| 45 | 0.022222222 |  |
| 67 | 0.014925373 |  |
| 58 | 0.017241379 |  |
| 53 | 0.018867925 |  |
| 42 | 0.023809524 |  |
| N=6 | 0.114923566 |  |
| H.M. | 52.20861322 |  |

## Table 8

In Table 8, the reciprocal of observation are grven in the second columm 0.114923566 represents the total of reciprocal of the observations and 52.2086 is the H M. of the data.

Similarly, one can keep the following steps in mind to find out the H.M. for observations given with frequencies. These are

Forungrouped frequency data, Steps are
(i) Find out the reciprocal of the observations.
(ii) Multiply the reciprocal observations with the corresponding frequencies values and take theirsum i.e $\sum f_{i} / x_{i}$.
(iii) Divide N by the term i.e $\sum f_{i} / x_{i}$ where N is the total frequency.
(iv) This will give you the value of H.M. for discrete frequency data.

## For grouped frequency data. Steps are

(i) First calculate the mid points $\left(m_{i}\right)$ of the class intervals.
(ii) Multiply the reciprocal observations with the corresponding frequencies values and take theirsum i.e $\sum f_{i} / m_{i}$.
(iii) Divide N by the term i.e $\sum f_{i} / m_{i}$ where N is the total frequency.
(iv) This will give you the value of H.M. for grouped frequency data.

Hence, one can evaluate the H.M. for three different situations i.e. simple, discrete frequency andgrouped frequency.

## Merits

(a) H.M. is defined in a rigid manner.
(b) H.M. is evaluated on all the observations in the data.
(c) H.M. gives less weight to large values and more weight to small values so this measure isappropriate in those case where such condition is required.
(d) H.M. is used to measure relative changes and is widely used for the averaging of ratio andrates.

## Demerits

(a) H.M. is very difficult measure to be understood by a non-mathematical background person.
(b) H.M. is comparatively difficult to calculate than other measure.
(c) H.M. cannot be evaluated if the value of the observation is zero.
(d) H.M. cannot be evaluated when some of the observations are positive and some are negative inthe data set.
(e) H.M. value does not exist in the data set.

## Relationship between A.M., G.M. and H.M.

A.M. is always greater than or equal to G.M. and G.M. is greater than or equal to H.M. for a givenobservations. In mathematical term

$$
A . M . \geq G . M . \geq H . M .
$$

These measures are only equal when all the observations are same in the data.

### 10.10.1.4. MEDIAN

Median is that positional value of the variable which divides the distribution into two equal parts, one part comprises all values greater than or equal to the median value and the other comprises all values less than or equal to it. The Median is the "middle" element when the data set is arranged in order of the magnitude. Since the median is determined by the position of different values, it remains unaffected if, say, the size of the largest value increases.

## Computation of median

The median can be easily computed by sorting the data from smallest to largest and finding out the middle value.

## Example 1

Suppose we have the followingobservation in a data set: $5,7,6,1,8,10,12$, 4 , and 3.
Arranging the data, in ascending orderyou have:

$$
1,3,4,5,6,7,8,10,12
$$

The "middle score" is 6, so the median is 6 . Half of the scores are larger than 6 and half of the scores are smaller. If there are even numbers in the data, there will be two observations

## Activities

- Find mean and median for all four values of the series. What do you observe?

TABLE 5.4
Mean and Median of different series

| Series | $X$ (Variable <br> Values) | Mean | Median |
| :--- | :--- | :---: | :---: |
| A | $1,2,3$ | $?$ | $?$ |
| B | $1,2,30$ | $?$ | $?$ |
| C | $1,2,300$ | $?$ | $?$ |
|  | 1.2 .3000 | $?$ | $?$ |

- Is median affected by extreme values? What are outliers?
- Is median a better method than mean? which fall in the middle. The median in this case is computed as the arithmeticmean of the two middle values.

The following data provides marks of 20 students. You are required to calculate the median marks.
$25,72,28,65,29,60,30,54,32,53,33,52,35,51,42,48,45,47$, 46, 33.

Arranging the data in an ascendingorder, you get
$25,28,29,30,32,33,33,35,42,45,46,47,48,51,52,53,54,60$, 65, 72.

You can see that there are two observations in the middle, 45 and 46. The median can be obtained by taking the mean of the two observations:

$$
\text { Median }=\frac{45+46}{2}=45.5 \text { marks }
$$

In order to calculate median it is important to know the position of the median i.e. item/items at which the median lies. The position of the mediancan be calculated by the following formula:

$$
\text { Position of the median }=\frac{(N+1) t h}{2} \text { item }
$$

Where $\mathrm{N}=$ number of items.
You may note that the above formula gives you the position of the median in an ordered array, not the median itself. Median is computed by the formula:

$$
\text { Median }=\text { size of } \frac{(N+1)+h}{2} \text { item }
$$

## Discrete Series

In case of discrete series the position of median i.e. $(\mathrm{N}+1) / 2^{\text {th }}$ item can be located through cumulative frequency. The corresponding value at this position is the value of median.

## Example 2

The frequency distributsion of the number of persons and their respective incomes (in Rs) are given below. Calculate the median income.


In order to calculate the median income, you may prepare the

frequency distribution as given below.

TABLE 5.5
Computation of Median for Discrete Series

| Income(in Rs) |  | No. of persons(f) |
| :--- | :--- | :--- |
| 10 | 2 | Cumulative frequency(cf) |
| 20 | 4 | 6 |
| 30 | 10 | 16 |
| 40 | 4 | 20 |

The median is located in the $(\mathrm{N}+1) / 2=(20+1) / 2=10.5^{\text {th }}$ observation. This can be easily located through cumulative frequency. The $10.5^{\text {th }}$ observation lies in the c.f. of 16 . The income corresponding to this is Rs 30 ,so the median income is Rs 30.

## Continuous Series

In case of continuous series you have to locate the median class where $N / 2^{\text {th }}$ item $\left[\operatorname{not}(N+1) / 2^{\text {th }}\right.$ item $]$ lies. Themedian can then be obtained as follows:

$$
\text { Median }=L+\frac{\left.\left(\frac{N}{2}-\text { e.f. } .\right)\right)_{t h}}{f} X h
$$

Where, $L=$ lower limit of the median class,
c.f. $=$ cumulative frequency of the class preceding the median class, $\mathrm{f}=$ frequency of the median class, $\mathrm{h}=$ magnitude of the median classinterval. No adjustment is required iffrequency is of unequal size or magnitude.

## Example 3

Following data relates to daily wages of persons working in a
factory. Compute the median daily wage.

| Daily wages (in <br> Rs.) | $55-60$ | $50-55$ | $45-50$ | $40-45$ | $35-40$ | $30-35$ | $25-30$ | $20-25$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Number <br> workers | 7 | 13 | 15 | 20 | 30 | 33 | 28 | 14 |

The data is arranged in descending order here. In the above illustration median class is the value of ( $\mathrm{N} / 2)^{\text {th }}$ item (i.e. 160/2) $=80^{\text {th }}$ item of the series, which lies in 35-40 class interval. Applying the formula of the median as:

TABLE 5.6
Computation of Median for Continuous Series

| Daily wages(in Rs) | No. of Workers (f) | Cumulative Frequency |
| :--- | :--- | :--- |
| $20-25$ | 14 | 14 |
| $25-30$ | 28 | 42 |
| $30-35$ | 33 | 75 |
| $35-40$ | 30 | 105 |
| $40-45$ | 20 | 125 |
| $45-50$ | 13 | 140 |
| $50-55$ | 7 | 153 |
| $55-60$ |  | 160 |

$$
\begin{aligned}
\text { Median } & =\mathrm{L}+\frac{(\mathrm{N} / 2-\text { c.f. })}{\mathrm{f}} \times \mathrm{h} \\
& =\frac{35+(80-75)}{30} \times(40-35) \\
& =\text { Rs } 35.83
\end{aligned}
$$

Thus, the median daily wage is Rs 35.83 . This means that $50 \%$ of the workers are getting less than or equal to Rs 35.83 and $50 \%$ of the workers are getting more than or equal to thiswage.

## Quartiles

Quartiles are the measures which divide the data into four equal parts, each portion contains equal number of observations. There are three quartiles. The first Quartile (denoted by $\mathrm{Q}_{1}$ ) or lower quartile has $25 \%$ of the items of the distribution below it and $75 \%$ of the items are greater
 than it. The second quartiles (denoted by $Q_{2}$ ) or median has $50 \%$ of the observations above it. The third quartiles (denoted by $Q_{3}$ ) or upper quartiles has $75 \%$ of the items of the distribution below it and $25 \%$ of the items above it. Thus, Q1 and Q3 denote the two limits within which central $50 \%$ of the data lies.

## Percentiles

Percentiles divide the distribution intohundred equal parts, so you can get 99 dividing positions denoted by $\mathrm{P}, \mathrm{P}, \mathrm{P}_{3}, \ldots, \mathrm{P} 99 . \mathrm{P}_{50}$ is the median value. If you have secured 82 percentile in a management entrance examination, it means that your position is below 18 per cent of total candidates appeared in the examination. If a total of one lakh students appeared, where do you stand?

## Calculation of Quartiles

The method for locating the Quartile issame as that of the median in case ofindividual and discrete series. The value of $Q_{1}$ and $Q_{3}$ of an ordered series can be obtained by the following formula where N is the number ofobservations.

$$
\begin{aligned}
& Q_{1}=\text { size of } \frac{(N+1) t h}{4} \text { item } \\
& Q_{3}=\text { size of } \frac{3(N+1) t h}{4} \text { item }
\end{aligned}
$$

## Example 4

Calculate the value of lower quartile from the data of the marks obtainedby ten students in an examination. 22, 26, 14, 30, 18, 11, 35, 41, 12, 32.

Arranging the data in an ascendingorder, $11,12,14,18,22,26,30,32,35,41$.
$Q_{1}=$ size of $\frac{(N+1) \text { th }}{4}$ item $=$ size of $\frac{(10+1) \text { th }}{4}$ item $=$ size of 2.75 th item $=2$ nd item $+0.75(3$ rd item -2 nd item $)=12+0.75(14-12)=13.5$ marks.

### 10.10.1.5. Mode

Sometimes, you may be interested inknowing the most typical value of a series or the value around which maximum concentration of items occurs. For example, a manufacturer would like to know the size of shoes that has maximum demand or style of the shirt that is more frequently demanded. Here, Mode is the most appropriate measure. The word mode has been derived from the French word "la Mode" which signifies the most fashionable values of a distribution, because it is repeated the highest number of times in the series. Mode is the most frequently observed data value. It is denoted by $\mathrm{M}_{\mathrm{O}}$.

## Computation of Mode

## Discrete Series

Consider the data set $1,2,3,4,4,5$. The mode for this data is 4 because 4 occurs most frequently (twice) in the data.

## Example 5

Look at the following discrete series:

| Variable | 10 | 20 | 30 | 40 | 50 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Frequency | 2 | 8 | 20 | 10 | 5 |

Here, as you can see the maximum frequency is 20, the value of mode is 30 . In this case, as there is a unique value of mode, the data is unimodal. But, the mode is not necessarily unique, unlike arithmetic
mean and median. You can have data with two modes (bi-modal) or more than two modes (multi-modal). It may be possible that there may be no mode ifno value appears more frequent thanany other value in the distribution. Forexample, in a series $1,1,2,2,3,3,4,4$, there is no mode.


