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Total Pages : 3 Roll No. ....

MT-501

Advanced Algebra-I

MA/MSC Mathematics (MAMT/MSCMT)

1st Semester Examination, 2023 (June)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A
(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only. (2×9½=19)

1. Let G_1 and G_2 is a group. Let H_1 and H_2 be a normal subgroup of G_1 and G_2 respectively then prove that

(a) $H_1 \times H_2$ is normal subgroup of $G_1 \times G_2$

(b) $\dfrac{G_1 \times G_2}{H_1 \times H_2} \cong \dfrac{G_1}{H_1} \times \dfrac{G_2}{H_2}$

2. If M and M' are two R-modules and if $f : M \to M'$ is a homomorphism, then

(a) $f(0) = 0 \in M'$

(b) $f(-x) = -f\{x)$

(c) $f(x - y) = f(x) - f(y)$ for all $x, y \in M$.

3. If $B = \{b_1 = (-1,1,1), b_2 = (1,-1,1), b_3 = (1,1,-1)\}$ is the usual basis of R^3. Find its dual basis.

4. Let K be a field extension of a field F. Then an element a in K is algebraic over F iff $F(a)$ is finite extension of F, i.e. $[F(a): F]$ is finite.

5. Conjugacy on a group G is an equivalence relation.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Define

(a) Image of Homomorphism.

(b) Isomorphism

2. Prove that the conjugacy on a group G is an equivalence relation.

3. Prove that every finite group G has a composition series.

4. Define

(a) Euclidean ring.

(b) Unique factorization domain.

5. Prove that every field is Euclidean ring.

6. Show that the following mapping is not linear $t : R^2 \rightarrow R^2$ given by $t(x, y) = (x^3, y^3)$ for all $(x, y) \in R^2$.

7. Let K be field extension of a field F and let $\alpha \in$ K be a algebraic over F. Then any two minimal monic polynomial for α over F are equal.

8. Let G_1 and G_2 be groups and if $H_1 = \{(a, e_2) | a \in G_1\} \in$ then $G_1 \times G_2 \cong G_2 \times G_1$.
