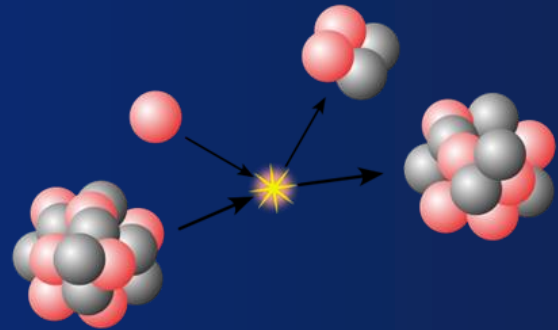
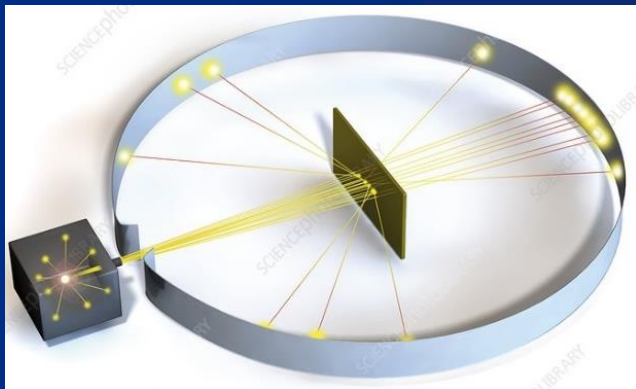




MSCPH 511

M. Sc. III SEMESTER NUCLEAR PHYSICS



DEPARTMENT OF PHYSICS

SCHOOL OF SCIENCE

UTTARAKHAND OPEN UNIVERSITY HALDWANI

NUCLEAR PHYSICS

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UNIT 1

NUCLEAR PROPERTIES

Structure of the Unit

- 1.1 Introduction
- 1.2 Objectives
- 1.3 Introduction to Nuclear Terminology
- 1.4 Discovery of Neutron
- 1.5 Rutherford Scattering and Nuclear Size Estimation
 - 1.5.1 Nuclear Radius
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1.1 INTRODUCTION

In this unit, we will study the basic concepts of the nucleus, constituents of the nucleus and some basic nuclear properties like nuclear size, nuclear mass, nuclear charge, etc. Before discussing the unit in detail we must have some knowledge about the atomic structure. Now question arises, what is the basic building block of all the matters? The answer is an “atom” and it is the smallest amount of matter that retains all the properties of an element and atomic theory also suggests that it is composed of smaller particles that no longer have the same properties as the overall element and consists of two main components which will be discussed in this unit. Now, when we look into the origins of nuclear physics, this can be traced back to atomic structure investigations, which began in 1896 with Henry Becquerel's discovery of radioactivity. It's helpful to review how the nuclear atom came to be in order to grasp the concept of the nucleus. The majority of scientists in the early nineteenth century agreed that chemical elements are made up of atoms, but they had no idea about atomic structure. The fact that all atoms contain negatively charged electrons was one of the first indications. So, in order for an atom to remain neutral, it must possess positively charged matter of some kind. But how did it come to be like that way? Rutherford discovered this when he used a thin gold foil to conduct his famous alpha-scattering experiment. The alpha particles were supposed to pass through the foil with hardly any deflection. This is based on the Thomson model, which assumes that the electric charge inside an atom is uniformly distributed throughout its volume. However, they were surprised to find that while the majority of the alpha particles did not deviate much, a few were distributed over quite wide angles. Some have even become scattered. This is when the nuclear atom notion was formed, in which the atom is claimed to be made up of a tiny nucleus carrying all of the positive charge and approximately all of its mass, with the electrons separated by some distance away.

1.2 OBJECTIVES

This unit introduces the basic concepts regarding nucleus and their basic properties. After studying this unit, you should be able to-

- Describe the basic terminology used in nuclear physics like atomic number, atomic mass, isobars, isotopes etc.
- Define fundamental properties of the nucleus such as nuclear size, nuclear mass and nuclear charge
- Know about Nuclear spin, magnetic moment and electric quadrupole moment.

1.3 INTRODUCTION TO NUCLEAR TERMINOLOGY

Probing the fundamental particles and their interactions, identifying and interpreting the features of nuclei, and making technological improvements that benefit society are the three main components of nuclear physics.

In nuclear physics a nuclear species is characterized by the total amount of positive charge contained by the nucleus and also by its total number of mass units. The value of the net nuclear charge is equal to $+Ze$, where Z is the *atomic number* and e denotes the magnitude of the electronic charge. The fundamental positively charged particle inside the nucleus is the *proton*, which is also the nucleus of the simplest atom, Hydrogen. A nucleus with the atomic number Z almost contains Z protons, and on the other hand an electrically neutral atom must therefore contain Z negatively charged electrons.

As we know that the mass of the electrons is negligible as compared to the proton mass ($m_p \cong 2000 m_e$), therefore the mass of the electron can often be ignored while we have the discussions about the mass of an atom. The *mass number* of a nuclear species is denoted by the symbol A , which is the integer nearest to the ratio between the nuclear mass and the fundamental mass unit.

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In general, A is greater than Z , nearly for all the nuclei and in most cases, it increases by a factor of two or more as compared to Z . Thus, there are more massive components in the nucleus.

Before 1932, it was supposed that the nucleus contained A protons, in order to provide the proper mass, along with $A - Z$ nuclear electrons to give a net positive charge of Ze . One question also arises here: Does electron exist inside the nucleus? The answer is provided with certain facts that are not satisfied for the electrons to remain inside the nucleus.

1. The nuclear electrons would have to be held to the protons by a strong force, maybe even stronger than the Coulomb force. Despite this, there is no evidence for a strong force between protons and atomic electrons.
2. If we try to confine the electrons in a region of a small space as small as a nucleus ($\Delta x \sim 10^{-14} m$) the uncertainty principle would require that these electrons have a momentum distribution with a range $\Delta p \sim \hbar/\Delta x = 20 MeV/c$. Electrons which emit from the nucleus in radioactive β decay have energies value generally less than $1 MeV$; never we have seen a decay of electrons with $20 MeV$ energies. Thus the existence of $20 MeV$ electrons in the nucleus is not confirmed by this observation.
3. The total intrinsic angular momentum (spin) of nuclei for which the value of $A - Z$ is odd would disagree with the observed values if A protons and $A - Z$ electrons were present in the nucleus. For example, consider the nucleus of deuterium with $A = 2, Z = 1$, which according to the proton-electron hypothesis would contain 2 protons and 1 electron. The proton and electron both have intrinsic spin angular momentum $\frac{1}{2}$ and from the quantum mechanical rules for adding spins of particles would require that these three spins of $\frac{1}{2}$ combine to give a total of value either $\frac{3}{2}$ or $\frac{1}{2}$. But the observed spin of the deuterium nucleus is 1.
4. Magnetic dipole moments in nuclei containing unpaired electrons should be far higher than those measured. If a single electron were present in a deuterium nucleus, we would anticipate the nucleus to have a magnetic dipole moment about the same size as an

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electron, however the observed value of the magnetic moment for the deuterium nucleus is $\frac{1}{2000}$ of the value of the magnetic moment of one electron.