## Continuous Series

In case of continuous frequency distribution, modal class is the class with largest frequency. Mode can be calculated by using the formula:

$$
M_{\mathrm{o}}=\mathrm{L}+\frac{\mathrm{D}_{1}}{\mathrm{D}_{1}+\mathrm{D}_{2}} \times \mathrm{h}
$$

Where $L=$ lower limit of the modal class $D_{1}=$ difference between the frequency of the modal class and the frequency of the class preceding the modal class (ignoring signs).
$\mathrm{D}_{2}=$ difference between the frequency of the modal class and the frequency ofthe class succeeding the modal class (ignoring signs). $\mathrm{h}=$ class interval of the distribution.

You may note that in case of continuous series, class intervals should be equal and series should be exclusive to calculate the mode. If mid points are given, class intervals are tobe obtained.

## Example 6

Calculate the value of modal workerfamily's monthly income from the following data:

| Income <br> per month <br> (in 000 Rs) | Less <br> than <br> 50 | Less <br> than <br> 45 | Less <br> than <br> 40 | Less <br> than <br> 35 | Less <br> than <br> 30 | Less <br> than <br> 25 | Less <br> than <br> 20 | Less <br> than <br> 15 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Cumulative <br> frequency | 97 | 95 | 90 | 80 | 60 | 30 | 12 | 4 |

As you can see this is a case of cumulative frequency distribution. In order to calculate mode, you will have to convert it into an exclusive series. Inthis example, the series is in the descending order. This table should be converted into an ordinary frequency table (Table 5.7) to determine the modal class.

| Income Group <br> (in '000 Rs) | Frequency |
| :--- | :--- |
| $45-50$ | $97-95=2$ |
| $40-45$ | $95-90=5$ |
| $35-40$ | $90-80=10$ |
| $30-35$ | $80-60=20$ |
| $25-30$ | $60-30=30$ |
| $20-25$ | $30-12=18$ |
| $15-20$ | $12-4=8$ |
| $10-15$ |  |

The value of the mode lies in 25-30 class intervals. By inspection also, it can be seen that this is a modalclass.
Now $L=25, D_{1}=(30-18)=12, D_{2}=(30-20)=10, h=5$
Using the formula, you can obtainthe value of the mode as:
$\mathrm{M}_{\mathrm{O}}$ (in '000 Rs)

$$
\begin{gathered}
\mathrm{M}_{\mathrm{o}}=\mathrm{L}+\frac{\mathrm{D}_{1}}{\mathrm{D} 1+\mathrm{D} 2} \times \mathrm{h} \\
=25+\frac{12}{12+10} \times 5=27.273
\end{gathered}
$$

Thus the modal worker family'smonthly income is Rs 27.273 .

## RELATIVE POSITION OF ARITHMETIC MEAN, MEDIAN AND MODE

Suppose we express, Arithmetic Mean $=M_{e}$ Median $=M_{i}$, Mode $=M_{0}$
The relative magnitude of the three are $M_{e}>M_{i}>M_{0}$ or $M_{e}<M_{i}<M_{0}$ (suffixes occurring in alphabetical order). The median is always between the arithmetic mean and the mode.

### 10.11. Dispersions:

## Definitions

Some authors have defined the measures of dispersion as: "Dispersion is the measure of variation of the items" by A.L. Bowley.
"Dispersion or spread is the degree of the scatter or the variation of the variable about a central value" by B.C. Brooks and W. F. L. Dicks.
In literature, the dispersion is also considered as synonym for heterogeneous in the data. As heterogeneous is basically used to understand the extent of variations among observations. Dispersion can only be zero if all the observations have same values. The dispersion is more when the difference between the observations is very large. So one can say that if the variation is small like in data set I then it is considered as insignificant but if the variation is large as shown from the data set II then it is considered as significant.
As measures of dispersion are basically used to discuss about the variation, scatterness of the observations from the central tendency measure. The measures of central tendency and measures of dispersion both together discuss about the characteristics of the data set but they do not able to demonstrate that, to what extent the observations deviate from the central value i.e. whether equal number of observations are dispersed from the central tendency or whether the data is symmetrical about the mean or not. It also give us an idea of how many observations have their value below the mean value or above the mean value. If one is interested to know about the concentration of the observations around a central tendency measure then it is essential to study two more measures. These are Skewness and Kurtosis. These two measure are considered as a supportive measures for better understanding the characteristics of the data. Skewness and Kurtosis is discussed in the module "Measure of Dipersion -II with Skewness and Kurtosis".

### 10.11.1. Importance of measures of dispersions

(a) The main motive behind using these measures to check the authenticity of the central tendency measures. It is used to see whether the value of central tendency measure are reliable or not.
(b) The second important thing about the measures of central tendency is that these are also used to compare the two datasets through relative values.
(c) This measure is also useful in identifying the reasons behind the variations in order to control them but these are not helpful in give the exact reason behind variations in the observations. For example, in electronic industry quality of an item cannot be judged only by through whether an item is produced under defined limits
but also through the variations between the characteristics of observations.
(d) Measures of dispersions are also used to help for further analysis of the data like correlation, regression, testing of hypothesis and ANOVA etc
As there are many measures of dispersion, the ideal measure prevails the following characteristics:
(a) The values of these measures should be rigid.
(b) It should be calculated on all the observations.
(c) The method should be easily calculated and understood by nonmathematical background person.
(d) The measure should be used for further algebraic treatment.
(e) The measure should not be affected by extreme values and fluctuation of the observations.
The measures of dispersion are further categorized into two types. These are shown in the following flowchart


Figure 1
From the Figure 1, the measure of dispersion is categorized into two measures: Absolute Measures and Relative Measures. As dispersion measure is used to detect the deviation of the observations from the central tendency. If the measures of dispersion express the dispersion of the observations in the original units then the measures are called absolute measures of dispersion. One can only compare the variations between two series if both series are in same units otherwise comparison is not meaningless. To overcome this problem, those measures that give the dispersion values in terms of ratio and percentageare called relative measures.

A relative measure of dispersion is the ratio of absolute measure of dispersion with its appropriate average. These measure are independent of units and these are termed as coefficient of dispersion. One important thing, while the calculation of
relative measures, is that the units of absolute measure and theappropriate average must be same.

### 10.11.2. Mean Deviation

## Mean deviation

Mean deviation is computed as the absolute value of difference of observations from a central tendency i.e. mean, median and mode. Mostly, mean deviation is calculated by taking deviation of observations from mean and median.
Let $x_{1}, x_{2}, \ldots, x_{n}$ be the $n$ observations to compute mean deviation. First assume a number A and take absolute deviation of each observations from the value A i.e. $\left|x_{1}-A\right|,\left|x_{2}-A\right|_{\text {,wew }}$
$\left|x_{n}-A\right|$. As some of the observations are more than A and some of them are less so modulus is taken to accumulate the total deviation of observation from a point $A$.
Now the sum of deviated observations from $A$ is $\sum\left|x_{i}-A\right|$ and the arithmetic mean of these. observation will give mean deviation about $A$.

$$
\text { Mean deviation about } A=\frac{1}{n} \sum\left|x_{i}-A\right|
$$

If A is chosen as mean value then the deviation is called mean deviation about mean and if A is chosen as median then the deviation is called mean deviation about median.
The above formula is used for simple series data. For ungrouped frequency distribution, the formula will be

$$
\text { Mean deviation about } A=\frac{1}{\bar{N}} \sum f_{\mathrm{i}}\left|x_{\mathrm{i}}-A\right|
$$

For grouped frequency distribution, the formula will be

$$
\text { Mean deviation about } A=\frac{1}{N} \sum f_{i}\left|m_{i}-A\right|
$$

where $N$ is the total frequency;
$f_{i}$ is the frequency corresponding to the ith observation; $m_{i}$ be the midpoint of the ith class interval.

Ques 1. Calculate the mean deviation about mean for the following observations.30, $40,50,55,60,65,70,80,90,100$

Ans: First find out the A.M. of the observations. It is 64 . Now take absolute deviation of each observation from 64 . The values are $34,24,14,9,4,1,6,16,26$. Take the sum of these observationand divide it by 10 i.e. number of observations.

## The answer is $134 / 10=13.4$.

Ques 2. Calculate the mean deviation about mean for the following dataset.

| Observations | frequency |
| :---: | :---: |
| 38 | 7 |
| 43 | 9 |
| 46 | 10 |
| 49 | 6 |
| 51 | 4 |
| 54 | 8 |
| 67 | 3 |
| 78 | 5 |

Ans. First compute A.M. of the data using the formula $\sum f_{\mathrm{i}} \mathrm{x}_{\mathrm{i}} / \mathrm{N}$ where $N=\Sigma f$.
. After that subtract this mean value from each observation and take modulus.
Now multiply this absolute value with the corresponding frequency. Now compute the A.M of these absolute observation. Hence 8.22 is the mean deviation about mean.

| Observations | frequency | $f_{i} x_{i}$ | deviation from mean $x_{\boldsymbol{i}}{ }^{-} \boldsymbol{x}$ | absolute deviation $f$ aboutmean $\mid x_{i}-\nmid x$ | $\left\|f i^{*}\right\| x_{i}-\not x$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 38 | 7 | 266 | -12.65 | 12.65 | 88.58 |
| 43 | 9 | 387 | -7.65 | 7.65 | 68.88 |
| 46 | 10 | 460 | -4.65 | 4.65 | 46.54 |
| 49 | 6 | 294 | -1.65 | 1.65 | 9.92 |
| 51 | 4 | 204 | 0.35 | 0.35 | 1.38 |
| 54 | 8 | 432 | 3.35 | 3.35 | 26.77 |
| 67 | 3 | 201 | 16.35 | 16.35 | 49.04 |
| 78 | 5 | 390 | 27.35 | 27.35 | 136.73 |
|  | $N=52$ | $\begin{aligned} & \sum_{=} f_{i} x_{i} \end{aligned}$ |  |  | 427.84 |
|  | mean | $\begin{aligned} & \sum f_{i} x_{i} \\ & = \\ & 50.65 \end{aligned}$ |  |  | $\begin{aligned} & \sum \boldsymbol{f}_{\underline{i}} \underline{x} \underline{\underline{i}} \underline{=x} \mid \\ & \boldsymbol{N} \\ & 8.22 \end{aligned}$ |

Ques 3 Calculate the mean deviation about mean for the following dataset.

| Class Interval | Frequency |
| :--- | :---: |
| $0.0-5.0$ | 4 |
| $5.0-10.0$ | 6 |
| $10.0-15.0$ | 8 |
| $15.0-20.0$ | 5 |
| $20.0-25.0$ | 7 |
| $25.0-30.0$ | 3 |

Ans: First compute A.M. of the data using the formula $\frac{\Sigma \text { fimi }}{N}$ where $N=\sum f_{\mathrm{i}}$ and $m_{l}$ is the mid point of the ith class interval. The values are shown in the Table 4. After
that subtract this mean value from midpoints and take modulus. Now multiply this absolute value with the corresponding frequency. Now compute the A.M. of these absolute observation. Hence the mean deviation about mean for grouped frequency distribution dataset is 6.556 .

