

**P-816**

Total Pages : 5

Roll No. ....

**MCA-09/MSCIT-09**

**Discrete Mathematics**

(MCA/MSCIT)

3rd Semester Examination, 2023 (June)

**Time : 2 Hours]**

**Max. Marks : 70**

**Note :** This paper is of Seventy (70) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answer to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

**SECTION–A**

**(Long Answer Type Questions)**

**Note :** Section 'A' contains Five (05) long answer type questions of Nineteen (19) marks each. Learners are required to answer any Two (02) questions only.

(2×19=38)

1. (a) Let  $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$  and a relation  $R$  on  $A$  is defined as : (10)

$R = \{(a, b) : (a, b) \in A \times A \text{ and } a - b \text{ is divisible by } 2\}$ .

Check whether  $R$  is an equivalence relation, if so, find the equivalence class of 2.

- (b) Show that the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined as  $f(x) = 3x + 4$  for all  $x \in \mathbb{R}$  is one to one onto. (9)

2. (a) Define logical equivalence. Prove that  $(P \vee Q) \rightarrow R \equiv (P \rightarrow R) \wedge (Q \rightarrow R)$  (10)

- (b) Write predicates for the following sentences:

- (i) All students are happy.
- (ii) Some politicians are clever.
- (iii) Some boys are not tall. (9)

3. (a) In how many ways can 6 students (3 boys and 3 girls) arrange themselves in a row, if

- (i) no restriction is given.
- (ii) 2 particular students always sit together.
- (iii) 2 particular students always occupy corner positions.
- (iv) boys and girls always sit together.
- (v) only girls sit together. (10)

(b) There are 5 girls and 4 boys in a group. Find the number of ways in which a committee of 5 students can be formed under the following conditions: (9)

(i) There are 2 boys in the committee.

(ii) There are at least 2 girls in the committee.

(iii) There are at most 2 girls in the committee.

(iv) There is no restriction on the number of boys and girls in the committee.

4. (a) Show that the set of all positive rational numbers  $Q^+$  forms an abelian group under the composition defined

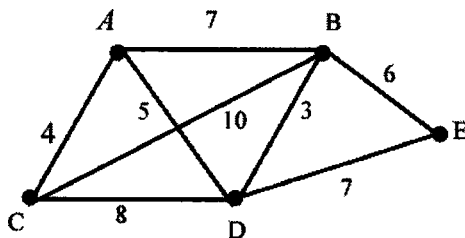
$$\text{by } a * b = \frac{ab}{2}. \quad (10)$$

(b) Prove that A non-empty subset H of a group G is a subgroup of G if (9)

(i)  $a \in H, b \in H \Rightarrow ab \in H$ .

(ii)  $a \in H \Rightarrow a^{-1} \in H$ .

5. (a) Define spanning tree in a graph. Find the minimal spanning tree using Prim's algorithm in the graph given below. (10)



- (b) Define a graph, simple graph and null graph. Prove that the number of vertices having odd degree in a graph is always even. (9)

## SECTION-B

### (Short Answer Type Questions)

**Note :** Section 'B' contains Eight (08) short answer type questions of Eight (08) marks each. Learners are required to answer any Four (04) questions only. (4×8=32)

1. Draw the Venn diagram of the following :
  - (a)  $(X \cup Y) \cap Z$
  - (b)  $X \cap Y \cap Z$
  - (c)  $(X - Y) \cup Z$
  - (d)  $(X \cup Y) - Z$
2. Define partial order relation. Let  $X = \{1, 2, 3, 4, 6, 12, 24, 36, 72\}$  and  $R = \{(x, y): x|y, \forall x, y \in X\}$ , where 'I' denotes the relation 'divides'. Check whether R is a partial order relation?
3. Let  $X = \{1, 2, 3, 4\}$  and  $f: X \rightarrow X, g: X \rightarrow X$  and  $h: X \rightarrow X$  defined as  $f = \{(1, 3), (2, 1), (3, 4), (4, 3)\}$ ,  $g = \{(1, 2), (2, 3), (3, 1), (4, 1)\}$  and  $h = \{(1, 2), (2, 3), (3, 4), (4, 1)\}$   
Find (i)  $f \circ g \circ h$  (ii)  $g \circ f \circ h$ .

4. Define principal disjunctive normal form (PDNF). Write the PDNF of  $P \vee (P \wedge Q)$ .
  5. Check the validity of the following argument : "If I go to market, then I buy a chocolate. I go to market. Therefore, I buy a chocolate."
  6. Define Pigeonhole principle. Find the minimum number of students in a class so that at least four students have same week-day of birth.
  7. Define lattice and bounded lattice with the help of suitable examples.
  8. Define different Tree traversal techniques with the help of suitable examples.
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