All sorts of above reasons were eliminated by the discovery of neutron in 1932 by Chadwick. The neutrons in general are electrically neutral and it has a mass almost equal to that of proton mass. Thus a nucleus with Z protons and $A - Z$ neutrons has the proper total mass *and* charge, without the need to introduce nuclear electrons.

In general, a nuclear species or nuclide is represented by A_ZX , where X represents the chemical symbol with the number of neutrons $A - Z$, where A is the atomic mass and Z represents the atomic number which have the same value as that of protons. For example: 1_1H , 4_2He , 6_3Li , ${}^{16}_8O$, ${}^{56}_{26}Fe$.

The nucleon family consists of two members: neutrons and protons. We use the term nucleons when we only want to talk about nuclear particles without specifying whether they're protons or neutrons. As a result, a nucleus with mass number A has A nucleons. Nuclides with the same proton number but different neutron numbers are called *isotopes*; for example, the element chlorine has two isotopes that are stable against radioactive decay, ${}^{35}Cl$ and ${}^{37}Cl$. It also has many other unstable isotopes that are artificially produced in nuclear reactions; these are the radioactive isotopes (or *radioisotopes*) of Cl. A nuclide with the same N but different Z ; these are called *isotones*. The stable isotones with $N = 1$ are 2H and 3He . Such Nuclides which have the same mass number A are known as *isobars*; thus, stable 3He and radioactive 3H are isobars.

1.4 DISCOVERY OF NEUTRON

The neutron was not found until 1932, when James Chadwick calculated the mass of this neutral particle using scattering data. Since the time of Rutherford, scientists have known that the atomic mass number A of nuclei is somewhat more than double that of most atoms' atomic number Z , and that the nucleus contains almost all of the atom's mass. Protons and electrons were thought to be the fundamental particles around 1930, however this required that a number

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of electrons be bound in the nucleus to partially negate the charge of A protons. However, by this time, the uncertainty principle and "particle-in-a-box" confinement calculations had established that there just wasn't enough energy available to confine electrons in the nucleus.

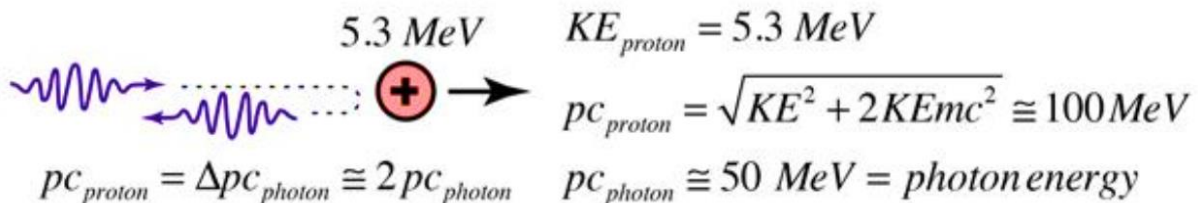
By putting the particle's De-Broglie wavelength equal to that dimension, a rough scale of the energy required for confinement to that dimension can be produced. If we assume a hydrogen atom has a diameter of 0.2 nm, the equivalent confinement energy is roughly 38 eV, which is the correct order of magnitude for atomic electrons. However, it takes around 250 MeV of energy to confine an electron to a nuclear dimension of about 5 fermis.

The maximum confinement energy available from the nucleus's electrical attraction is given by

$$\frac{Zke^2}{r} = \frac{79(1.44 \text{ MeV} \cdot \text{fm})}{5\text{fm}} \approx 23 \text{ MeV} \ll 250 \text{ MeV} \quad \dots \dots \dots (1)$$

So, from above result there are no electrons in the nucleus.

Bothe and Becker observed in 1930 that bombarding beryllium with alpha particles from a radioactive source created neutral radiation that was penetrating but non-ionizing, which was an experimental breakthrough. They assumed it was gamma rays, but Curie and Joliot demonstrated that when this radiation was used to bombard a paraffin target, it expelled protons with an energy of roughly 5.3 MeV. As can be seen through momentum and energy analysis, that wasn't the case with gamma rays:



5.3 MeV

$KE_{proton} = 5.3 \text{ MeV}$

$pc_{proton} = \sqrt{KE^2 + 2KEmc^2} \cong 100 \text{ MeV}$

$pc_{proton} = \Delta pc_{photon} \cong 2pc_{photon}$ $pc_{photon} \cong 50 \text{ MeV} = \text{photon energy}$

Fig. 01: Curie and Joliot experiment

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This analysis is similar to that of a head-on elastic collision in which a small particle collides with a considerably larger one. The required energy for the gamma ray explanation was significantly higher than any energy measured in the nucleus, thus the neutral radiation must be some form of neutral particle.

If the neutral particle had a mass comparable to that of the proton, the 5.3 MeV energy of the expelled protons could be easily explained. This would only require 5.3 MeV from the neutral particle in head-on collisions, which is within the range of observed nuclear particle emissions.

By bombarding targets other than hydrogen, such as nitrogen, oxygen, helium, and argon, Chadwick was able to demonstrate that the neutral particle could not be a photon. Not only were these interactions incompatible with photon emission on energy grounds, but their cross-section was orders of magnitude larger than that of photon Compton scattering.

Chadwick was left with the problem of estimating the mass of the neutral particle. He decided to bombard boron with alpha particles and study how the neutral particles interacted with nitrogen.