| Class Interval | mid value $(m i)$ | $\begin{aligned} & \text { Frequency } \\ & \qquad f_{i} \end{aligned}$ | $\begin{aligned} & f \\ & i \\ & m \\ & i \end{aligned}$ | deviatio <br> n from <br> mean $[0]$ | absolu te <br> deviati on about mean | $\eta^{*} \mid m_{i}-\bar{k}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0-5.0 | 2.5 | 4 | 10 | -12.12 | 12.12 | 48.48 |
| 5.0-10.0 | 7.5 | 6 | 45 | -7.12 | 7.12 | 42.72 |
| 10.0-15.0 | 12.5 | 8 | 100 | -2.12 | 2.12 | 16.96 |
| 15.0-20.0 | 17.5 | 5 | 87.5 | 2.87 | 2.87 | 14.39 |
| 20.0-25.0 | 22.5 | 7 | 157.5 | 7.87 | 7.87 | 55.15 |
| 25.0-30.0 | 27.5 | 3 | 82.5 | 12.87 | 12.87 | 38.63 |
|  |  | $N=33$ | $\sum f_{i} m_{i}=482.5$ |  |  | 216.36 |
|  |  |  | $\begin{gathered} \quad \bar{x}= \\ \sum f_{i} m_{i} \\ =14.62 \\ N \end{gathered}$ |  |  | $\begin{gathered} \sum \underline{f} \underline{i} \underline{\underline{m}} \underline{i-x} \mid \\ \boldsymbol{N} \\ =6 . \\ 55 \\ 6 \end{gathered}$ |

## Merits and Demerits of Mean deviation

## Merits

There are some merits of mean deviation measure. These are
(i) Mean deviation is defined in a rigid manner.
(ii) Mean deviation is an easy method for a non-mathematical background person.
(iii) Mean deviation is based on all the observations so it is better than range and quartiledeviation measures.
(iv) Mean deviation is less influenced by the extreme values than range, standard deviation.
(v) Mean deviation is appropriate for comparison purpose because it is derived from thedeviation of observations from central values i.e. mean, median or mode.
(vi) Mean deviation is basically an average of absolute deviation from central value. Hence it remove biasness to some extent that occur due to observations while calculating and provide an accurate value for dispersion.

## Demerits

There are few demerits of mean deviation measure. These are
(i) The major drawback of mean deviation measure is that while computing it we ignore thesigns of deviation and take absolute of it. So one cannot get any information about the concentration of the observations above the mean or below the mean. So this measure is not useful for further mathematical treatment.
(ii) Mean deviation cannot be computed for open end classes distributions.
(iii) Mean deviation is not an accurate measure of dispersion for highly skewed data.

Although mean deviation has some limitations. Due to its simplicity and easy to understand features it has great importance in accountancy, economics, forecasting and business sectors.

### 10.11.3. Standard Deviation

Karl Pearson in 1823 first introduced this measure and after that it is the most widely used measure of dispersion till date. Standard deviation is the measure that prevails all the features that other measures lack of. So basically it is considered as an ideal measure of deviation. It is also know as square root of variance, root mean square deviation etc and denoted by the Greek letter $\sigma$ (sigma). Standard deviationvalue is high or low when there is more or less variation among observations respectively.

Let $x_{1}, x_{2}, \ldots, x_{n}$ be the n observations and $\bar{x}$ be the average value, then the standard deviation (S.D.) isevaluated as

$$
\text { S.D. }=\frac{\sqrt{\sum\left(x_{i}-\bar{x}\right)^{2}}}{n}
$$

For frequency data, the formula is

$$
\text { S.D. }=\mathrm{v} \frac{\overline{\overline{\sum f_{i}\left(x_{i}-\bar{x}\right)^{2}}}}{N}
$$

For grouped frequency data, the formula is

$$
\text { S.D. }=\sqrt{\frac{\sqrt{\sum f_{i}\left(m_{i}-\bar{x}\right)^{2}}}{N}}
$$

where $N$ is the total frequency;
$f_{i}$ is the frequency corresponding to the ith observation;
$m_{i}$ be the midpoint of the ith class interval.
Ques 6: Calculate standard deviation of the following dataset. $30,40,50,55,60,65$, 70, 80, 90, 100

Ans. First calculate the mean of the observations i.e. 64. Now subtract each value from 64 and take square. Add the square for all observation i.e. ( $30-$ $64)^{2}+(40-64)^{2}+\cdots+(100-64)^{2}$ which is 4290 . Divide this sum by 10 and take a square root.

Standard deviation value is $\frac{\sqrt{4290}}{10}=20.712$.

As the observations are very far away from the mean value. Hence the standard deviation is coming outto be very high.

Ques 7: Calculate the standard deviation for the following dataset.

| Observations | frequency |
| :---: | :---: |
| 38 | 7 |
| 43 | 9 |
| 46 | 10 |
| 49 | 6 |
| 51 | 4 |
| 54 | 8 |
| 67 | 3 |
| 78 | 5 |

Ans. Calculate the mean of the observations as shown in Table 6. Subtract the mean from theobservations as shown in $4^{\text {th }}$ column. Now take square of these observations as shown in the $5^{\text {th }}$ columnin Table 6 . Column 6 shows the product of frequencies with values from column 5 . Now take sum of the observations in column 5 and divide it by $N$ i.e. total frequency. Take square root of the value 125.22 and the require standard deviation is 11.190 .

| Observations <br> $\left(\boldsymbol{x}_{\boldsymbol{i}}\right)$ | Frequency <br> $(\boldsymbol{f} \boldsymbol{i})$ | $\boldsymbol{f}_{\boldsymbol{i}} \boldsymbol{x} \boldsymbol{i}$ | $\boldsymbol{x}_{\boldsymbol{i}}-\overline{\boldsymbol{x}}$ | $\left(\boldsymbol{x}_{\boldsymbol{i}}-\overline{\boldsymbol{x}}\right)^{\mathbf{2}}$ | ${\boldsymbol{f} \boldsymbol{i} *\left(\boldsymbol{x}_{\boldsymbol{i}}-\overline{\boldsymbol{x}}\right)^{\mathbf{2}}}^{338}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 7 | 266 | -12.65384 | 160.11 | 1120.838757 |  |
| 43 | 9 | 387 | -7.653846 | 58.58 | 527.2322485 |
| 46 | 10 | 460 | -4.653846 | 21.65 | 216.5828402 |
| 49 | 6 | 294 | -1.653846 | 2.735 | 16.4112426 |


| 51 | 4 | 204 | 0.346153 | 0.119 | 0.479289941 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 54 | 8 | 432 | 3.346153 | 11.19 | 89.5739645 |
| 67 | 3 | 201 | 16.34615 | 267.19 | 801.5902367 |
| 78 | 5 | 390 | 27.34615 | 747.81 | 3739.060651 |
|  | $N=52$ | $\sum_{2634} f_{i} x_{i}$ |  |  | $\begin{aligned} & \sum_{i f} *\left(x_{i}-\bar{x}\right)^{2}= \\ & 6511.769231 \end{aligned}$ |
|  | Mean | $\begin{aligned} & \sum f_{i} x_{i} \\ & N \\ & =50.653 \end{aligned}$ |  |  | $\begin{array}{ll} 2 & f i(x i-\bar{x}) \\ \sum_{125.22} & N \end{array}$ |
|  |  |  |  |  | $\begin{aligned} & \sqrt{ } \frac{f i(x i \underline{i}-\bar{x}) 2}{N}= \\ & 11.19 \end{aligned}$ |

Ques 8. Calculate the standard deviation for the following dataset.

| Class Interval | Frequency |
| :--- | ---: |
| $0.0-5.0$ | 4 |
| $5.0-10.0$ | 6 |
| $10.0-15.0$ | 8 |
| $15.0-20.0$ | 5 |
| $20.0-25.0$ | 7 |
| $25.0-30.0$ | 3 |

Ans. First calculate the mid point of the class intervals then calculate the mean of the observations as shown in Table 8. Subtract the mean from the midpoint as shown in 5th column. Now take square of these observations as shown in the 6th column in Table 8. Column 7 shows the product of frequencies with values from column 6 . Now take sum of the observations in column 7 and divide it by $N$ i.e. total frequency. Take square root of the value 57.62 and the require standard deviation is 7.59 .

| Class <br> Interval | mid <br> value <br> $(\boldsymbol{m} \boldsymbol{i})$ | Frequency <br> $(\boldsymbol{f} \boldsymbol{i})$ | $\boldsymbol{f i m i}$ | $\left(\boldsymbol{m i}_{\boldsymbol{i}}-\overline{\boldsymbol{x}}\right.$ | $\left(\boldsymbol{m} \boldsymbol{i}-\bar{\chi}^{2}\right.$ | $\boldsymbol{f} \boldsymbol{i}\left(\boldsymbol{m} \boldsymbol{i}-\bar{\chi}^{2}\right.$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $0.0-5.0$ | 2.5 | 4 | 10 | -12.1212 | 146.92 | 587.6939578 |
| $5.0-10.0$ | 7.5 | 6 | 45 | -7.1212 | 50.71 | 304.2689366 |
| $10.0-15.0$ | 12.5 | 8 | 100 | -2.1212 | 4.499 | 35.99591552 |
| $15.0-20.0$ | 17.5 | 5 | 87.5 | 2.8788 | 8.287 | 41.4374472 |


| 20.0-25.0 | 22.5 | 7 | 157.5 | 7.8788 | 62.07 | 434.5284261 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 25.0-30.0 | 27.5 | 3 | 82.5 | 12.8788 | 165.86 | 497.5904683 |
|  |  | $N=33$ | $\sum_{482.5}^{f_{i m}}=$ |  |  | $\begin{aligned} & \sum \boldsymbol{f}_{\boldsymbol{i}} *\left(\boldsymbol{m}_{\boldsymbol{i}}-\overline{\boldsymbol{x}}{ }^{\mathbf{2}}\right. \\ & =1901.515152 \end{aligned}$ |
|  |  |  | $\begin{aligned} & \bar{x} \\ & \sum f_{i} m_{i}= \\ & N \\ & 14.6212 \end{aligned}=$ |  |  | $\begin{aligned} & \sum_{\boldsymbol{N}} f_{i}(m i-x)= \\ & 57.62167126 \end{aligned}$ |
|  |  |  |  |  |  | $\sum_{\boldsymbol{N}}^{\boldsymbol{f}_{i}\left(\boldsymbol{m}_{\boldsymbol{i}}-\bar{x}\right)^{2}}=$ |

Table 8

## Merits and Demerits of Standard deviation

 MeritsAs standard deviation is considered as the best absolute measure of dispersion.
These are the merits ofstandard deviation measure
(i) Standard deviation is rigidly defined measure.
(ii) Standard deviation is based on all the observations.
(iii) Standard deviation is evaluated by squaring of deviation of observations from the mean. This makesthe standard deviation for further mathematical treatment.
(iv) Standard deviation has less influenced by the change in the sample observations.
(v) It is also possible to compute the combined standard deviation of two or more dataset fromindividual standard deviation.
(vi) Standard deviation is used for further statistical analysis like skewness and correlation.

## Demerits

(i) Although standard deviation is considered as the best absolute measure of dispersion but it is not freefrom demerits. These are Standard deviation is very difficult to understand for a non-mathematical background person.
(ii) Standard deviation gives more weight to extreme values and less weight to values close to mean. As while squaring the deviation this bias will be increased.

Although, standard deviation has some demerits or limitations but this is the widely used method for theabsolute measure of dispersion.

### 10.12. Correlation and regression.

## Meaning:

Correlation is a statistical technique to ascertain the association or relationship between two or more variables. Correlation analysis is a statistical technique to study the degree and direction of relationship between two or more variables.

A correlation coefficient is a statistical measure of the degree to which changes to the value of one variable predict change to the value of another. When the fluctuation of one variable reliably predicts a similar fluctuation in another variable, there's often a tendency to think that means that the change in one causes the change in the other.

