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MT-506

Advanced Algebra-II

MA/MSC Mathematics (MAMT/MSCMT)

2nd Semester Examination, 2023 (June)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A
(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only. (2×9½=19)

1. Let $t : R^3 \to R^3$ be a linear transformation such that $t(a. b, c) = (3a + c, - 2a + b, - a + 2b + 4c)$. What is the matrix of t in ordered basis $\{\alpha_1, \alpha_2, \alpha_3\}$, where $\alpha_1 = (1,0,1)$, $\alpha_2 = \{-1, 2, 1\}$ $\alpha_3 = \{2, 1, 1\}$.

2. Find the Characteristic roots and characteristic spaces of the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \\ 0 & 0 & 4 \end{bmatrix}$.

3. State and Prove Gram Schmidt orthogonalization process.

4. Prove that if K be a Galois extension of a field F. Then there exists a one-to-one correspondence between the set of all subfields of K containing F and the set of all subgroups of G(K\F). Further, if E is any subfiels of K which contains F, then

   (i) [K: E] = o [G(K\E)] and [E: F] = index of G(K\E) in G(K\F),

   (ii) E is normal extension of F iff G(K\E) is normal subgroup of G(K\F).

   (iii) If E is a normal extension of F, then G (E\F) $\cong$ G (K\F)\G(K\E).

5. Let V be a finite dimensional inner product space. Then a linear transformation t: V $\rightarrow$ V is orthogonal if and only if its matrix relative to an orthonormal basis is orthogonal.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Define :

(a) Normal Extension.

(b) Automorphism.

2. Show that the Galois group of $x^4 + 1 \in Q[x]$ is the Klein four group.

3. Prove that for any matrix A, the row rank of A, equals to the column rank of A.

4. Let A be a matrix of order $n \times n$ over a field F. Prove that a scalar $\lambda \in$ F is an eigenvalue of A iff $\det(A - \lambda I) = 0$.

5. Define real inner product space with example.

6. If W is any subspace of a finite dimensional inner product space V, then $(W^{\perp})^{\perp} = W$

7. Prove that the eigenvalues of a self-adjoint linear transformation are real.

8. Let K be a normal extension of a field F and L is an intermediate field, so prove that $F \subset L \subset K$, then K is also a normal extension of L.