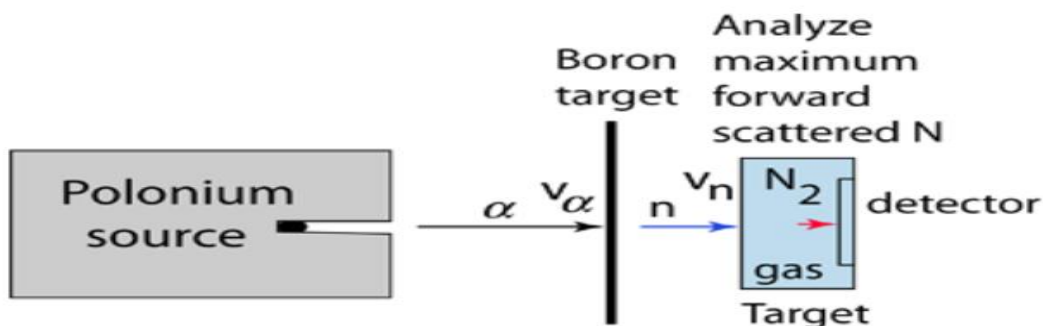


Figure 02: Discovery of neutron

The masses of boron and nitrogen were well known; thus, these specific targets were chosen. When energy conservation is applied to the combined interactions, the following formulas result.

$$\frac{1}{2}m_\alpha v_\alpha^2 + m_\alpha c^2 + m_B c^2 = \frac{1}{2}m_N v_N^2 + m_N c^2 + \frac{1}{2}m_n v_n^2 + m_n c^2 \dots \dots \dots (2)$$

Solving for the mass energy of the neutron gives

$$m_n c^2 \approx \frac{\frac{1}{2} m_\alpha v_\alpha^2 + m_\alpha c^2 + m_B c^2 - m_N c^2}{1 + \frac{v_n^2}{2c^2}} \dots \dots \dots (3)$$

The speed of the neutron is the last remaining unknown on the right hand side of the equation. Chadwick blasted hydrogen atoms with his created neutrons, assuming that the neutron mass was near to that of the proton, in order to determine the speed of the protons following the impacts. He then used the above energy expression to generate a neutron mass of 9381.8 MeV by setting the neutron speed equal to those proton speeds. Chadwick obtained the initial value for the neutron mass, which matched the current accepted estimate of 939.57 MeV by using a consistent set of measurements.

1.5 RUTHERFORD SCATTERING AND NUCLEAR SIZE ESTIMATION

A Preliminary idea about the nuclear model was first suggested by Rutherford in 1910. The diagram of Rutherford's experimental set up is shown in the figure 03. Obviously the apparatus has to be closed in vacuum. A thin film of gold foil is made to strike by a beam of alpha particles. The scattered alpha particles were detected by a fluorescent screen-telescope arrangement. Rutherford observed that many of the alpha particles went straight through the foil or they were deflected by a very small angle. But a few alpha particles were deflected by a very large angle. Further some were even got scattered back. This observation was not expected on the basis of idea of atomic structure at that time: that the positive charge is uniformly smeared over the entire volume of the atom.

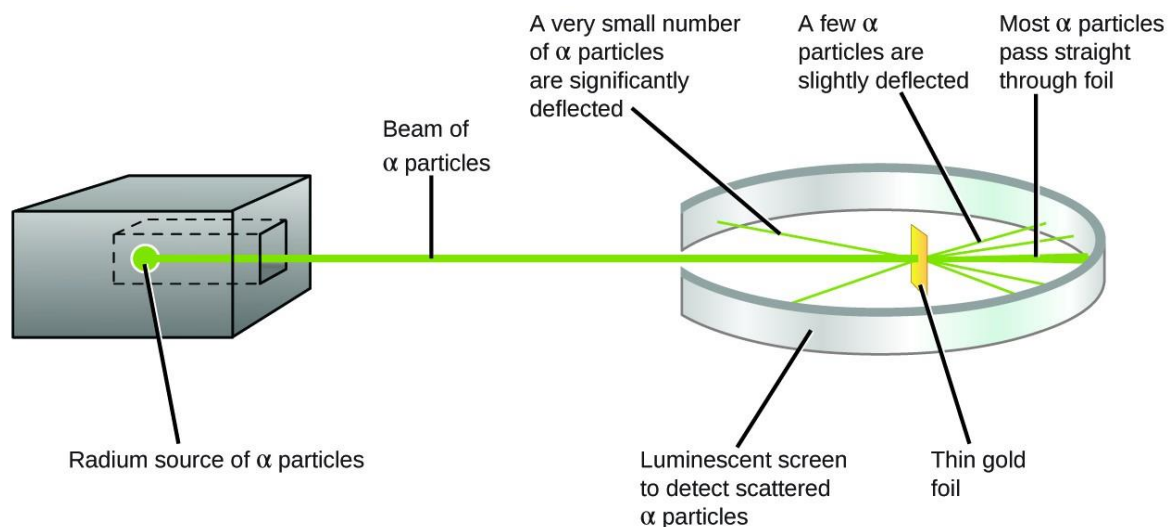


Figure 03: Rutherford's gold foil experiment

For explanation of these deflections, Rutherford assumed that the positive part of the atom was concentrated in a very small volume at the center of the atom. This *core*, which is surrounded by the cloud of electrons, which makes the entire atom electrically neutral, is now called as *nucleus*. This nuclear model of the atom accounts for the scattering of alpha particles at large angles in the following way. An alpha particle approaching the center of the atom experiences an increasingly large Coulomb repulsion. Since the atom is mostly empty space, most of the alpha particles do not go sufficiently close to the nucleus to get sufficiently deflected and so they pass through the foil without any deviation. However, an alpha particle that passes close to the nucleus is subjected to a very large Coulomb repulsive force exerted by the massive positive core and is deflected at a large angle in a single encounter.

1.5.1 Nuclear Radius

From the above discussion we have a basic idea about the nucleus. Now we shall discuss about the estimation of Nuclear Radius by considering Rutherford's α -Scattering experiment using classical approach. Let us consider that the gold nucleus in the scattering experiment has a radius R . An α -particle trajectory can be specified by its impact parameter b as shown in the figure

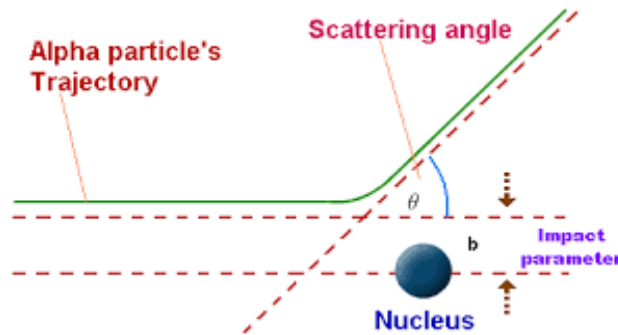


Figure 04: Deflection of an alpha particle by a gold nucleus

For the less coulomb reflection i.e for b is greater than R the deflection of alpha particle is also less. Also when b is less than R , the alpha particle goes through the nuclear positive charge distribution and so the nuclear charge above the trajectory and the one below the trajectory work in opposite directions and hence again the deflection produced in the alpha particle trajectory is small. When the alpha particle just almost touches the nucleus (which is considered to be a positive charge), this is the case when coulomb repulsion becomes maximum and correspondingly for $b \sim R$, the deflection of alpha particle also becomes maximum. For this qualitative discussion, let us consider a trajectory corresponding to small deflection. As the particle approaches near the gold nucleus, it is slowed by the Coulomb repulsion and is speeded up on its way out i.e away from the nucleus as shown in the figure below