## Uses of correlations:

1. Correlation analysis helps inn deriving precisely the degree and the direction of such relationship.
2. The effect of correlation is to reduce the range of uncertainity of our prediction. The prediction based on correlation analysis will be more reliable and near to reality.
3. Correlation analysis contributes to the understanding of economic behaviour, aids in locating the critically important variables on which others depend, may reveal to the economist the connections by which disturbances spread and suggest to him the paths through which stabilizing farces may become effective
4. Economic theory and business studies show relationships between variables like price and quantity demanded advertising expenditure and sales promotion measures etc.
5. The measure of coefficient of correlation is a relative measure of change.

### 10.12.1. Types of Correlation

Correlation is described or classified in several different ways. Three of the most important are:
I. Positive and Negative
II. Simple, Partial and Multiple
III. Linear and non-linear

## I. Positive, Negative and Zero Correlation:

Whether correlation is positive (direct) or negative (in-versa) would depend upon the direction of change of the variable.
(A) Positive Correlation:

If both the variables vary in the same direction, correlation is said to be positive. It means if one variable is increasing, the other on an average is also increasing or if one variable is decreasing, the other on an average is also deceasing, then the correlation is said to be positive correlation. For example, the correlation between heights and weights of a group of persons is a positive correlation.

| Height (cm): X | 158 | 160 | 163 | 166 | 168 | 171 | 174 | 176 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Weight (kg): Y | 60 | 62 | 64 | 65 | 67 | 69 | 71 | 72 |

## (B) Negative Correlation:

If both the variables vary in opposite direction, the correlation is said to be negative. If means if one variable increases, but the other variable decreases or if one variable decreases, but the other variable increases, then the correlation is said to be negative correlation. For example, the correlation between the price of a product and its demand is a negative correlation.

| Price of Product (Rs. Per Unit) : X | 6 | 5 | 4 | 3 | 2 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Demand (In Units) : Y | 75 | 120 | 175 | 250 | 215 | 400 |

(C) Zero Correlation:

Actually it is not a type of correlation but still it is called as zero or no correlation. When we don't find any relationship between the variables then, it is said to be zero correlation. It means a change in value of one variable doesn't influence or change the value of other variable. For example, the correlation between weight of person and intelligence is a zero or no correlation.

## II. Simple, Partial and Multiple Correlation:

The distinction between simple, partial and multiple correlation is based upon the number of variables studied.
(A) Simple Correlation:

When only two variables are studied, it is a case of simple correlation. For example, when one studies relationship between the marks secured by student and the attendance of student in class, it is a problem of simple correlation. Partial Correlation: In case of partial correlation one studies three or more variables but considers only two variables to be influencing each other and the effect of other influencing variables being held constant. For example, in above example of relationship between student marks and attendance, the other variable influencing such as effective teaching of teacher, use of teaching aid like computer, smart board etc are assumed to be constant.

## (B) Multiple Correlation:

When three or more variables are studied, it is a case of multiple correlation. For example, in above example if study covers the relationship between student marks, attendance of students, effectiveness of teacher, use of teaching aids etc, it is a case of multiple correlation.

## (C) Linear and Non-linear Correlation:

Depending upon the constancy of the ratio of change between the variables, the correlation may be Linear or Non-linear Correlation.

Linear Correlation: If the amount of change in one variable bears a constant ratio to the amount of change in the other variable, then correlation is said to be linear. If such variables are plotted on a graph paper all the plotted points would fall on a straight line. For example: If it is assumed that, to produce one unit of finished product we need 10 units of raw materials, then subsequently to produce 2 units of finished product we need double of the one unit.

| Raw material : X | 10 | 20 | 30 | 40 | 50 | 60 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Finished Product : Y | 2 | 4 | 6 | 8 | 10 | 12 |

(D) Non-linear Correlation:

If the amount of change in one variable does not bear a constant ratio to the amount of change to the other variable, then correlation is said to be non-linear. If such variables are plotted on a graph, the points would fall on a curve and not on a straight line. For example, if we double the amount of advertisement expenditure, then sales volume would not necessarily be doubled.

| Advertisement Expenses : $X$ | 10 | 20 | 30 | 40 | 50 | 60 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| Sales Volume : $Y$ | 2 | 4 | 6 | 8 | 10 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Illustration 01:
State in each case whether there is
(a) Positive Correlation
(b) Negative Correlation
(c) No Correlation

| S.No | Particulars | Solution |
| :--- | :--- | :--- |
| 1 | Price of commodity and its demand | Negative |
| 2 | Yield of crop and amount of rainfall | Positive |
| 3 | No of fruits eaten and hungry of a person <br> Negative | Negative |
| 4 | No of units produced and fixed cost per unit | Negative |
| 5 | No of girls in the class and marks of boys | No correlation |
| 6 | Ages of Husbands and wife | Positive |
| 7 | Temperature and sale of woollen garments | Negative |
| 8 | Number of cows and milk produced | Positive |
| 9 | Weight of person and intelligence | No correlation |
| 10 | Advertisement expenditure and sales volume | Positive |

### 10.12.2. Methods of Measurement of Correlation

## Methods of measurement of correlation:

Quantification of the relationship between variables is very essential to take the benefit of study of correlation. For this, we find there are various methods of measurement of correlation, which can be represented as given below:


Among these methods we will discuss only the following methods:

1. Scatter Diagram
2. Karl Pearson's Coefficient of Correlation
3. Spearman's Rank Coefficient of Correlation

## Scatter Diagram:

This is graphic method of measurement of correlation. It is a diagrammatic representation of bivariate data to ascertain the relationship between two variables. Under this method the given data are plotted on a graph paper in the form of dot. i.e. for each pair of $X$ and $Y$ values we put dots and thus obtain as many points as the number of observations. Usually an independent variable is shown on the X -axis whereas the dependent variable is shown on the Y -axis. Once the values are plotted on the graph it reveals the type of the correlation between variable $X$ and $Y$. A scatter diagram reveals whether the movements in one series are associated with those in the other series.

- Perfect Positive Correlation: In this case, the points will form on a straight line falling from the lower left hand corner to the upper right hand corner.
- Perfect Negative Correlation: In this case, the points will form on a straight line rising from the upper left hand corner to the lower right hand corner.
- High Degree of Positive Correlation: In this case, the plotted points fall in a narrow band, wherein points show a rising tendency from the lower left hand corner to the upper right hand corner.
- High Degree of Negative Correlation: In this case, the plotted points fall in a narrow band, wherein points show a declining tendency from upper left hand corner to the lower right hand corner.
- Low Degree of Positive Correlation: If the points are widely scattered over the diagrams, wherein points are rising from the left hand corner to the upper right hand corner.
- Low Degree of Negative Correlation: If the points are widely scattered over the diagrams, wherein points are declining from the upper left hand corner to the lower right hand corner.
- Zero (No) Correlation: When plotted points are scattered over the graph haphazardly, then it indicate that there is no correlation or zero correlation between two variables.




## Illustration 02:

Given the following pairs of values:

| Capital Employed (Rs. In <br> Crore) | 1 | 2 | 3 | 4 | 5 | 7 | 8 | 9 | 11 | 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Profit (Rs. In Lakhs) | 3 | 5 | 4 | 7 | 9 | 8 | 10 | 11 | 12 | 14 |

(a) Draw a scatter diagram
(b) Do you think that there is any correlation between profits and capital employed? Is it positive or negative? Is it high or low?

## Solution:

From the observation of scatter diagram we can say that the variables are positively correlated. In the diagram the points trend toward upward rising from the lower left hand corner to
 the upper right hand corner,
hence it is positive correlation. Plotted points are in narrow band which indicates that it is a case of high degree of positive correlation.

## Karl Pearson's Coefficient of Correlation:

Karl Pearson's method of calculating coefficient of correlation is based on the covariance of the two variables in a series. This method is widely used in practice and the coefficient of correlation is denoted by the symbol " $r$ ". If the two variables under study are X and $Y$, the following formula suggested by Karl Pearson can be used for measuring the degree of relationship of correlation.


$$
\begin{array}{r}
r=\frac{\sum(X-\bar{X})(Y-\bar{Y})}{\sqrt{\sum(X-\bar{X})^{2}} \sqrt{(Y-\bar{Y})^{2}}}
\end{array} \text { Where, } \bar{X} \text {. mean of } X \text { variable } \bar{Y} \text { mean of } Y \text { variable }
$$

$$
r=\frac{\sum f(d x)(d y)-\frac{\left.\sum f d x\right)\left(\sum f d y\right)}{N}}{\sqrt{\sum(f d x) \wedge 2-\frac{\left.\sum f d x\right) \wedge 2}{N}} \sqrt{\sum(d y) \wedge 2-\frac{\left(\sum f d y\right) \wedge 2}{N}}} \quad d_{x}=X-A
$$

Above different formula's can be used in different situation depending upon the information given in the problem.

Illustration 03:
From following information find the correlation coefficient between advertisement expenses and sales volume using Karl Pearson's coefficient of correlation method.

| Firm | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Advertisement Exp. (Rs. <br> In Lakhs) | 11 | 13 | 14 | 16 | 16 | 15 | 15 | 14 | 13 | 13 |


| Sales Volume (Rs. In <br> Lakhs) | 50 | 50 | 55 | 60 | 65 | 65 | 65 | 60 | 60 | 50 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Solution:

Let us assume that advertisement expenses are variable $X$ and sales volume are variable $Y$.

| Calculation of Karl Pearson's coefficient of correlation |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \mathrm{F} \\ & \mathrm{i} \\ & \mathrm{r} \\ & \mathrm{~m} \\ & \hline \end{aligned}$ | X | Y | $\begin{gathered} x= \\ x \cdot \dot{x} \end{gathered}$ | $\mathrm{x}^{2}$ | $\begin{aligned} & \mathrm{y}=\dot{\mathrm{Y}} \\ & \mathrm{Y}-\dot{\mathrm{Y}} \end{aligned}$ | $\mathrm{v}^{2}$ | xy |
| 1 | 11 | 50 | -3 | 9 | -8 | 64 | 24 |
| 2 | 13 | 50 | -1 | 1 | -8 | 64 | 8 |
| 3 | 14 | 55 | 0 | 0 | -3 | 9 | 0 |
| 4 | 16 | 60 | 2 | 4 | 2 | 4 | 4 |
| 5 | 16 | 65 | 2 | 4 | 7 | 49 | 14 |
| 6 | 15 | 65 | 1 | 1 | 7 | 49 | 7 |
| 7 | 15 | 65 | 1 | 1 | 7 | 49 | 7 |
| 8 | 14 | 60 | 0 | 0 | 2 | 4 | 0 |
| 9 | 13 | 60 | -1 | 1 | 2 | 4 | -2 |
| $1$ | 13 | 50 | -1 | 1 | -8 | 64 | 8 |
|  | $\begin{gathered} 14 \\ 0 \\ \hline \end{gathered}$ | 580 |  | 22 |  | 360 | 70 |
|  | $\sum_{\mathrm{X}}$ | $\Sigma^{Y}$ |  | $\sum_{\mathrm{x} 2}$ |  | Ey2 | $\sum_{x y}$ |
| $\dot{X}=\frac{\Sigma X}{n}=\frac{140}{10}=14 \quad \dot{Y}=\frac{\Sigma Y}{n}=\frac{580}{10}=58$ |  |  |  |  |  |  |  |



Interpretation: From the above calculation it is very clear that there is high degree of positive correlation i.e. $r=0.7866$, between the two variables. i.e. Increase in advertisement expenses leads to increased sales volume.

