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MT-606

Analysis and Advanced Calculus-II

MA/MSC Mathematics (MAMT/MSCMT)

4th Semester Examination, 2023 (June)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A
(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only. (2×9½=19)

1. Prove that an operator T on a finite-dimensional Hilbert space H is singular if and only if there exist a non-zero vector x in H such that $Tx = 0$.

2. Prove that if f be a regulated function on a compact interval $[a, b]$ of R into R such that $a < b$ and for all t in $[a, b]$, $f(t) \geq 0$. Then $\int_b^a f(t)dt \geq 0$.

3. Prove that every scalar multiple of self- adjoint operator is also normal.

4. Prove that If T be an operator on a Hilbert space H, then there exist a unique linear operator T^* on H such that $(Tx, y) = (x, T^*y) \forall x, y \in H$ obviously T^* is the adjoint operator H.

5. State and prove Implicit function Theorem.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Prove that A closed linear subspace M of a Hilbert space H reduces an operator if and only if M is invariant under both T and T^*.

2. Show by an example that it is not necessary for an arbitrary operator on Hilbert space H to possess an eigen value.

3. Write a short note on Positive Operator, Unitary Operator.

4. Prove that If T is a normal operator on a Hilbert space H, then x is an eigenvector of T with eigen value λ iff x is an eigenvector of T^*. with $\overline{\lambda}$ as eigenvalue.

5. Define orthogonal Projection, Reduciblity and Invariance of an operator on a Hilbert space.

6. Define integral of a step function and also defined regulated function.

7. State and Prove Global Uniqueness Theorem.

8. State a spectral theorem.