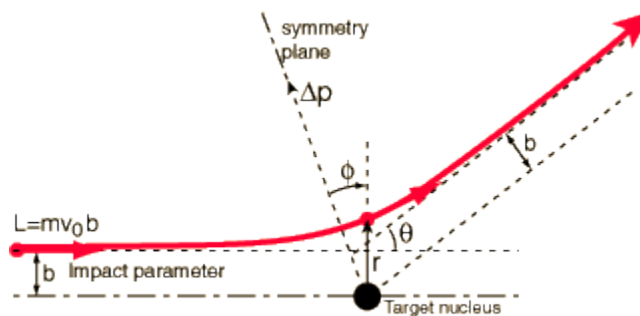


Fig.05: Particle trajectory with impact parameter b and small deflection θ

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The Coulomb repulsion in the region close to the scattering gold nucleus is given by the formula

$$F = \frac{1}{4\pi\epsilon_0} \frac{(2e)(Ze)}{b^2} \dots\dots\dots(4)$$

Where $2e$ represents the charge on α particle. Z represents the atomic number and for gold ($Z=79$).

Now this repulsive force can be approximately considered to be operating in the direction perpendicular to the direction of incidence, over a distance b . If we consider the particle's velocity to be v then the time during which the force acts is given by

$$\Delta t = \frac{b}{v} \dots\dots\dots(5)$$

By this force there is a momentum Δp in the direction perpendicular to the incident direction. By Newton's law

$$F = \frac{\Delta P}{\Delta t}$$

$$\Delta p = F \Delta t = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{b^2} \frac{b}{v} \dots\dots\dots(6)$$

From fig.05 the deflection θ is

$$\theta \sim \frac{\Delta p}{p} = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2/bv}{mv} \dots\dots\dots(7)$$

Where m denotes the mass of the α -particle.

So, we have

$$b = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{mv^2} \frac{1}{\theta} \dots\dots\dots(8)$$

Above equation shows the approximate relation between deflections and impact parameter.

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Rutherford in his experiment observed the maximum deflections were of the order of 1 radian. This doesn't mean that large deflections were not observed by Rutherford. Occasionally, deflections close to π radians were observed by him.

In above equation if we put the values corresponding to maximum deflections i.e $\theta = 1$ and $b = R$, we have

$$R \sim \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{mv^2} \dots\dots\dots (9)$$

If we put other values $Z= 79$ for gold, mass of α particle $m = 6 \times 10^{-27} \text{ kg}$

$v \sim 10^7 \text{ m/s}$ for speed of α particle

$$e = 1.6 \times 10^{-19} \text{ coulombs}$$

We obtain the radius has order of 10^{-14} m .

This distance is extremely small as compared to the atomic radius value 10^{-10} m , which is smaller by a factor of 10^4 .

Since protons are positively charged and the gravitational force is negligible inside the nucleus, the query then arranges us to how all these protons state together in a nucleus of size having value 10^{-14} m . There is certainly a nuclear interaction acting between protons and neutrons in bracket nucleons inside the nucleus over dispenses of about 10^{-14} m that is strong enough to counteract the massive column repulsion between the closely packed protons. Because the nuclear interaction's attraction force has no effect on the additional nuclear atomic structure, it must be a short-range force, effective only across distances of 10^{-14} m . This new force must, by definition, be more dependent on distance than an inverse square force. With very high energy particles generated by accelerators, the magnitude of the region where this nuclear short-range force is considerable (relative to the Coulomb force) may be examined. When such particles are fired at thin foils, the distribution of scattering angles reveals that the radius R of the nucleus is

proportional to the cube root of the scattering nuclei's mass number, i.e. $R = R_0 A^{1/3}$, where R_0 has the value of 1.07×10^{-15} m.

1.5.2 Measurement of Nuclear Radius

In the previous section we have estimated the nuclear size by the use of Rutherford's α -scattering experiment. There are many experimental methods for measuring nuclear size. In this section we shall study about the experimental method of measuring the nuclear radius which was first employed by R. Hofstadter and his colleagues by the important bearing that nuclear size has our understanding about the nuclear force.

This method is based on measurement of nuclear charge distribution and it assumes that charge distribution and mass distribution within nuclei are essentially the same. By analyzing scattering of high-energy electrons from various nuclei we have obtained the charge distribution in nuclei.

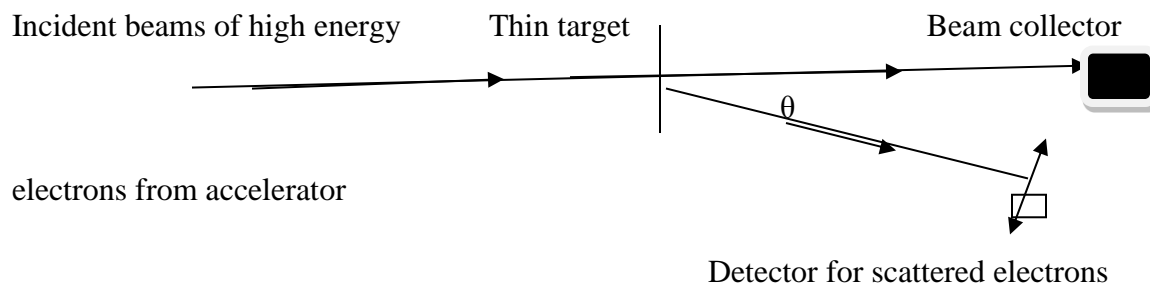


Fig.06: Experimental Setup for nuclear radius measurement

In this experiment electrons are chosen as bombarding particles and its interaction within the nucleus is well known electro-magnetic interaction. As shown in the fig.06 the elastically scattered electrons are observed by a detector as a function of angles.

It is easy to see that the de Broglie wavelength of 200 MeV electrons is about 10^{-14} m (by using the relation $\lambda = h/p$) which is the size of the nucleus. Thus, to get information about

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nuclear charge distribution, the incident electron beam energy has to be the order of 200 MeV. The assumption that the nuclear charge is uniformly spread over a spherical volume and not a point charge leads one to expect diffraction effects.

