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MT-604

Integral Transforms

MA/MSC Mathematics (MAMT/MSCMT)

3rd Semester Examination, 2023 (June)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answer to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

SECTION–A
(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only. (2×9½=19)

1. Solve $(D^2 + 9)y = \cos 2t$, if $y(0) = 1$, $y\left(-\frac{\pi}{2}\right) = -1$.

2. Find $f(t)$, if its Fourier sine transform is $\dfrac{P}{(1+P)^2}$.

3. The temperature $U(x, t)$ in the semi-infinite rod $0 \le x \le \infty$ is determined by the differential equation :

$$\frac{\partial U}{\partial t} = k \frac{\partial^2 U}{\partial x^2}.$$

Subject to the conditions :

(a) $U = 0$ when $t = 0, x \ge 0$.

(ii) $\dfrac{\partial U}{\partial x} = -\mu$ (constant), when $x = 0, t > 0$. Making use of cosine transform, show that $U(x, t) = \dfrac{2\mu}{\pi} \displaystyle\int_0^\infty \frac{\cos pu}{p^2}$ $(1 - e^{-kp^2 t})dp$.

4. Apply convolution theorem to prove that

$$B(m, n) = \int_0^1 u^{m-1} (1-u)^n - 1\ du = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)},\ m, n > 0.$$

Hence deduce that

$$\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2n-1}\theta\ d\theta = \frac{1}{2} B(m, n) = \frac{2\Gamma(m)\Gamma(n)}{\Gamma(m+n)},$$

where $B(m, n)$ is called Beta function.

5. An infinite long string having one end $x = 0$ is initially at rest on the x-axis. The end $x = 0$ undergoes a periodic transverse displacement given by $A_0 \sin\omega$, $t > 0$. Find the displacement of any point on the string at any time.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Find the Laplace transform of :

   (a) $(1 + te^{-t})^3$

   (b) $t^2 e^t \sin 4t$.

2. Find the inverse Laplace transform of

   (a) $\dfrac{a^2}{p(p+a)^2}$.

   (b) $\dfrac{1}{p} \log \dfrac{p+2}{p+1}$.

3. Prove that if n is a positive integer, $M\left[\left(x\dfrac{d}{dx}\right)^n f(x);\, p\right]$ $= (-1)^n p^n F(p)$ where $M\{f(x);\, p\} = F(p)$.

4. If F(p) and G(p) are the Mellin transform of $f(x)$ and $g(x)$ respectively. Find the mellin transform of

$$x^{\lambda}\int_{0}^{\infty} u^{\mu} f\left(\frac{x}{\mu}\right) g(u)\, du$$ where λ and μ are constants.

5. Find the Hankel transform of $\frac{\cos ax}{x}$ taking $xJ_0(px)$ as the kernel.

6. Show that $f(t) = e^{t^3}$ is not of exponential order.

7. Prove that

(a) If M $\{f(x); p\}$ = F(p), then M $\{f(x); p\} = a^{-p}$ F(p).

(b) If M $\{f(x); p)$ = F(p), then M$\{x^n f(x); p\}$ = F($p + a$).

8. Find the Hankel transform of

$$f(x) = \begin{cases} 1, \; 0 < x < a, \; v > 0 \\ 0, \; x > a, \; v = 0. \end{cases}$$
