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MAMT-06

Analysis and Advanced Calculus

A/M.Sc. Mathematics (MAMT/MSCMT)

2nd Year Examination, 2023 (June)

Time : 2 Hours] **Max. Marks : 70**

Note : This paper is of Seventy (70) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein. Candidates should limit their answer to the questions on the given answer sheet. No additional (B) answer sheet will be issued.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nineteen (19) marks each. Learners are required to answer any Two (02) questions only.

(2×19=38)

1. State and prove Reisz Lemma.

2. Let p be a real number such that $1 \leq p < \infty$. Show that the space l_p^n of all n – tuples of scalars with the norm defined by

$$\|x\|_p = \left\{ \sum_{i=1}^{n} |x_i|^p \right\}^{1/p}$$ is a Banach space.

3. Let N and N$'$ be normed linear spaces and D be a subspace of N. Then show that a linear transformation T : D $\rightarrow$ N$'$ is closed iff its graph T_G is closed.

4. Let M be a closed linear subspace of Hilbert space H and x be a vector not in M, suppose that $d = d(x, M)$. Then show that there exists a unique vector y_0 in M such that $\|x - y_0\| = d$.

5. If $\{e_1, e_2, ..., e_n)$ be finite orthonormal set in a Hilbert space H and x be any vector in H then prove that

(a) $\sum_{i=1}^{n} |(x, e_i)|^2 \leq |x|^2$.

(b) $x - \sum_{i=1}^{n} (x, e_i)\, e_i \perp e_j \;\forall\, j.$

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Eight (08) marks each. Learners are required to answer any Four (04) questions only. (4×8=32)

1. Define with examples

(a) Normed Space.

(b) Inner product space.

2. Prove that every compact subset of a normed space is bounded.

3. Prove that if x and y are any two vectors in a Hilbert space H, then

$$4(x,\ y) = \|x + y\|^2 - \|x - y\|^2 + i\,\|x + iy\|^2 - i\|x - iy\|^2$$

4. Give an example of inner product space which is not a Hilbert space.

5. State and prove Bessel's inequality in a Hilbert space.

6. Prove that an operator T on a Hilbert space H is unitary iff it is an isometric isomorphism of H onto itself.

7. If T_1 and T_2 are normal operators on H with the property that either commutes with adjoint of the other then prove that $T_1 + T_2$ and T_1T_2 are normal.

8. Let M be a linear subspace of a Hilbert space H then prove that M is closed iff $M = M^{\perp\perp}$.