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MT-609

Integral Equations

MA/MSC Mathematics (MAMT/MSCMT)

4th Semester Examination, 2023 (June)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A
(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only. (2×9½=19)

1. Show that the function $g(x) = xe^x$ is a solution of the Volterra integral equation $g(x) = \sin x + 2\int_0^x \cos(x-t)g(t)dt$.

2. Find the eigen values and eigen functions of the homogeneous integral equation

$$g(x) = \lambda \int_1^2 \left(xt + \frac{1}{xt} \right) g(t)dt.$$

3. Solve the following integral equations :

(a) $g(x) = e^x + \lambda \int_0^1 2e^x . e^t g(t)dt.$

(b) $g(x) = e^x + \lambda \int_0^{10} xt \; g(t)dt.$

4. Slove the Volterra integral equation of the first kind

$$\int_0^x a^{x-1} g(t)dt = f(x). \; f(0) = 0.$$

5. Slove the integral equation with the aid of resolvent kernel:

$$g(x) = e^{x^2} + \int_0^x e^{x^2 - t^2} g(t)dt.$$

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Show that the function $g(x) = xe^{-x}$ is a solution of the equation

$$g(x) - 4\int_0^{\infty} e^{-(x+t)} g(t)dt = (x-1)e^{-x}.$$

2. Find the eigen values and eigen functions of the integral equation

$$g(x) - \lambda \int_0^1 \left(2xt - 4x^2\right) g(t)dt.$$

3. Convert the following BVP into integral equation

$$\frac{d^2y}{dx^2} + \lambda y = 0;\ y(0) = 0 = y(l).$$

4. Solve the integral equation by the method of resolvent kernal

$$g(x) = 1 + \lambda \int_0^{\pi} \sin\left(x - t\right) g(t)dt.$$

5. Define following with example :

(a) Integral equation.

(b) Linear integral equation.

(c) Fredholm integral equation of first and second kind.

(d) Volterra integral equation of first and second kind.

6. Solve the integral equation :

$$g(x) = x + \lambda \int_0^1 \mathrm{K}(x, t)\, g(t)dt$$

$$\mathrm{K}(x, t) = \begin{cases} x(t-1), & 0 \le x \le t \\ t(x-1), & t \le x \le 1 \end{cases}$$

7. Solve the integral equation :

$$g(x) + 5 \int_0^x \cos 2\,(x-t)\, g(t)dt = 10, \text{ where } g(0) = 2.$$

8. Solve the following integral equation by the method of successive approximations to third order

$$g(x) = 1 + \lambda \int_0^1 (x+t) g\,(t)dt, \text{ by taking } g\,(0) = 1.$$
