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MT-509

Differential Geometry and Tensor-II

MA/MSC Mathematics (MAMT/MSCMT)

2nd Semester Examination, 2023 (June)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answer to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

SECTION–A
(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only.

(2×9½=19)

1. Calculate the Christoffel symbols $[ij, k]$ and $\left\{ \begin{matrix} i \\ jk \end{matrix} \right\}$

Corresponding to the metric

$ds^2 = (dx^1)^2 + (x^1)^2 (dx^2)^2 + (x^1 \sin x^2)^2 (dx^3)^2.$

2. In Covariant differentiation, the fundamental tensors g_{ij}, g^{ij} and the kronecker delta δ^i_j behave like constants. Prove it.

3. Prove that the curves $u + v$ = Constant; are geodesies on a surface with metric

$(1 + u^2)du^2 - 2uvdudv + (1 + v^2)dv^2$.

4. Prove that a necessary and sufficient condition that a curve on a developable surface be a geodesic is that the surface be the rectifying developable of the curve.

5. State and prove Gauss-Bonnet theorem.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Let A_{ij} and B_{ij} be any two covariant tensors of rant 2, then prove that $(A_{ij} + B_{ij}), k = A_{ij},_k + B_{ij, k}$.

2. Prove that $\delta^i_j \delta^j_k = \delta^i_k$.

3. If two tensors are equal in a co-ordinate system then prove that, they are also equal in every other co-ordinate system.

4. Prove that $\frac{\partial A_i}{\partial x^j}$ is not a tensor though A_i is a tensor of type (0,l).

5. A Particle is constrained to move on a smooth surface under no forces except the normal reaction. Show that its path is a geodesic.

6. Prove that geodesies on a right circular cylinder are helics.

7. Write a short note on 'Bonnet's formula, for geodesic Curvature.

8. If two families of geodesies on a surface intersect at a constant angle, prove that the surface has Zero Gaussian Curvature.