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## **MT-602**

### **Viscous Fluid Dynamics-I**

MA/MSc Mathematics (MAMT/MSCMT)

3rd Semester Examination, 2023 (June)

**Time : 2 Hours]**

**[Max. Marks : 35**

**Note :** This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

### **SECTION-A**

#### **(Long Answer Type Questions)**

**Note :** Section 'A' contains Five (05) long answer type questions of Nine and Half ( $9\frac{1}{2}$ ) marks each. Learners are required to answer any Two (02) questions only.

$$(2 \times 9\frac{1}{2} = 19)$$

1. Show that for each state of stress at a point, there exists at least one set of three mutually perpendicular principal directions.

2. Show that the vorticity vector  $\Omega$  of an incompressible viscous fluid moving under no external forces satisfies the differential equation  $\frac{D\Omega}{Dt} = (\Omega \cdot \nabla)q + \nu \nabla^2 \Omega$  where  $\nu$  is the kinematic viscosity.
3. Show that in the dynamics of compressible fluids, there are only five independent dimensionless groups.
4. State and prove the Navier-Stokes equations of motion for a viscous fluid compressible fluid.
5. Discuss unsteady flow of a viscous incompressible fluid due to an oscillating plane wall.

### SECTION-B

#### (Short Answer Type Questions)

**Note :** Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Let the stress tensor at a point in the fluid be 
$$\begin{bmatrix} 5 & 2 & 2 \\ 2 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}.$$

Obtain the principal stresses.

2. Show that the two principal directions corresponding to any two distinct principal stresses are orthogonal.
3. The resistance force  $R$  of a supersonic plane during flight can be considered as dependent upon the length of the aircraft  $l$ , velocity  $V$ , air velocity  $\mu$ , air density  $\rho$  and bulk modulus of air  $K$ . Find an expression for  $R$ .
4. Define and give the physical importance of the followings: Grashoff Number, Eckert Number, Reynold's Number, Brinkman Number.
5. Derive the velocity distribution in plane poiseuille flow.
6. Write short notes on :
  - (a) Limitations of the Navier-Stokes equations.
  - (b) Exact Solutions of the Navier-Stokes equations.
7. What type of the motion do the following velocity components constitute?  
$$u = a + by - cz; \quad v = d - bx + ez; \quad w = f + cx - ey;$$
where  $a, b, c, d, e, f$  are arbitrary constants.
8. Write Navier-Stokes equations in cartesian co-ordinates. Simplify these equations when the fluid is incompressible and viscous effects are negligible.

