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Total Pages : 3 Roll No. ....

MT-508

Special Functions

MA/MSC Mathematics (MAMT/MSCMT)

2nd Semester Examination, 2023 (June)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A
(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only. (2×9½=19)

1. Solve in series $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + y = 0$.

2. Prove that $J_n(x)$ is the coefficient in the expansion of $e^{\frac{x}{2}\left(z-\frac{1}{z}\right)}$.

3. Solve Hypergeometric equation

$$x(1-x)\frac{d^2y}{dx^2}+\{\gamma-(\alpha+\beta+1)x\}\frac{dy}{dx}-\alpha\beta\gamma=0\text{; about } x=0.$$

4. Prove that $(1 - 2xz + z^2)^{-½} = \sum_{n=0}^{\infty} z^n P_n(x), |x| \leq 1, |z| < 1$.

5. Show that the Laguerre polynomials are orthogonal over $(0, \infty)$ with respect to the weight function e^{-x}.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Prove that $(n+1)\, L_{n+1}(x) = (2n+1-x)\, L_n(x) - n\, L_{n-1}(x)$

2. Define Hypergeometric function and write its different forms.

3. Express $H(x) = x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Hermite's polynomials.

4. Show that $L_n(0) = 1$.

5. State and prove orthogonality property of Laguerre polynomials.

6. Prove that $H_{2n}(0) = (-1)^{n\frac{(2n)!}{n!}}$.

7. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\left(\frac{2}{\pi x}\right)} \sin x$

8. Prove that $(2n+1)\, P_n(x) = P'_{n+1}(x) - P'_{n-1}(x)$.