### 10.12.3. Properties of Coefficient of Correlation:

1. The coefficient of correlation always lies between -1 to +1 , symbolically it canwritten as $-1 \leq r \leq 1$.
2. The coefficient of correlation is independent of change of origin and scale.
3. The coefficient of correlation is a pure number and is independent of the units of measurement. It means if $X$ represent say height in inches and $Y$ represent say weights in kgs, then the correlation coefficient will be
neither in inches nor in kgs but only a pure number.
4. The coefficient of correlation is the geometric mean of two regression coefficient, symbolically $r=\sqrt{ } b x y *$ byx
5. If $X$ and $Y$ are independent variables then coefficient of correlation is zero.

### 10.13. Regression

## Meaning:

A study of measuring the relationship between associated variables, wherein one variable is dependent on another independent variable, called as Regression. It is developed by Sir Francis Galton in 1877 to measure the relationship of height between parents and their children.
Regression analysis is a statistical tool to study the nature and extent of functional relationship between two or more variables and to estimate (or predict) the unknown values of dependent variable from the known values of independent variable.
The variable that forms the basis for predicting another variable is known as the Independent Variable and the variable that is predicted is known as dependent variable. For example, if we know that two variables price $(\mathrm{X})$ and demand $(\mathrm{Y})$ are closely related we can find out the most probable value of $X$ for a given value of $Y$ or the most probable value of $Y$ for a given value of $X$. Similarly, if we know that the amount of tax and the rise in the price of a commodity are closely related, we can find out the expected price for a certain amount of tax levy.

### 10.13.1. Uses of Regression Analysis

1. It provides estimates of values of the dependent variables from values of independent variables.
2. It is used to obtain a measure of the error involved in using the regression line as a basis for estimation.
3. With the help of regression analysis, we can obtain a measure of degree of association or correlation that exists between the two variables.
4. It is highly valuable tool in economies and business research, since most of the problems of the economic analysis are based on cause and effect relationship.

### 10.13.2. Difference between Correlation and Regression

| S.No. | Correlation | Regression |
| :---: | :---: | :---: |
| 1 | It measures the degree and direction of relationship between the variables. | It measures the nature and extent of average relationship between two or more variables in terms of the originalunits of the data |
| 2 | It is a relative measure showing association between the variables. | It is an absolute measure of relationship. |
| 3 | Correlation Coefficient is independent of change of both origin and scale. | Regression Coefficient is independent of change of origin but not scale. |
| 4 | Correlation Coefficient <br> independent <br> measurement. is <br> of <br> of   |  |
| 5 | Expression of the relationship between the variables ranges from -1 to +1 | Expression of the relationship between the variables may be in any of the forms like: $Y=a+b X+c X^{2}$ |
| 6 | It is not a forecasting device | It is a forecasting device which can be used to predict the value of dependent variable from the given value of independent variable. |
| 7 | There may be zero correlation such as weight of wife and income of husband | There is nothing like zero regression |

### 10.13.3. Regression Lines and Regression Equation

Regression lines and regression equations are used synonymously. Regression equations are algebraic expression of the regression lines. Let us consider two variables: X \& $Y$. If $y$ depends on $x$, then the result comes in the form of simple regression. If we take the case of two variable $X$ and $Y$, we shall have two regression lines as the regression line of $X$ on $Y$ and regression line of $Y$ on $X$. The regression line of $Y$ on $X$ gives the most probable value of $Y$ for given value of $X$ and the regression line of $X$ on $Y$ given the most probable value of $X$ for given value of Y . Thus, we have two regression lines. However, when there is either perfect positive or perfect negative correlation between the two variables, the two regression line will coincide, i.e. we will have one line. If the variables are independent, $r$ is zero and the lines of regression are at right angles i.e. parallel to $X$ axis and $Y$ axis.

Therefore, with the help of simple linear regression model we have the following two regression lines

1. Regression line of Y on X : This line gives the probable value of Y (Dependent variable) for any given value of $X$ (Independent variable).

$$
\begin{array}{cll}
\text { Regression line of } Y \text { on } X & : & Y-\dot{Y}=b_{y x}(X-\dot{X}) \\
0 R & : & Y=a+b X
\end{array}
$$

2. Regression line of $X$ on $Y$ : This line gives the probable value of $X$ (Dependent variable) for any given value of $Y$ (Independent variable).
Regression line of $X$ on $Y \quad: \quad X-\dot{X}=b_{x y}(Y-\dot{Y})$
OR : $X=a+b Y$

In the above two regression lines or regression equations, there are two regression parameters, which are "a" and " b ". Here " a " is unknown constant and " b " which is also denoted as "byx" or "bxy", is also another unknown constant popularly called as regression coefficient. Hence, these "a" and "b" are two unknown constants (fixed numerical values) which determine the position of the line completely. If the value of either or both of them is changed, another line is determined. The parameter "a" determines the level of the fitted line (i.e. the distance of the line directly above or below the origin). The parameter "b" determines the slope of the line (i.e. the change in Y for unit change in X ).

If the values of constants "a" and "b" are obtained, the line is completely determined. But the question is how to obtain these values. The answer is provided by the method of least squares. With the little algebra and differential calculus, it can be shown that the following two normal equations, if solved simultaneously, will yield the values of the parameters "a" and "b".
Two normal equations:

| Xon $Y$ | $Y$ on $X$ |
| :--- | :--- |
| $\Sigma X=N a+b \sum Y$ | $\sum Y=N a+b \Sigma X$ |
| $\Sigma X Y=a \sum Y+b \Sigma Y^{2}$ | $\Sigma X Y=a \Sigma X+b \sum X^{2}$ |
|  |  |

This above method is popularly known as direct method, which becomes quite cumbersome when the values of $X$ and $Y$ are large. This work can be simplified if
instead of dealing with actual values of $X$ and $Y$, we take the deviations of $X$ and $Y$ series from their respective means. In that case:

## Regression equation Y on X :

$Y=a+b X \quad$ will change to $(Y-Y)=b_{y x}(X-X)$ regression
$X=a+b Y \quad$ will change to $(X-X)=b_{x y}(Y-Y)$ regression

In this new form of regression equation, we need to compute only one parameter i.e. "b". This "b" which is also denoted either "byx" or "bxy" which is called as regression coefficient.

### 10.13.4. Regression Coefficient:

The quantity "b" in the regression equation is called as the regression coefficient or slope coefficient. Since there are two regression equations, therefore, we have two regression coefficients.
1.Regression Coefficient X on Y , symbolically written as "bxy"
2.Regression Coefficient Y on X , symbolically written as "byx" Different formula's used to compute regression coefficients:

| Method | Regression Coefficient X on Y | Regression Coefficient Y on X |
| :---: | :---: | :---: |
| Using the correlation <br> coefficient (r) and <br> standard deviation $(\sigma)$ | $\mathrm{b}_{\mathrm{xy}}=r \frac{\sigma x}{\sigma y}$ | $\mathrm{~b}_{\mathrm{yx}}=r \frac{\sigma y}{\sigma x}$ |
| Direct Method: Using <br> sum of X and Y | $\mathrm{b}_{\mathrm{xy}}=\frac{\mathrm{N} \sum \mathrm{XY}-\sum \mathrm{X} \sum \mathrm{Y}}{\mathrm{N} \sum \mathrm{Y}^{2}-\left(\sum \mathrm{Y}\right)^{2}}$ | $\mathrm{~b}_{\mathrm{yx}}=\frac{\mathrm{N} \sum \mathrm{XY}-\sum \mathrm{X} \sum \mathrm{Y}}{\mathrm{N} \sum \mathrm{X}^{2}-\left(\sum \mathrm{X}\right)^{2}}$ |
| When deviations are <br> taken from arithmetic <br> mean | $\mathrm{b}_{\mathrm{xy}}=\frac{\sum x y}{\sum y^{2}}$ |  |
| where $x=\mathrm{X}-\dot{\mathrm{X}}$ and $y=\mathrm{Y}-\dot{\mathrm{Y}}$ | where $x=\mathrm{X}-\dot{\mathrm{X}}$ and $y=\mathrm{Y}-\dot{\mathrm{Y}}$ |  |

## Properties of Regression Coefficients:

1. The coefficient of correlation is the geometric mean of the two regression coefficients. Symbolically $r=\sqrt{ }$ bxy $*$ byx
2. If one of the regression coefficients is greater than unity, the other must be less than unity, since the value of the coefficient of correlation cannot exceed unity. For example if bxy $=1.2$ and byx $=1.4$ "r" would be $=\sqrt{ } 1.2 * 1.4=1.29$, which is not possible.
3. Both the regression coefficient will have the same sign. i.e. they will be either positive or negative. In other words, it is not possible that one of the regression coefficients is having minus sign and the other plus sign.
4. The coefficient of correlation will have the same sign as that of regression coefficient, i.e. if regression coefficient have a negative sign, "r" will also have negative sign and if the regression coefficient have a positive sign, "r" would also be positive. For example, if bxy $=-0.2$ and byx $=-0.8$ then $r=-\sqrt{ } 0.2 * 0.8=-0.4$ 5. The average value of the two regression coefficient would be greater than the value of coefficient of correlation. In symbol (bxy +byx$) / 2>r$. For example, if bxy $=0.8$ and byx $=0.4$ then average of the two values $=(0.8+0.4) / 2=0.6$ and the value of $r=r=\sqrt{ } 0.8 * 0.4=0.566$ which less than 0.6
5. 

R
egression coefficients are independent of change of origin but not scale.

## Illustration 01:

Find the two regression equation of X on Y and Y on X from the following data:

| $\mathrm{X}:$ | 10 | 12 | 16 | 11 | 15 | 14 | 20 | 22 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Y}:$ | 15 | 18 | 23 | 14 | 20 | 17 | 25 | 28 |

Solution:
Calculation of Regression Equation

| $\mathbf{X}$ | $\mathbf{Y}$ | $\mathbf{X}^{2}$ | $\mathbf{Y}^{2}$ | $\mathbf{X Y}$ |
| :---: | :---: | :---: | :---: | :---: |
| 10 | 15 | 100 | 225 | 150 |
| 12 | 18 | 144 | 324 | 216 |
| 16 | 23 | 256 | 529 | 368 |
| 11 | 14 | 121 | 196 | 154 |
| 15 | 20 | 225 | 400 | 300 |
| 14 | 17 | 196 | 289 | 238 |
| 20 | 25 | 400 | 625 | 500 |
| 22 | 28 | 484 | 784 | 616 |
| $\mathbf{1 2 0}$ | $\mathbf{1 6 0}$ | $\mathbf{1 , 9 2 6}$ | 3,372 | $\mathbf{2 , 5 4 2}$ |
| $\mathbf{X}$ | $\mathbf{\Sigma} \mathbf{Y}$ | $\mathbf{\Sigma} \mathbf{X}^{2}$ | $\Sigma \mathbf{Y}^{2}$ | $\mathbf{\Sigma X Y}$ |

Here $\mathrm{N}=$ Number of elements in either series X or series $\mathrm{Y}=8$
Now we will proceed to compute regression equations using normal equations.

