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MT-510

Mechanics-II

MA/MSC Mathematics (MAMT/MSCMT)

2nd Semester Examination, 2023 (June)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answer to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

SECTION–A
(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only. (2×9½=19)

1. A symmetrical top is set in motion on a rough horizontal plane with an angular motion about its axis of figure, the axis being inclined at an angle i to the vertical. Show that

between the greatest approach to and recess from the vertical, the centre of gravity describes an arc

$$h \tan^{-1}\left(\frac{\sin i}{p - \cos i}\right)$$

where p and h have their usual meanings.

2. Deduce the Lagrange's Equations from Hamilton's Principle.

3. Find the equation of the stream lines for the flow $\vec{q} = x\hat{i} - y\hat{j}$.

4. Show that for an incompressible fluid, the equation of continuity becomes $div\,\vec{q} = 0$.

5. State and prove Euler's dynamical equations of motion in Cartesian coordinate.

SECTION–B
(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. If $u = 2xy$ and $v = a^2 + x^2 - y^2$ are the velocity components of a fluid motion, then find the stream function.

2. Show that $u = \dfrac{-2xyz}{(x^2+y^2)^2}, v = \dfrac{(x^2-y^2)z}{(x^2+y^2)^2}$ and $w = \dfrac{y}{(x^2+y^2)}$ are the velocity components of a possible fluid motion. Is this motion irrotational?

3. A mass of the fluid is in motion so that the lines of motion lie on the surface of coaxial cylinders, show that the equation of continuity is

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho u)}{\partial \theta} + \frac{\partial(\rho v)}{\partial z} = 0.$$

where u, v are the velocity perpendicular and parallel to z.

4. To determine the image of the doublet with respect to a circle.

5. In irrotational motion in two dimension, prove that

$$\left(\frac{\partial q}{\partial x}\right)^2 + \left(\frac{\partial q}{\partial y}\right)^2 = q\nabla^2 q.$$

6. A stream in a horizontal pipe, after passing a contraction in the pipe at which its sectional area is A is delivered at atmospheric pressure at a place, where the sectional area is

B. Show that if a side tube is connected with the pipe at the former place, water will be sucked up through it into the pipe from a reservoir at a depth

$$\frac{s^2}{2g}\left(\frac{1}{A^2}-\frac{1}{B^2}\right)$$

below the pipe, s being the delivery per second.

7. The velocity components for a two dimensional flow system can be given in the Eulerian system by $u = 2x + 2y + 3t$; $v = x + y + \frac{1}{2}t$. Find the displacement of a fluid particle in the Lagrangian system.

8. Define the following :

(a) Complex Potential.

(b) Source and sinks.