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MAMT-09

Integral Transforms and Integral Equations

MA/M.Sc. Mathematics (MAMT/MSCMT)

2nd Year Examination, 2023 (June)

Time : 2 Hours] **Max. Marks : 70**

Note : This paper is of Seventy (70) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answer to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

SECTION–A
(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nineteen (19) marks each. Learners are required to answer any Two (02) questions only.

(2×19=38)

1. Solve the differential equation by Laplace Transform:

$\frac{d^4y}{dx^4} - y = 1$, subject to conditions; $y(0) = y'(0) = y''(0)$
$= y'''(0) = 0.$

2. Find the solution of $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, where $0 < x < 1$, $t > 0$ together with the conditions.

$$u(x, 0) = 3 \sin 2\pi x,\ u(0, t) = 0,\ u(1, t) = 0.$$

3. Find the Fourier sine and cosine transform of $f(t)$, if

$$f(t) = \begin{cases} t, & 0 < t < 1 \\ 2 - t, & 1 < t < 2 \\ 0, & t > 2. \end{cases}$$

4. Solve the integral equation

$$g(x) = x + \lambda \int_{-\pi}^{\pi} (x \cos t + t^2 \sin x + \cos x \sin t) g(t) dt.$$

5. Solve the symmetric integral equation by using Hilbert - schmidit theorem

$$g(x) = (x + 1)^2 + \int_{-1}^{1} (xt + x^2 t^2) g(t) dt.$$

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Eight (08) marks each. Learners are required to answer any Four (04) questions only. (4×8=32)

1. Find the Laplace transform of

(i) $(1 + te^{-t})^3$

(ii) $\cosh^2 4t$.

2. Evaluate the inverse Laplace transform of $\dfrac{p}{(p+3)^2}$.

3. If $M(f(x): p\} = F(p)$, then show that

(i) $$M\left\{\int_0^x f(u)du;\ p\right\} = -\frac{1}{p}F(P+1)$$

(ii) $$M\left\{\int_0^x dy \int_0^y f(u)du;\ p\right\} = \frac{1}{p(p+1)}F(P+2)$$

4. Find the Hankel transform of

(i) $\dfrac{\cos ax}{x}$.

(ii) $\dfrac{\sin ax}{x}$.

5. Solve the homogeneous Fredholm integral equation

$$\phi(x) = \lambda \int_0^1 e^{x+t} \phi(t) dt.$$

6. Define following :

(i) Singular integral equation.

(ii) The Abel integral equation.

(iii) Integro-differential equation.

(iv) Integral equation of convolution type.

7. Using iterative method, solve

$$g(x) = f(x) + \lambda \int_0^1 e^{x-t} g(t) dt.$$

8. Convert the differential equation $\frac{d^2y}{dx^2} + \lambda y = 0,$ with the conditions; $y(0) = 0$, $y(1) = 0$, into Fredholm integral equation of second kind. Also recover the original differential equation from the integral equation you obtain.