## Regression equation of X on $\mathrm{Y}: \quad \mathrm{X}=\mathrm{a}+\mathrm{bY}$

The two normal equations are:

$$
\begin{aligned}
& \sum X=N a+b \sum Y \\
& \sum X Y=a \sum Y+b \sum Y^{2}
\end{aligned}
$$

Substituting the values in above normal equations, we get

| $120=$ | $8 \mathrm{a}+160 \mathrm{~b}$ | $\ldots .$. (i) |
| :--- | :--- | :--- |
| 2542 | 160 a | 3372 b |$\quad$.... (ii)

Let us solve these equations (i) and (ii) by simultaneous equation methodMultiply equation

$$
\text { (i) by } 20 \text { we get } \quad 2400=160 a+3200 b
$$

Now rewriting these equations:
$2400=16 \varnothing \mathrm{a}+3200 \mathrm{~b}$
$2542=160 \mathrm{a}+3372 \mathrm{~b}$
$-142=0$

Therefore nowwe have $-142=-172 \mathrm{~b}$, this can rewrittenas $172 \mathrm{~b}=142$
Now, $b=\frac{122}{172}=0.8256$ (rounded off)
Substituting the value of $b$ in equation ( i ), we get
$\left.\begin{array}{l}120=8 \mathrm{a}+(160 * 0.8256) \\ 120=8 \mathrm{a}+132 \text { (rounded off) } \\ 8 \mathrm{a}=120-132 \\ 8 \mathrm{a}=-12 \\ \mathrm{a}\end{array}\right)$

Thus we got the values of $a=-1.5$ and $b=0.8256$
Hence the required regression equation of $X$ on $Y$ :

$$
X=a+b Y \quad \Rightarrow \quad X=-1.5+0.8256 Y
$$

## Regression equation of $\mathbf{Y}$ on $\mathbf{X}: \quad \mathbf{Y}=\mathbf{a}+\mathbf{b X}$

The two normal equations are:

$$
\begin{array}{ll}
\sum Y & =\mathrm{Na}+\mathrm{b} \sum \mathrm{X} \\
\sum X Y= & =\mathrm{a} \sum \mathrm{X}+\mathrm{b} \sum \mathrm{X}^{2}
\end{array}
$$

Substituting the values in above normal equations, we get
$160=8 a+120 b$
$2542=120 a+1926 b$
..... (iv)
Let us solve these equations (iii) and (iv) by simultaneous equation method
Multiply equation (iii) by 15 we get $2400=120 a+1800 b$
Now rewriting these equations:

| $2400=12 \emptyset a+$$1800 b$ <br> $2542=$ <br> $120 a+$ <br> $1926 b$ <br> $-142=$ |
| :--- |
| $-(-)$ |

Therefore now we have $-142=-126 b$, this can rewritten as $126 b=142$
Now, $b=\frac{142}{126}=1.127$ (rounded off)
Substituting the value of $b$ in equation (iii), we get

```
160 = 8a + (120*1.127)
160=8a + 135.24
```

$$
\begin{array}{rlr}
8 \mathrm{a} & =160 \cdot 135.24 \\
8 \mathrm{a} & =24.76 \\
\mathrm{a} & =24.76 / 8 \\
\mathrm{a} & =3.095
\end{array}
$$

Thus we got the values of $\mathrm{a}=3.095$ and $\mathrm{b}=1.127$
Hence the required regression equation of $Y$ on $X$ :

$$
Y=a+b X \quad \Rightarrow \quad Y=3.095+1.127 X
$$

## Illustration 02:

From the following data obtain the two regression lines:

| Capital Employed (Rs. in lakh): | 7 | 8 | 5 | 9 | 12 | 9 | 10 | 15 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Sales Volume (Rs. In <br> lakh) | 4 | 5 | 2 | 6 | 9 | 5 | 7 | 12 |

## Solution:

## Calculation of Regression Equation

| X | Y | $\mathrm{X}^{2}$ | $\mathrm{Y}^{2}$ | XY |
| :---: | :---: | :---: | :---: | :---: |
| 7 | 4 | 49 | 16 | 28 |
| 8 | 5 | 64 | 25 | 40 |
| 5 | 2 | 25 | 4 | 10 |
| 9 | 6 | 81 | 36 | 54 |
| 12 | 9 | 144 | 81 | 108 |
| 9 | 5 | 81 | 25 | 45 |
| 10 | 7 | 100 | 49 | 70 |
| 15 | 1 | 225 | $\begin{aligned} & 1 \\ & 4 \end{aligned}$ | 180 |
| 75 | $\begin{aligned} & \hline 5 \\ & 0 \end{aligned}$ | 769 | $\begin{aligned} & 3 \\ & 8 \\ & 0 \end{aligned}$ | 535 |
| $\Sigma \mathrm{X}$ | $\sum_{\mathrm{Y}}$ | $\Sigma \mathbf{X}^{2}$ | $\sum_{\substack{Y \\ 2}}$ | $\Sigma X Y$ |

Regression line/equation of $X$ on $Y$ :

$$
\begin{aligned}
& (X-\dot{X})=b_{x y}(Y-\dot{Y}) \\
& \dot{X}=\frac{\sum X}{n}=\frac{75}{8}=9.375 \\
& \dot{Y}=\frac{\sum Y}{n}=\frac{50}{8}=6.25
\end{aligned}
$$

Regression coefficient of X on Y :

$$
\begin{aligned}
& b_{x y}=\frac{n \sum X Y-\sum X \sum Y}{n \sum Y^{2}-\left(\sum Y\right)^{2}} \\
& b_{x y}=\frac{(8+5355)-(75+50)}{(8+380)-(50)^{2}} \\
&=\frac{4280-3750}{3040-2500} \\
&=\frac{530}{540}=\underline{0.9815} \\
&(X-\dot{X})=b_{x y}(Y-\dot{Y}) \\
& \Rightarrow X-9.375=0.9815(Y-6.25) \\
& \Rightarrow X-9.375=0.9815 Y-6.1344 \\
& \Rightarrow X=9.375-6.1344+0.9815 Y \\
& \Rightarrow X=3.2406+0.9815 Y
\end{aligned}
$$

Regression line/equation of Y on X :

$$
\begin{aligned}
& (Y-\dot{Y})=b_{y x}(X-\dot{X}) \\
& \dot{X}=\frac{\sum X}{n}=\frac{75}{8}=9.375 \\
& \dot{Y}=\frac{\sum Y}{n}=\frac{50}{8}=6.25
\end{aligned}
$$

Regression coefficient of Y on X :
$\mathrm{b}_{\mathrm{yx}}=\frac{\mathrm{n} \sum \mathrm{XX}-\sum \mathrm{X} \sum \mathrm{Y}}{\mathrm{n} \sum \mathrm{X}^{2}-\left(\sum \mathrm{X}\right)^{2}}$
$\mathrm{b}_{\mathrm{xy}}=\frac{(8.535)-(75+50)}{(8.769)-(75)^{2}}$
$=\frac{4280-3750}{6152-5625}$
$=\frac{530}{527}=\underline{1.0057}$
$(\mathrm{Y}-\dot{\mathrm{Y}})=\mathrm{b}_{\mathrm{yx}}(\mathrm{X}-\dot{\mathrm{X}})$
$\Rightarrow \mathrm{Y}-6.25=1.0057(\mathrm{X}-9.375)$
$\Rightarrow \mathrm{Y}-6.25=1.0057 \mathrm{X}-9.4284)$
$\Rightarrow \mathrm{Y}=6.25-9.4284+1.0057 \mathrm{X}$
$\Rightarrow \mathrm{Y}=-3.1784+1.0057 \mathrm{X}$

### 10.13.5. Review of Correlation and Regression Analysis:

In correlation analysis, when we are keen to know whether two variables under study are associated or correlated and if correlated what is the strength of correlation. The best measure of correlation is proved by Karl Pearson's Coefficient of Correlation. However, one severe limitation of this method is that it is applicable only in case of a linear relationship between two variables. If two variables say X and Y are independent or not correlated then the result of correlation coefficient is zero.

Correlation coefficient measuring a linear relationship between the two variables indicates the amount of variation one variable accounted for by the other variable. A better measure for this
purpose is provided by the square of the correlation coefficient, known as "coefficient of determination". This can be interpreted as the ratio between the explained variance to total variance:

$$
\mathrm{r}^{2}=\frac{\text { Explained variance }}{\text { Total Variance }} \text { Similarly, Coefficient of non-determination }=\left(1-\mathrm{r}^{2}\right) \text {. }
$$

Regression analysis is concerned with establishing a functional relationship between two variables and using this relationship for making future projection. This can be applied, unlike correlation for any type of relationship linear as well as curvilinear. The two lines of regression coincide i.e. become identical when $r=-1$ or +1 in other words, there is a perfect negative or positive correlation between the two variables under discussion if $r=0$, then regression lines are perpendicular to each other.

### 10.13.6. Difference between Correlation and Regression

Correlation and regression are both used as statistical measurements to get a good understanding of the relationship between variables. If the correlation coefficient is negative (or positive) then the slope of the regression line will also be negative (or positive). The table given below highlights the key difference between correlation and regression.

| Correlation | Regression |
| :--- | :--- |
| Correlation is used to determine <br> whether variables are related or not. | Regression is used to numerically <br> describe how a dependent variable <br> changes with a change in an <br> independent variable |
| Correlation tries to establish a linear <br> relationship between variables. | It finds the best-fitted regression line <br> to estimate an unknown variable on <br> the basis of the known variable. |
| The variables can be used <br> interchangeably | The variables cannot be interchanged. |
| Correlation uses a signed numerical <br> value to estimate the strength of the | Regression is used to show the <br> impact of a unit change in the |


| Correlation | Regression |
| :--- | :--- |
| relationship between the variables. | independent variable on the <br> dependent variable. |
| The Pearson's coefficient is the best <br> measure of correlation. | The least-squares method is the best <br> technique to determine the regression <br> line. |

### 10.14. Introduction to remote sensing and its application in forestry.

To meet the various information requirements in forest management different data sources, like field survey, aerial photography and satellite imagery is used, depending on the level of detail required and the extension of the area under study. Before aerial photography was used for forest management purposes, information was generally obtained by means of field surveys, identifying and measuring forest types and stands. This is still by far the most accurate and detailed way of measurement, although the lack of geographical positioning systems did not allow accurate location of the forests classified. The method is, however very elaborate, time consuming and expensive, and it is nowadays used predominantly for research purposes and for intensive sustainable production purposes. The traditional aerial photograph resulting from different film types was and still is an important remote sensing tool. Knowledge of photogrammetry and photography is essential for its proper use. For many decades the use of aerial photographic data has been accepted by many forest institutions as a tool in various forest activities, such as planning, mapping, inventory, harvesting, area determination, road lay-out, and registration of declined and dead trees etc. on a local, regional or national scale. For the purpose of consistently and repeatedly monitor forests over larger areas, it is preferable to use remote sensing data and automated image analysis techniques. Several types of remote sensing data, including aerial photography, multispectral scanner (MSS), radar (Radio Detection and Ranging), Lidar (Light Detection and Ranging) laser and videography data have been used by forest agencies to detect, identify, classify, evaluate and measure various forest.