Portions of electron waves incident on different parts of the nucleus will be scattered in a particular direction, with phase differences resulting in constructor for destructive interference at some angle. Fig.07 is taken from the work of Hofstadter and his collaborators and shows the angular distribution of elastically scattered 200 MeV electrons from nuclei assumed to have spherical uniform charge distribution (having uniform density up to radius r as shown in fig.08. The actual nuclear charge distribution is not exactly given by a step function, but is shown by dotted line. One can clearly see in fig.07 that experiments do show diffraction maxima and minima as expected from a uniform charge distribution. For comparison, the expected result (which is not obtained experimentally) from a point charge assumption is also shown in fig.07. For theoretical calculations Hofstadter and his colleagues assumed a charge density of this form

$$\rho(r) = \frac{\rho_0}{1+e^{(r-R)/b}} \quad \dots\dots\dots(10)$$

Where ρ_0 stands for density at the nuclear center, R is the radius at which ρ falls to $\frac{\rho_0}{2}$ and b measures how rapidly ρ falls to zero at the nuclear surface.

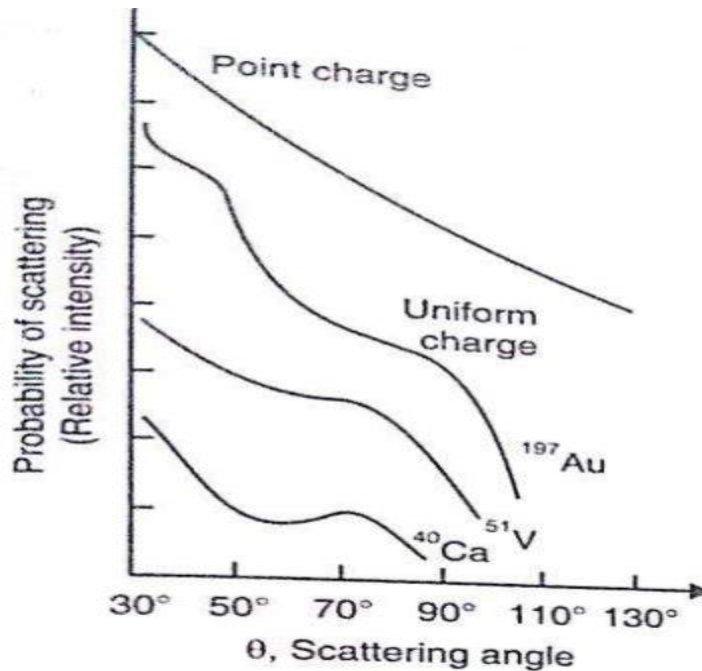


Fig.07: Shows experimental angular distribution of scattered electrons from three types of nuclei

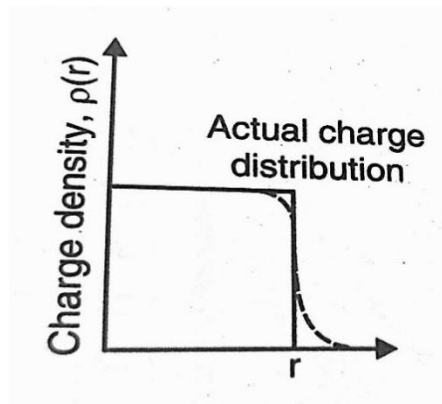


Fig. 08: step function shows here the uniform charge distribution

It is clear from the distribution relation given by equation 4 possesses a property that for $R \gg b$, ρ is close to ρ_0 , until $(R-r)$ is a few times b , at which point ρ decreases to very small values in a distance determined by b , but not by R . If we put the following values, then this charge distribution agrees with the experimental data involving many nuclei

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$$\rho_0 \sim 1.65 \times 10^{44} \frac{\text{nucleons}}{\text{m}^3}, \quad a \sim 0.55 \text{ fermi}$$

$$\text{and } R \sim 1.07A^{1/3} \text{ fermi} \quad \dots\dots\dots(11)$$

Where 1 Fermi = 1 F = 10^{-15} m.

If we vary the values of R and b, then we obtain the charge distribution which represents the observed angular distribution shown in Fig. 07.

The fig. 09 shown below represents that the charge density is essentially the same for almost all nuclei(except lighter nuclei H and He) decreasing slowly for the increasing values of A. There exists a general relationship for the nuclear radius, which has been experimentally verified, as given by equation 11, i.e.

$$R = R_0A^{1/3}$$

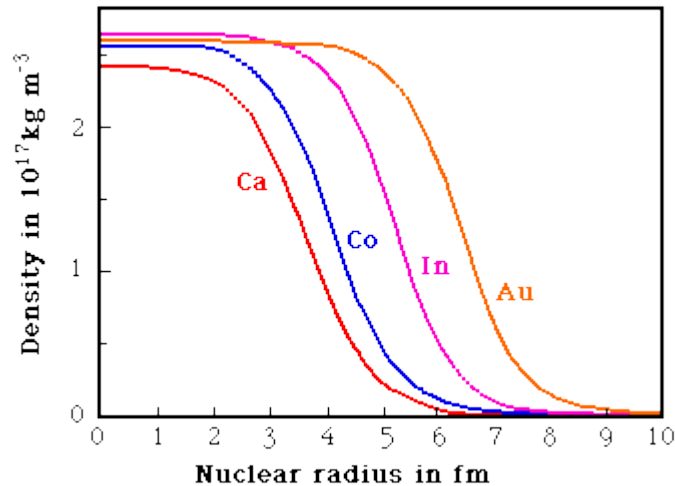


Fig. 09: Nuclear charge distribution found by using high-energy electrons as probes in some nuclei

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Other methods are also there for obtaining the nuclear radius. An important method probes the extent of the nuclear force rather than the nuclear charge. We have the same general relationship

$$R = R_0 A^{1/3}.$$

Example : 01 : Determine the radii of O^{16} and Pb^{206} nucleus. (given that $R_0 = 1.4 \text{ fm}$)

Solution : Using the relation $R = R_0 A^{1/3}$ and substituting the values of A in the following formula, we have

$$R(O^{16}) = 1.4 \times (16)^{1/3} = 3.33 \text{ fm.}$$

$$R(O^{206}) = 1.4 \times (206)^{1/3} = 3.29 \text{ fm.}$$

Example : 01 : Determine the stable nucleus that has a radius one third that of Os^{189} .

Solution : By Using the relation $R = R_0 A^{1/3}$

$$\frac{1}{3} = \frac{R}{R_{Os}} = \left(\frac{A}{A_{Os}} \right)^{1/3} = \left(\frac{A}{189} \right)^{1/3}$$

$$A = \frac{189}{27} = 7$$

The element with the mass number ($A=7$) is Lithium (Li^7).

1.6 ANGULAR MOMENTUM

The angular momentum of an isolated system is known to be conserved according to elementary quantum mechanics. Since the nucleus is an isolated system, its angular momentum is a constant quantity. Most commonly, the total nuclear angular momentum J is denoted by symbol I corresponding to nuclear spin term.

