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MT-507

Topology

MA/MSC Mathematics (MAMT/MSCMT)

2nd Semester Examination, 2023 (June)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only.

(2×9½=19)

1. (a) If τ_1 and τ_2 are topologies on the same set X, then prove that $\tau_1 \cap \tau_2$ is also a topology on X.

(b) Prove that a subset A of a topological space is open iff $\mathring{A} = A$, where $\mathring{A}$ is interior of A.

2. Prove that one-one onto map $f : (X, \tau) \rightarrow (Y, \mu)$ is a Homeomorphism iff $\overline{f(A)} = f(\overline{A})$, for any $A \subset X$.

3. (a) Prove that the property of a space being T_1 – space is a topological property.

(b) Prove that every metric space is T_2 – space.

4. Prove that a subset of R is connected iff it is an interval.

5. (a) Show that a closed subset of a compact space is compact.

(b) Show that T_∞ is a topology on X_∞.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Prove that for a subset A of a topological space (X, τ),

$\overline{A} = A \cup A'$

where $\overline{A}$ = closure of A and A' = Derived set of A.

2. Let (X, τ) be a topological space and $x \in X$ be arbitrary and if N_1 and N_2 are neighborhoods of x, then show that $N_1 \cap N_2$ is also a neighborhoods of x.

3. Prove that a function $f : X \to Y$ is continuous iff the inverse image under f of every closed subset of Y is closed in X.

4. Prove that the property of space being T_2 –space is a hereditary property.

5. Prove that every T_3 – space is T_2 – space.

6. Show that two open subsets of a topological space are separated iff they are disjoint.

7. Prove that the product space $X \times Y$ is connected iff X and Y are connected.

8. Show that every compact topological space is locally compact but converse is not necessarily true.