Cover types and their changes. Over the past decades tremendous progress has been made in demonstrating the potentials and limitations for identifying and mapping various earth surface features using optical remote sensing data. For large areas, satellite imagery has been shown effective for forest classification, and consequently mapping. It is emphasized that one of the advantages of the use of remote sensing in forest survey is the relative short time in which most of the required information can be obtained. Gradually other types of remote sensing tools were developed with which forest object properties were registered from the air or from space. The new technologies, integrating satellite imagery, analytical photogrammetry and geoinformation systems (GIS) offer new possibilities, especially for general interpretation and mapping and will be a challenge for future research and application. The analogue photographic data of aerial photographs as well as the satellite scanning data can be digitized and used for multispectral or multi-temporal classification and corrections, geometrical or radiometrical. Scanning techniques are also applicable in airplanes. Nowadays the products of this aerospace
 technology are considered to be superior to and a replacement of the old fashioned analogue aerial photography. However, this technology is additional and complementary to the aerial photography. Sometimes the products are used alone, but in most cases a combination with aerial photographs is applied. Also fieldwork remains essential when applying remote sensing techniques. Various factors can be mentioned to explain why in managed forests the operational application of remote sensing in the estimation of a number of stand parameters, is relatively low. Foresters are in general conservative, in the beginning they were reserved in applying aerial photography and nowadays other remote sensing techniques are not embraced whole-heartily. There is a hesitation to take risks when departing from traditional data sources. Lack of knowledge of access to data of the specialized technology is and other reason for the limited application.

The Earth's atmosphere contains carbon dioxide (CO2) and other greenhouse gases (GHGs) that act as a protective layer, causing the planet to be warmer than it would otherwise be. This heat retention is critical in maintaining habitable temperatures. If there were significantly less CO2 in the atmosphere, global temperatures


Fig 2. Holding the stock of $\mathrm{CO}_{2}$ concentration would drop below levels to which ecosystems and human society have adapted. As CO2 levels rise, mean global temperatures are also expected to rise as increasing amounts of solar radiation are trapped inside the greenhouse. The concentration of CO 2 in the atmosphere is determined by a continuous flow among the stores of carbon in the atmosphere, the ocean, the earth's biological systems, and its geological materials (Fig. 1). As long as the amount of carbon flowing into the atmosphere as CO2 and out (in the form of plant material and dissolved carbon) are in balance, the level of carbon in the atmosphere remains constant.

Photosynthesis leads to the conversion of carbon dioxide into organic carbon in growing plants, and some of the carbon thus sequestered as plant biomass is subsequently lost through respiration. A large net flux of carbon from atmosphere to tree accompanies early tree growth. Over time, the net rate of exchange decreases due to increasing carbon loss through respiration or the loss and subsequent decomposition of plant material as litter and woody debris. A large amount of carbon is released to the atmosphere as trees die and decompose (Fig. 2). Other mechanism of carbon loss from forest systems includes physical removal of organic matter or rapid loss through natural disturbance, such as fire. A significant form of removal in the United States is harvest of wood, but carbon can also be removed through runoff or leaching through soil. Subsequent forest regeneration and growth can then re establish the section of forest as a sink of atmospheric carbon dioxide.

Forests are influenced by natural and human causes, including harvesting, over harvesting and degradation, large scale occurrence of wildfire, fire control, pest and disease outbreaks, and conversion to non forest use, particularly agriculture and pastures. These disturbances often cause forests to become sources of


Fig 3. Global carbon cycle CO2 because the rate of net primary productivity is exceeded by total respiration or oxidation of plants, soil, dead organic matter and net ecosystem production (NEP < 0). At the same time however, some areas of harvested and degraded forests or agricultural and pasture lands are abandoned and revert naturally to forests or are converted to plantations, thus becoming carbon sinks, i.e. the rate of respiration from plants, soil and dead organic matter is exceeded by net primary productivity (NEP > 0 ). The current role of forests in the global carbon cycle is not only a function of present forest land use, but also of past use and disturbance. Prior to this century CO2 emissions from changes in forest land use, mainly caused by agricultural expansion in mid and high latitude countries, were higher than emissions from the combustion of fossil fuels (Houghton and Skole, 1990). From the turn of the century until about the 1930s, global CO2 emissions from changes in forest land use were similar in magnitude to those from fossil fuel combustion (Fig. 3). After about the 1940s, CO2 emissions from the changes in forest land use in the tropics dominated the flux from the biota to the atmosphere. Since then, worldwide fossil fuel use has soared, biotic emissions from the mid and high latitude regions has declined greatly as forests expanded into abandoned agricultural lands and as logged stands redrew, and deforestation in the tropics has accelerated (Houghton et al., 1987).

### 10.14.1. Role of Forests

Forests play an important role in global carbon cycles. Policies that influence the rate of Conversion of forest to other land use or encourage afforestation and reforestation of deforested lands have the potential to have a large impact on concentrations of atmospheric CO2 (IPCC, 2001). Forest conversion is the second largest global source of anthropogenic carbon dioxide emissions and is likely responsible for 10-25\% of carbon dioxide emissions worldwide (Houghton 2003; Santilli et al., 2005).Within the U.S. forests are net carbon sinks, sequestering approximately $780 \mathrm{Tg} / \mathrm{yr}$ CO2 Eq. (latest data for 2004), which is approximately $11 \%$ of U.S. greenhouse gas emissions. A number of existing and proposed policy instruments specifically include the use of forests to capture CO2. Global climate change is increasingly recognized as the greatest global threat facing humanity. For the majority of the world's population, the persistent problems of food insecurity, rural poverty, the struggle to develop, sustain new sources of economic growth must now be considered against a backdrop of uncertainty and change in historical climatic patterns. Separately and together, governments, and both international and domestic organizations not only need to continue responding to the immediate concerns of extreme poverty, environmental degradation and social unrest but in addition must now begin to prepare communities and entire regions to adopt to uncertain future climatic regimes as well as to make tangible contributions in first slowing, and ultimately reestablishing a balance in greenhouse gas exchanges at a planetary scale.

## Role of Remote Sensing for Assessment of Biomass/Carbon

A variety of approaches and data sources have been used to estimate forest above ground biomass. A comprehensive review of remote sensing based estimates of AGB has been completed, categorized by data source: (i) field measurement; (ii) remotely sensed data; or (iii) ancillary data used in GIS based modeling. Estimation from field measurements may entail destructive sampling (Miksys et al., 2007) or direct measurement and the application of algometric equations Above ground biomass is necessary for studying productivity, carbon cycles, nutrient allocation and fuel accumulation in terrestrial ecosystems (Alban et al., 1978; Brown et al., 1999; Crow, 1978).Remote sensing techniques allow scientists to examine properties and
processes of ecosystems and their interannual variability at multiple scales because satellite observations can be obtained over large areas of interest with high revalidation frequencies. Many studies have demonstrated that indices such as Spectral Vegetation index (SVI), Simple Ratio (SR), Normalized Difference Vegetation Index (NDVI), and Corrected Normalized Difference Vegetation Index (NDVIc) obtained from satellite data are useful predictors of Leaf Area Index (LAI), biomass and productivity in grasslands and forests (Cheng \& Zhao, 1990; Diallo et al., 1991). Stand level biomass is frequently calculated from linear and non linear regression modeOls established by species with field measurements (Crow \& Schlaegel, 1988). Although Oestimates of AGB vary with species composition, tree height, basal area and stand structure, bole diameter at breast height (DBH) is the most commonly used and widely available variable for calculating AGB (Crow \& Schlaegel,1988). Numerous regression models have been developed to estimate AGB in the Great Lakes Region, while these models are accurate at tree, plot and stand levels, they are limited when considering spatial pattern analysis of AGB across the landscape. In order to scale AGB estimates to the landscape level, the estimates have to be linked with various vegetation indices derived by remote sensing data. Past studies have shown varying degrees of success in estimating forest biomass and primary production from remote sensing data in temperate and tropical forests worldwide (Brown et al., 1999; Gower et al., 1999). Recent studies suggest that such relationships very temporally and spatially; however, biomass estimates at the landscape level are necessary for understanding processes of the target landscapes and provide baseline data for future studies (Foody et al., 2003; Woodcock et al., 2001).Models derived from remote sensing need further calibration with ground data before they can be used appropriately to predict AGB for a given landscape.

| 1. ForestryInformation <br> needs and Remote |  |
| :--- | :--- | ---: |
| Sensing |  |

In practice, researchers choose one or several types of remotely sensed data according to their information needs. The


Fig 4. Given a limited bandwidth, trade-offs have to be made between spectral, temporal, and spatial properties of the imagery acquired.
Source: T., T.A. Warner, J.B McGraw, and M.A. Fajvan. 2001. Remote Sensing of Environment 75: 100-112 information needs are converted to specific properties of remotely sensed data,
such as spatial resolution, spectral resolution, temporal resolution, etc. Tables 1 list commonly used sensors on Earth observation satellite that are still operational. These sensors provide diverse remotely sensed data with a unique configuration of image resolutions, such as spatial resolution, spectral resolution and temporal resolution (Fig. 5).

Jensen (2007) defined spatial resolution as "a measure of the smallest angular or linear separation between two objects that can be resolved by the remote sensing system". In other words, the spatial resolution stands for how detailed information the remotely sensed data could provide. Temporal resolution can be defined as "how often the sensor records imagery of a particular area" (Jensen, 2007). For example, the well-known Landsat TM has 16-day temporal resolution. Another key property of a remote sensing system is spectral resolution, which is defined as "the number and dimension (size) of specific wavelength intervals (referred to as bands or channels) in the electromagnetic spectrum to which a remote sensing instrument is sensitive". Likewise, take the Landsat TM as an example. The Landsat TM imagery has 7 bands ( 6 optical bands plus 1 thermal band). It is noteworthy that the spectral resolution is mainly applied to describe optical imagery, and it cannot be used for Radar or Lidar remotely sensed data. In fact, the aforementioned data collection can be described as selecting the unique configuration of image resolutions (or properties), which can be used to meet certain research needs. Wulder et al. (2009) endeavored to clearly demonstrate the relationship of information needs and the selection of appropriate data and processing methods in remote sensing for studies of vegetation condition. The issues need to be taken into account, including "the scale at which the target must be measured (e.g. landscape-level or tree level information); the attributes of interest (change, condition, spatial extent); cost; timeliness; and, repeatability"
(Wulder et al., 2009)
Table 1. The Current Commonly-used Optical and Radar Sensors

| Satellite Program | Satellite Platform | Sensor | Data <br> Operator |
| :---: | :---: | :--- | :---: |
| Data Operator Optical Remote |  |  |  |
| POES (Polar Orbiting <br> Environmental Satellites) | NOAA 18 |  |  |
| EOS (Earth <br> Observing System) | TERRA/AQUA | MODIS | NASA/USGS |
| Landsat | LANDSAT5 | TM | NASA/USGS |


| SPOT (Satellite <br> Pour I' Observationde la Terre)  | SPOT 4 | HRVIR VEGETATION | Spot Image |
| :---: | :---: | :---: | :---: |
|  | SPOT 5 | HRG VEGETATION | Spot Image |
| IRS (Indian <br> Remote SensingSatellites) | $\begin{array}{\|l\|} \hline \text { RSS } \\ \text { (ResourceSat-1) } \end{array}$ | $\begin{gathered} \hline \text { LISS III LISS IV } \\ \text { AWiFS } \end{gathered}$ | ISRO (India Space Rese |
| $\begin{array}{cc}\text { DMC } & \begin{array}{c}\text { (DisasterMonitoring } \\ \text { Constellation) }\end{array}\end{array}$ | Beijing-1 | SLIM-6 DMC | Inter <br> nati <br> onal |
| CBERS (China-BrazilEarthResources Satellite) | CBERS-2B | CCD HRC IRMSS WFI | CAST (Chin a)/IN |
| Digital Constellation | World View 2 | WV110 | DigitalGlobe Corporate |
|  | Quick Bird 2 | BGIS 2000 | DigitalGlobe Corporate |
| Geo Eye | GeoEye-1 GIS MS Geo Eye Inc. |  |  |
| RADARSAT Constellation |  |  |  |

Under remote sensing technologies in forest studies optical sensors have been commonly used in forestry studies. However, the use of hyperspectral sensors, Radar and Lidar is still relatively underdeveloped. It is worth paying more attention to the application of hyperspectral sensors, Radar and Lidar in forestry studies.

