P-133

Total Pages : 4 Roll No. ....

MT-502

Real Analysis

MA/MSC Mathematics (MAMT/MSCMT)

Ist Semester Examination, 2023 (June)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only.

(2×9½=19)

1. If $E_1, E_2, \ldots$ are disjoint measurable sets and $E := E_1 + E_2 + \ldots$, then show that E is measurable and

$$\mu(E) = \sum_{k=1}^{\infty} \mu(E_k),$$

where $\mu(E)$ represents the measure of E.

2. Show that there exists a non-measurable set.

3. Let f be a non-negative bounded measurable function defined over a measurable set E such that

$$\int_E f(x)dx = 0.$$

Show that $f(x) = 0$ almost everywhere in E.

4. What is Lebesgue integral of a function f in an interval? Show that every bounded Riemann integrable function over an interval is Lebesgue integrable and both the integrals are equal. Does the converse hold true?

5. Let f_n be a sequence in L^P, $1 \le p \le \infty$, such that $f_n \to f$ *a.e.* and that $f \in L^p$. Show that

if $\lim_{n\to\infty} \|f_n\|_p = \|f\|_p$, then $\lim_{n\to\infty} \|f_n - f\|_p = 0$.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Let E_1, E_2, ... be a sequence of subsets of real numbers. Show that

$$\mu^*(E_1 + E_2 + \ldots) \leq \mu^*(E_1) + \mu^*(E_2) + \ldots,$$

where $\mu^*(E)$ denotes the Lebesgue outer measure of E.

2. Is the set of rational numbers measurable? If yes, find its measure.

3. If f is a continuous function and g is a measurable function, then show that the composite function $f \circ g$ is measurable.

4. Show that the function $\frac{\sin x}{x}$ is not Lebesgue integrable over $[0, \infty)$.

5. If A_1, A_2 are outer measurable sets and $A_2 \subset A_1$, then show that $A_1 \setminus A_2$ is an outer measurable set.

6. Show that the intersections of two outer measurable sets is outer measurable.

7. Show that the function ψ defined on $\mathbb{R}$ by

$$\psi(x) = \begin{cases} x+5\,; & x < -1 \\ 2\,; & -1 \le x < 0 \\ x^2\,; & x \ge 0 \end{cases}$$

is a measurable function.

8. Is L^p, $1 \le p \le \infty$, a metric space? Justify your answer briefly.
