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MT-601

Analysis and Advanced Calculus-I

MA/MSC Mathematics (MAMT/MSCMT)

3rd Semester Examination, 2023 (June)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A
(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only.

(2×9½=19)

1. Prove that the if N and N' be normed linear space over the same field and if T be a linear transformation from a normed linear space N into normed space N'. Then T is bounded if it is continuous.

2. Prove that Let N be a normed linear space, and suppose the set $S = \{x \in N: \|x\| = 1\}$ is compact then N is finite dimensional.

3. State and prove Reisz Lemma.

4. State and prove Minkowski's Inequality.

5. Prove that Every Banach space is a Normed space but converse may not be true.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. $(4\times4=16)$

1. Define an orthonormal set and orthogonal set in Hilbert space with example.

2. Define normed spaces, Banach spaces. Give two examples of Banach spaces.

3. Prove that if N be a real normed linear space and suppose $f(x) = 0$ for all $f \in N^*$. Show that $x = 0$.

4. Prove that if M is a closed linear subspace of a Hilbert space, then $H = M \oplus M^{\perp}$.

5. Prove the Parallelogram Law in Hilbert space.

6. State Hahn Banach Theorem and Uniform Boundedness Theorem.

7. Prove that An orthonormal set S in a Hilbert space H is complete iff $x \perp S \Rightarrow x = O \forall x \in H$.

8. Prove that If N be a normed linear space and $x, y \in N$, then $\|\| x \| - \| y \|\| < \|x - y\|$.
