

COURSE NAME: ADVANCED REAL ANALYSIS
COURSE CODE: MAT-502
SYLLABUS

Real number System

Finite and infinite sets, Natural numbers and Induction Principle, Countable and Uncountable sets, Cardinality, Integers, Rational Numbers, Ordered Field, Complete Ordered Field, Real Numbers, Dedekind Cuts. Sequences, limit point of sequences, limit-inferior and superior, convergent sequences, Cauchy's General principle of convergence, positive term series, Cauchy's root test, D'Alembert Ratio test, Raabe's test, series with arbitrary terms Rearrangements of Terms of a Series, limit, Continuity, Uniform Continuity, Derivative, Darboux's theorem, Rolle's theorem, Lagrange's mean Value Theorem, Cauchy's mean Value Theorem.

Riemann Integral

Refinement of partitions, Conditions of Integrability, Riemann Sums and Riemann Integral, Fundamental theorem, Improper Integrals. Monotonic Functions, Definition and Existence of Riemann-Stieltjes Integral, Improper Integral.

Uniform Convergence and Lebesgue Integral

Pointwise Convergence, Uniform Convergence on Interval, Cauchy's Criterion for Uniform Convergence, Test for Uniform Convergence, Test for Uniform Convergence of Series, Weierstrass M-test, Abel's and Dirichlet's test, Properties of Uniform Convergence, Uniform convergence and integrity. Measurable sets, Borel sets, Lebesgue Integral, Comparison with Riemann Integral for unbounded sets.

Metric Spaces

Definition and Examples of Metric Spaces, Definition and Examples of Metric Spaces, open sphere and closed sphere, Neighbourhoods, open sets and closed sets, limit points and Boundary points, subspace of metric space and Product of Metric space Bases, Convergent sequence, Cauchy sequence, Complete spaces, Dense sets and separable spaces, Baire's Category theorem, Continuity, Uniform continuity, Homeomorphism, Compact spaces and sets, sequential compactness, Finite Intersection Property, Separated Sets, Disconnected and Connected sets, Fixed point theorem, Contraction, Lipschitzian Map, Non-expansive Maps, Contractive Maps, Continuation Methods for Contractive and Non-expansive Mappings, Banach Contraction Principle.

REFERENCES:

1. R.G. Bartle and D.R. Sherbert (2000) Introduction of real analysis, John Wiley and Sons (Asia) P. Ltd., Inc.
2. W. Rudin (2019) Principles of Mathematical Analysis, McGraw-Hill Publishing, 1964.
3. Tom M. Apostol (1996). Mathematical Analysis (2nd edition), Narosa Book Distributors Pvt Ltd-New Delhi.
4. Pawan K. Jain and Khalil Ahmad (2005). Metric spaces, 2nd Edition, Narosa.

SUGGESTED READINGS:

1. Gerald G. Bilodeau, Paul R. Thie & G. E. Keough (2015). An Introduction to Analysis (2nd edition), Jones and Bartlett India .
2. K. A. Ross (2013). Elementary Analysis: The Theory of Calculus (2nd edition). Springer.