So, the total angular momentum J of a nucleon is the vector sum of its orbital and spin angular momenta values

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$$J = L + S \quad \dots\dots\dots(13)$$

where L is orbital angular momentum and S is spin angular momentum.

The magnitude of the total angular momentum of a nucleon is given by

$$J = \sqrt{j(j+1)} \hbar \quad \dots\dots\dots(14)$$

J can take on the half integral values, 1/2, 3/2, 5/2,.....

The magnitude of the spin angular momentum is given by

$$S = \sqrt{s(s+1)} \hbar \quad \dots\dots\dots(15)$$

Where $S = \frac{1}{2}$ is the spin angular momentum quantum number and the magnitude of the orbital angular momentum is given by

$$L = \sqrt{l(l+1)} \hbar \quad \dots\dots\dots(16)$$

L can take up integral values 0,1,2,3,

Quantum mechanical considerations show that the total orbital and spin angular moments of the nucleus is given by

$$\begin{aligned} P_I^2 &= I(I+1)\hbar^2 \\ P_L^2 &= L(L+1)\hbar^2 \quad \dots\dots\dots(17) \\ P_S^2 &= S(S+1)\hbar^2 \end{aligned}$$

It is determined during measurement which part of the angular momentum is larger along the direction of the applied electric or magnetic field. These have the magnitudes I, L, and S, respectively, for the three examples indicated above.

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The nuclear spin is always zero ($I=0$) for even Z and even N nuclei, according to measurements of the ground state spin of nuclei. This demonstrates that the nucleons inside the nucleus have a tendency to pair off into nucleon-like pairs when their angular momenta are equal and oppositely aligned.

The measured values of the ground state spins of the nuclei are small integers or half odd integers; the highest measured value is $9/2$, which is small when compared to the sum of the absolute values of I_i and S_i of all the individual nucleons contained in the nucleus. This is an important point to keep in mind. This is consistent with what was said earlier about pair creation inside the nuclei. The majority of nucleons of either kind appear to assemble into pairs of zero spin and orbital angular momentum protons and neutrons, giving the core itself zero total angular momentum. The nuclear spin, which is thus determined by the few remaining nucleons outside the core is therefore a small number, integral or half odd integral.

1.7 NUCLEAR STATISTICS

Nuclear statistics can be either Bose-Einstein statistics or Fermi-Dirac statistics, provide a description of a system with many particles, such as the nucleus, in terms of quantum mechanics. Thus, it is possible to divide all fundamental particles into two types

- **Bose-Einstein Statistics:** The wave function of a system of two identical subatomic particles is either symmetric or anti-symmetric in the exchange of the two particles' coordinates in the case of a subatomic particle subject to the laws of quantum mechanics. If the sign of the wave function is not altered by such an interchange, we have a symmetric wave function, and for that Bose-Einstein Statistics holds. Bosons, which are defined as particles with integral spin or zero spin, are subject to Bose-Einstein statistics. Photon, meson, and deuteron are what they are. Even mass number nuclei are subject to Bose-Einstein statistics.
- **Fermi-Dirac Statistics:** But when the exchange results in a change in the wave function's sign, we get an asymmetric wave function, and the associated statistics are

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known as Fermi-Dirac statistics. Fermions are all particles with half integral spin, which all correspond to Fermi-Dirac statistics. Protons, neutrons, and electrons also comes into this category. All odd mass number (A) nuclei correspond to F.D. statistics. No two fermions can exist in the same quantum state because all fermions obey the Pauli exclusion principle. On the other hand, Boson particles do not follow to the exclusion principle and a number of Bosons can exist in the same quantum state.

1.8 PARITY AND SYMMETRY

Another very important property that particles possess is Parity. The particle's parity is a characteristic of the wave function representing its quantum mechanical state. Positive parity refers to a wave function that represents a single particle that does not undergo a sign change upon reflection through the origin, whereas negative parity refers to a wave function that undergoes a sign change. So we have

$$\Psi(x, y, z) = \Psi(-x, -y, -z) = \Psi(x, y, z), \quad \text{for positive parity(18)}$$

$$\Psi(x, y, z) = \Psi(-x, -y, -z) = -\Psi(x, y, z), \quad \text{for negative parity(19)}$$

A wave function that describes multiple particles can be expressed as the sum of the wave functions of each individual particle or as a composite of those wave functions.

Since the parities of the individual particle wave functions are a product, the parity of the entire system is evidently determined by this. The mass density and charge density of nuclei are symmetric no matter what the parity is since they are always equal.

Symmetry is an important concept in atomic and nuclear physics. Schrodinger's equation for two identical, non-interacting particles travelling in the same potential has a straightforward solution that is

$$\Psi_{AB} = \Psi_A(1)\Psi_B(2) \quad \text{.....(20)}$$

where Ψ_A and Ψ_B stand for the positions of particles 1 and 2, and are two solutions to the same one-body wave equation. However, the wave function Ψ_{AB} is not an acceptable function for two

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identical particles because since it assumes that we can label the particles and distinguish between those in state A and those in state B. This difficulty can be avoided by writing the wave function in either one or the other of the two following forms, both of which are solutions of Schrodinger's equation if Eqn. 20 is a solution

$$\Psi_S = \frac{1}{\sqrt{2}} [\Psi_A(1)\Psi_B(2) + \Psi_A(2)\Psi_B(1)] \quad \text{.....(21)}$$

$$\Psi_A = \frac{1}{\sqrt{2}} [\Psi_A(1)\Psi_B(2) - \Psi_A(2)\Psi_B(1)] \quad \text{.....(22)}$$

When labels 1 and 2 are switched in both equations, the probability density is symmetric. As a result, both are valid solutions to the two-body problem. Because 1 and 2 can be switched around, Ψ_S does not change and it remains symmetric, while Ψ_A is anti-symmetric and sign changes.