## A. Hyperspectral Sensors

Optical sensors mentioned above, which are divided from the dimension of spatial resolution, are categorized into multispectral sensors. By contrast, there is a group of sensors called hyperspectral sensors, which accordingly generate hyperspectral data. Wang et al. (2010) stated that "hyperspectral data have the ability to collect ample spectral information across a continuous spectrum generally with 100 or more contiguous spectral bands". Shippert (2004) listed the existing hyperspectral sensors acquiring imagery from space, including the Hyperion sensor on NASA's EO-1 (National Aeronautics and Space Administration's Earth Observing-1), the CHRIS (Compact High Resolution Imaging Spectrometer) sensor on the European Space Agency's PROBA (Project for On-Board Autonomy) satellite, and the FTHSI (Fourier

Transform Hyperspectral Imager) sensor on the U.S. Air Force Research Lab's MightySat II satellite.

## B. Radar and Lidar

Besides optical sensors, Radar and Lidar play more and more important roles in remote sensing of forest studies. Radar, the acronym of radio detection and ranging, is based on the transmission of long-wavelength microwaves (e.g., 3-25 cm) through the atmosphere and then recording the amount of energy backscattered from the terrain (Jensen, 2007). Wang et al. (2009) briefly introduced the Phased Array type L-band Synthetic Aperture Radar (PALSAR) on board Advanced Land Observing Satellite (ALOS), and RADARSAT- 2 operated by the Canadian Space Agency (CSA) and MacDonald Dettwiler and Associates Ltd (MDA). Both could provide fully polarized SAR data to support PoISAR (Polarimetric SAR) technology (i.e., PoISAR decomposition), which has achieved promising results in many environmental researches (e.g., Lee et al., 2001; McNairn et al., 2009; Shimoni et al., 2009). Light detection and ranging (Lidar), also called Laser altimetry, is an active remote sensing technology that utilizes a laser to illuminate a target object and a photodiode to register the backscatter radiation (Lim et al., 2003; Hyyppa et al., 2009). It has been widely accepted that Lidar is capable of accurate (or even precise) vertical information (Wang et al., 2010). Therefore, it is believed that Lidar will bring forestry studies into an unprecedented age.

## Specific applications of remote sensing in Forest studies

Young and Giese (2003) summarized forest science and management into three categories: A. forest biology and ecology (e.g. forest biomes of the world, forest ecophysiology, forest soils, forest ecosystem ecology, landscape ecology, and forest trees: disease and insect interactions); B. forest management and multiple uses (e.g. forest management and stewardship, nonindustrial private forests, measuring and monitoring forest resources, silviculture and ecosystem management, forest-wildlife management, forest and rangeland management, forest and watershed management, forest and recreation behavior, behavior and management of forest fires, timber harvesting, wood products, and economics and the management of forests for wood and amenity values); and C . forests and society (e.g. urban forest, and social forestry: the community-based management of natural resources). As a matter of fact, remote
sensing has more or less served all the three categories. Several examples in remote sensing of forestry studies are provided as follows. The selected examples were included in the papers that were either highly cited or newly published Science Citation Index (SCI) papers.

## 1. Timber Volume Estimation

Timber volume is simply function of Tree height (ht) and diameter of tree at breast height (DBH). There are some adopted techniques to calculate the volume of standing trees i.e. volume table, volume equation. These techniques also depend upon the species in the forest stand and the region where the stand is located. Thus the timber volume can be calculated by using any of the above technique depending upon desired accuracy, money, time and labour. In Malii-Celebes (Indinesia), before estimating timber volume from aerial photographs, the relationship between dbh and crown diameter of upper canopy of trees was first investigated. As species identification was impossible on 1:10,000 scale photographs, all species were included in the test. The regression equation was found to be:
$\mathrm{d}=3.5 \mathrm{C}+12.3$ where, $\mathrm{d}=\mathrm{dbh}$ and $\mathrm{C}=$ crown diameter

## 2. Species Composition (biodiversity)

Turner et al. (2003) stated that the recent advances in remote sensing, such as the availability of remotely sensed data with high spatial and spectral resolutions, make it possible to detect key environmental parameters, which can be applied to determine the distribution and abundance of species across landscapes via ecological models. This approach, in general referred to as indirect remote sensing of biodiversity, plays a major role in this research area. For example, Defries et al. (2000) applied the 1 km Advanced Very High Resolution Radiometer (AVHRR) to estimate and map percentage tree cover and associated proportions of trees with different leaf longevity (evergreen and deciduous)

## 3. Forest Ecophysiology

Kokaly and Clark (1999) developed an approach to estimate the concentrations of nitrogen, lignin, and cellulose in dried and ground leaves using ban-depth analysis of absorption features (centered at $1.73 \mu \mathrm{~m}, 2.10 \mu \mathrm{~m}$, and $2.30 \mu \mathrm{~m}$ ) and stepwise multiple linear regression. As mentioned above, hyperspectral remote sensing was used to
estimate the leaf pigment of sugar maple (Acer saccharum) in the Algoma Region, Canada, and promising results were obtained (Zarco-Tejada et al., 2001).

## 4. Forest Ecosystem

Jin et al. (2011) developed an algorithm based on a semi-empirical Priestley- Taylor approach to estimate continental-scale evapotranspiration (ET) using MODIS satellite observations. The seasonal variation in ET has been indicated as a key factor to the soil moisture and net ecosystem CO2 exchange through water loss from an ecosystem. Lefsky et al. (2002) reviewed Lidar remote sensing for ecosystem studies. Lidar is capable of accurately measuring vertical information besides the horizontal dimension, such as the three dimensional distribution of plant canopies and subcanopy topography (Lefsky et al., 2002). More specifically, Lidar can provide accurate estimates vegetation height, cover, canopy structure, leaf area index (LAI), aboveground biomass, etc (Lefsky et al., 2002).

## 5. Measuring and Monitoring Forest Resources

Cohen et al. (1995) stated that "remote sensing can play a major part in locating mature and old-growth forests", and applied a number of remote sensing techniques to estimate forest age and structure. Over a 1,237,482 ha area was investigated and an accuracy of 82 per cent was obtained. Maps of species richness have been recognized as a useful tool for biodiversity conservation and management due to its capability of explicitly describing information on the spatial distribution and composition of biological communities (Hernandez-Stefanoni et al., 2011). Hernandez-Stefanoni et al. (2011) tested remotely sensed data with regression kriging estimates for improving the accuracy of tree species richness maps, and concluded that this research will make a great step forward in conservation and management of highly diverse tropical forests.

## 6. Damage Assessment

The use of remote sensing in the detection of the effects of damaging agents on a forest precedes most other remote sensing forestry uses. Forset damage is defined as any type and intensity of an effect, on one or more trees, produced by an external agent, that temporarily or permanently reduces the financial value, or impairs or removes the biological ability of growth and reproduction. In the United States, insects and diseases account for a timber loss equal to our annual growth and this loss
exceeds that from fire by seven times. Because the damaging agents are dynamic forces, entomologists and pathologists find that remote sensing techniques are most valuable when they are used at critical periods of stress. One damage causing agent may produce a number of damage syndromes conversely syndrome may have been caused by any number of agents.

## 7. Insects

Forest insects cause symptoms of tree and forest injury which are more easily recognized than those caused by forest diseases or air pollution. For example defoliators of coniferous or hardwood trees frequently cause the foliage to change color from a normal green-yellow to yellow or dark yellow-red. These changes are readily visible, occur over large areas, and can be mapped by direct observation. When many trees are attacked at one time and begin showing signs of stress by changes in foliage color, they can be differentiated from healthy trees by remote sensing methods.

## 8. Disease

Most visible symptoms of forest disease are evident only when the disease is far advanced in the host tree. As with insect damage, manifestations of disease show as discolorations and thinness of foliage. Oak was affected by fungus which occludes the water conducting tissues of oak. The symtoms show up as dying back of the top and discoloration of wilting oaks. Damages caused by cronartium ribicola Fischer on Pinus strobes are easy to detect on medium scale color and color infrared photos.

## 9. Deforestation

Since deforestation is a continuing process, efforts to inventory and monitor changes are very closely related. There are many uncertainties about actual rates of deforestation (Sader et al.1990), hence the need for accurate, up-to-date monitoring schemes. Techniques used to inventory these areas also can be applied in their systematic monitoring to create a time-serie sof data describing rates and magnitudes of deforestation. In Rondonia Brazil, for example, Landsat MSS (1980) and TM (1986) imagery were used to define the area and deforestation rates for a study area of
approximately 30,000 square kilometers (Stone et al. 1991). The researchers found that 3168 square kilometers ( 528 squarekm/year) of new clearing occurred between 1980 and 1986. Earlier research (Woodwell et al.1987) had revealed a rate of clearing of 14 square km/year from 1972-1978 and 79 square km/year from 1978-1980. Historical records have also been used in GIS to identify changes in forest cover. Between 1979and 1984, a land resource inventory project was completed in the Jhikhu Khola watershed in Nepal (see Schreier et al. 1989). Land use information was digitized using 1:50,000 scale topographic maps as the base for information collected by surveying 1980. Land use data that had been divided into three broad categories in the original 1950 topographic map were also digitized. The area of each land use type was calculated in the GIS and then the two layers were subtracted. "Although somewhat crude, this information was found to be very useful in producing a land use change overview map" (Schreier et al. 1989). The thirty-year interval revealed that about 50 percent of the forestland has been lost to shrub and agriculture. A second three-year project was initiated in 1988 to "examine processes relating to soil erosion, sediment transport, soil fertility changes and land use changes in a quantitative way" in the Jhikhu Khola watershed (Schmidt and Schreier 1991). Forest and agricultural land uses were mapped and digitized using 1:20,000 scale and aerial photographs taken in 1972 and 1989. Changes in the area of four land uses were calculated for each date: forest, grassland, irrigated agriculture and sloping terraces. In this case, using a larger scale and a different land cover scheme, the researchers found that the forest area had not decreased substantially (only 1 percent) during these 17 years.

## 10. Forest Fires

Fire is one of the disasters causing threats to the forests and the ecosystem throughout the world. Fires have adverse effects on soil, forests and humans. During the process of burning, the soil nutrients are reduced and the soil is left bare making it more susceptible to both soil and water erosion. The forest cover is drastically reduced through the death of fire intolerant tree species. Fire also leads to an increase in green house gas emissions (Fig. 6). Air pollution due to smoke causes prolonged effects on human health such as respiratory and cardiovascular problems. Mongolia has a serious increase in forest fires.

## Summary:

Measures of central tendency or averages are used to summarise the data. It specifies a single most representative value to describe the data set. Arithmetic mean is the most commonly used average. It is simple to calculate and is based on all the observations. But it is unduly affected by the presence of extreme items. Median is a better summary for such data. Mode is generally used to describe the qualitative data. Median and mode can be easily computed graphically. In case of open-ended distribution they can also be easily computed. Thus, it is important to select an appropriate average depending upon the purpose of analysis and the nature of the distribution.

