P-134

Total Pages : 3 Roll No. ....

MT-503

Differential Equation and Calculus of Variation

MA/MSC Mathematics (MAMT/MSCMT)

1st Semester Examination, 2023 (June)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only. (2×9½=19)

1. Solve $r = a^2t$ by Monge's method.

2. Use the method of separation of variables to solve the equation

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u. \text{ given that } u(x, 0) = 6e^{-3x}.$$

3. Reduce $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form.

4. Find the eigenvalues and eigenfunction for the boundary value problem

$$y'' - 2y + \lambda y = 0; y(0) = 0, y(\pi) = 0.$$

5. Find the extremal of the functional

$$I[y(x)] = \int_0^{\pi/2} [y''^2 - y^2 + x^2]\, dx$$

under the conditions

$$y(0) = 1, y'(0) = 0, y(\pi/2) = 0, y'(\pi/2) = -1.$$

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Show that 1, x, x^2 are three particular integrals of $x(x^2 - 1)y_1 + x^2 - (x^2 - 1)y - y^2 = 0$, and hence obtain the general solution $y(x + k) = x + kx^2$, k being an arbitrary constant.

2. Solve : $(2xz - yz)dx + (2yz - xz)dy - (x^2 - xy + y^2)dz = 0.$

3. Find the characteristics of $4r + 5s + t + p + q - 2 = 0.$

4. Solve $2s + rt - s^2 = 1.$

5. Test for extremum of the functional

$$F(y(x)) = \int_0^1 \sqrt{1 + y'^2}\, dy, \qquad y(0) = 0,\ y(1) = 2.$$

6. Solve $rx = (n - 1)p.$

7. Find the nature of following PDE

$$3\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + 5\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial y} = 0.$$

8. Obtain the Euler-Lagrange equation for the extremals of the functional

$$\int_{x_1}^{x_2} [y^2 - yy' + y'^2]\, dy.$$