1.9 MAGNETIC DIPOLE MOMENT

Now in this unit we shall study about the magnetic dipole moment μ which is associated with a current loop of area A and having a current I

$$\mu = IA \quad \text{.....(23)}$$

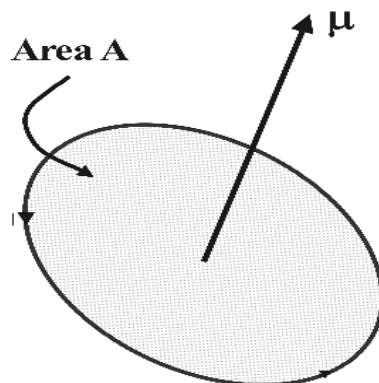


Fig. 10: Magnetic dipole with circular loop of area A

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A spinless particle revolving with with electric charge e generates a magnetic dipole moment which is equal to the number of revolutions per second times the charge times the area enclosed by the orbit. Now for this circular loop with area $A(=\pi r^2)$, we get the value of magnetic dipole moment by substituting the values of I and A

$$\mu = \left(\frac{ev}{2\pi r} \right) \pi r^2 = \frac{evr}{2} \dots\dots\dots(24)$$

where v is the velocity of the particle and r is the radius of the orbit. The orbital angular momentum of this particle is mvr . The ratio of the magnetic dipole moment to the angular momentum is called the gyromagnetic ratio which is given by

$$g = \frac{\mu}{p_1} = \frac{e}{2m} \dots\dots\dots(25)$$

Naturally, this relationship will hold for the two vectors' components as well as for p , in any direction, such as the direction of a magnetic field. We know from quantum physics that the mechanical moment p 's component along a specified direction (the z -direction) is $m h$, which can be either a positive or negative integer or zero. Consequently, we obtain for the magnetic dipole moment's z -component

$$\mu_z = \left(\frac{e\hbar}{2m} \right) m_1 \dots\dots\dots(26)$$

This expression yields the correct result for the z -component of the magnetic dipole moment that is due to orbital motion of electrons in an atom. It is believed to give the correct result also for the orbital motion of protons in a nucleus. The mass in the denominator of Eqn. 26 should then be the proton mass m_p . In contrary to magnetic dipole moments, we can introduce the nuclear magneton defined by

$$\mu_N = \frac{e\hbar}{2m_p} = 5.0505 \times 10^{-27} \text{ Jm}^2\text{wb}^{-1} \dots\dots\dots(27)$$

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μ_N shows its analogy with the Bohr magneton which is given by the formula $\frac{e\hbar}{2m_e}$ which is the unit of atomic magnetic moment. μ_N is much smaller than μ_B , being only 1/1836 part of the later.

The measured values of the magnetic moments of the electron, proton and the neutron are given below

$$\begin{aligned}\mu_e &= -1.001145358\mu_B \\ \mu_p &= 2.7927\mu_N \\ \mu_n &= -1.9131\mu_N \dots\dots\dots(28)\end{aligned}$$

In the above equation 28 negative sign results from because of the direction of the angular momentum vector and the magnetic dipole moment vector are opposite to each other. The total magnetic dipole moment for a nucleus is the sum of the moments associated with the spins of the protons and neutrons and the moments associated with the orbital motion of the proton. Since the z-component of the dipole moment is the only component which can be observed through its interaction with a magnetic field.

1.10 ELECTRIC QUADRUPOLE MOMENT

The nuclear electric quadrupole moment is a parameter which describes the effective shape of the ellipsoid of nuclear charge distribution. A non-zero quadrupole moment Q indicates that the charge distribution is not spherically symmetric. By convention, the value of Q is taken to be positive if the ellipsoid is prolate and negative if it is oblate.

First we have the value of electric potential V of any distribution of electric charges at a distance R in the Z-direction given by equation 29

$$V = \frac{1}{4\pi\epsilon_0} \left[\frac{1}{R} \int \rho dV + \frac{1}{R^2} \int \rho z dV + \frac{1}{R^3} \int \rho (3z^2 - r^2) dV + \dots\dots \right] \dots\dots\dots(29)$$

where, ρ is the charge density. The integration is performed over the region containing the electric charge. In the above expression we have a rapidly convergent series and as the value of R increases, only the first two or three terms become important in this series. The first term corresponds to the net charge and for larger R this is the only important term. The integral in the second term is called the dipole moment and the third term is called the quadrupole moment. It turns out that a nucleus must have a zero electric dipole moment and so the lowest order contribution to V due to a point charge arises from the quadrupole moment. Two examples of non-zero electric quadrupole moments are shown in Fig. 11

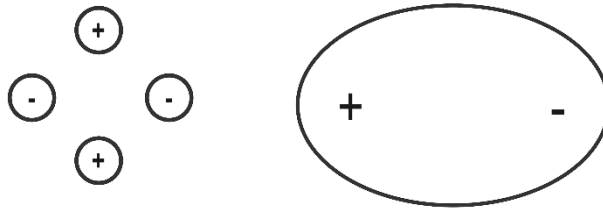


Fig. 11. Electric Quadrupole Moment

The four-charge (quadrupole) system in the left part of the Fig. 11 has net charge and dipole moment zero and so the entire electric field is produced by the electric quadrupole moment. An ellipsoidal charge distribution shown on the right leads to a highly deformed nucleus; it has a quadrupole moment; however, its dipole moment is zero. The charge density at point $r(x, y, z)$ is given by $Ze\rho(r)$ where Ze is nuclear charge.

The quadrupole moment Q is given by

$$\begin{aligned} Q &= Z \int (3z^2 - r^2) \rho(r) d^3r \\ &= Z \int r^2 (3\cos^2 \theta - 1) \rho(r) d^3r \end{aligned} \quad \dots\dots\dots(30)$$

For a spherically symmetric charge distribution $\rho(r)$, the quadrupole moment vanishes, for a prolate nucleus, the charge is concentrated along the z-axis and so Q is positive. Q is negative for an oblate nucleus as shown in Fig. 12

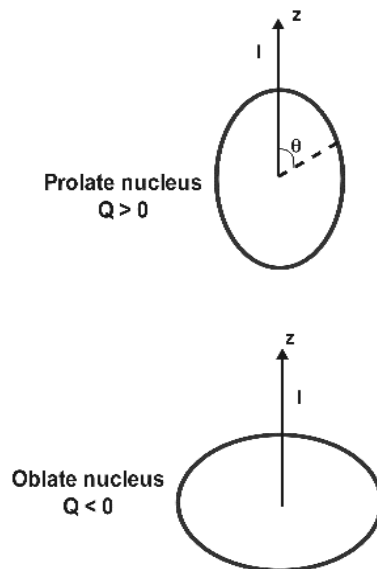


Fig. 12 Oblate and Prolate shapes

From Eq. 30, it is clear that Q has same dimensions as that of area and is given in terms of m^2 or barns (10^{-24} cm^2) or F^2 (fermi²).

In a quantum-mechanical definition of the quadrupole moment, the charge density ρ replaced by the probability density and the expression is summed over all the protons. In principle, these calculations are simple and directly yield the quadrupole moment referred to the z-axis. In classical or semiclassical calculations, one should consider the fact that the nuclear symmetry axis is not, in general, the space-fixed z-axis to which measured quadrupole moments are referred. Let us assume that the J-axis can be regarded as a symmetry axis, or at least that the time average of the charge distribution has a rotational symmetry about the J-axis. It can be

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defined as a quadrupole moment Q given by Eqn.30 with the z-direction along the J-vector. The angle θ between this vector and the space fixed z-axis in the state for which $m_J = J$ is given by

$$\cos \theta = \frac{J}{\sqrt{J(J+1)}} = \sqrt{\frac{J}{J+1}} \quad \text{.....(31)}$$

It can be shown by simple classical calculations that the relationship between Q_J and the observed quadrupole moment Q , which relates to the space-fixed z-axis in the state $m_J = J$, is

$$Q = \frac{\left(J - \frac{1}{2}\right)}{(J+1)} Q_J \quad \text{.....(32)}$$

This shows that a nucleus with $J=1/2$ has a zero-quadrupole moment with reference to a space-fixed axis. The same is true for a nucleus with $J=0$ for which there are no constraints on the orientation, and therefore the time-averaged charge distribution is symmetric. An important aspect of quadrupole moment measurement is that it makes it possible to determine nuclear deformation. Knowledge of nuclear deformation is important for studying collective behavior of nuclear particles.

1.11 SUMMARY

In this unit basic properties of Nucleus have been discussed in the simpler manner. In this unit Rutherford's scattering and nuclear size estimation have also been explained to understand the nuclear radius. Learners will also be able to solve the numerical based on nuclear size determination. The relation between mass and binding energy are discussed. The nuclear wave electrical properties like nuclear statistics, parity and symmetry have also been explained in this unit. The concepts of magnetic dipole moment and electrical Quadrupole moment have also been explained for the nucleus with suitable examples.

1.12 GLOSSARY

Nucleus	the central region of an atom where the majority of the mass is concentrated.
Isotopes	atoms of the same element that have different numbers of neutrons but the same number of protons and electrons.
Isobars	are elements that have the same number of nucleons (sum of protons and neutrons).
Mirror Nuclei	atomic nucleus that contains a number of protons and a number of neutrons that are mutually interchanged in comparison with another nucleus.
Scattering	a change in the direction of motion of a particle because of a collision with another particle
Angular Momentum	The property of any rotating object given by moment of inertia times angular velocity.
Parity	property important in the quantum-mechanical description of a physical System
Magnetic dipole moment	the measure of the objects tendency to align with the magnetic field.
Electric Qudrupole moment	a parameter which describes the effective shape of the ellipsoid of nuclear charge distribution

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1.15 TERMINAL QUESTIONS

1. Explain the term nuclear radius.
2. Explain the term nuclear magnetic dipole moment.
3. Define the term statistics and parity for the nucleus.
4. Show that a nucleus has a zero electric dipole moment.

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5. Show that nuclear density is constant for all nuclei.
6. Mention various methods for determining the size of the nucleus and describe any one in detail.
7. Explain the term electric quadrupole moment and find out an expression for quadrupole moment.
8. Estimate the ratios of the major to the minor axes of ${}_{51}\text{Sb}^{123}$. The quadrupole moment is $-1.2b$. Take $R = 1.5A^{1/3}\text{fm}$.
9. Explain in brief the different properties associated with the nucleus.
- 10..Discuss one method for the determination of the size of the nucleus.
- 11.. Explain the relationship between the nuclear mass with nuclear size.

UNIT 2

NUCLEAR STABILITY

Structure of the Unit

2.1 Introduction

2.2 Objectives

2.3 Binding Energy

2.3.1 Mass Defect

2.3.2 Binding Energy per Nucleon

2.3.3 Packing Fraction

2.4 Nuclear Reaction

2.5 Types of Nuclear Reactions

2.6 Mass Balance and Energy in Nuclear Reaction

2.7 Q-Value of Equation

2.8 Solution of the Q-Value of Equation

2.8.1 Exoergic Reactions

2.8.2 Endoergic Reactions

2.9 Glossary

2.10 Summary

2.11 References

2.12 Suggested Readings

2.13 Terminal Questions

2.1 INTRODUCTION

In the previous unit we have obtained the basic information of the nucleus and its properties. Now, in this unit we shall study about the binding energy which is responsible for the stability of the nucleus and we know that lighter nuclei are more stable than heavier nuclei in which the number of neutrons are in excess as compared to the number of protons. In this consequence we will also study about mass defect and packing fraction. Our current understanding of the nuclear structure is mostly based on studies in which a chosen nucleus is blasted with various particles, such as protons, neutrons, and deuterons. When these particles are close enough to the target nuclei to interact, either elastic or inelastic scattering may occur, one or more completely distinct particles may be ejected from the nucleus, or the incident particle may be trapped and generate a gamma ray. After the bombardment, a nuclear reaction is said to have occurred when the mass number and/or atomic number of the target nuclei changes.

2.2 OBJECTIVES

After studying the unit the learners will be able to understand

- Binding energy, mass defect and packing fraction
- Nuclear Reaction and its types
- Q-value and its solution for nuclear reaction
- Endoergic and Exoergic nuclear reactions
- Solve the numerical problems of finding the Q-value

2.3 BINDING ENERGY

2.3.1 Mass Defect

Now in this unit we shall study about mass defect which is a very important aspect while we discuss binding energy responsible for nuclear stability. Mass of an atom is concentrated in the central part called nucleus, which is made up of neutrons and protons. It has been observed that the mass of a nucleus is always less than the sum of the individual masses of the protons and the neutrons, which constitute it. This difference is a measure of the nuclear binding energy which holds the nucleus together, is known as the mass defect Δm .

If $m(Z, N)$ is the mass of the nucleus of an atom consisting of Z protons and N neutrons, the mass defect Δm is given as

$$\Delta m = Zm_p + Nm_n - m(Z, N)$$

Or
$$\Delta m = Zm_p + (A - Z)m_n - m(Z, N) \dots \dots \dots (1)$$

where m_p and m_n are the masses of a proton and a neutron.

It is convenient to talk in terms of the atomic masses, therefore, adding and subtracting the mass of Z atomic electrons on the RHS of the above equation, we get

$$\begin{aligned} \Delta m &= Z(m_p + m_e) + (A - Z)m_n - m(Z, N) - Zm_e \\ \Delta m &= ZM_H + (A - Z)m_n - M(Z, N) \dots \dots \dots (2) \end{aligned}$$

where m_e is the mass of one electron, $M(Z, N)$ is the atomic mass and M_H is the mass of neutral hydrogen atom.

This missing mass may be regarded as the mass, which would be converted into energy, if a particular atom is to be formed from the requisite number of electrons, protons and neutrons.

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This is also the amount of energy required to break up the atom into its constituents. Therefore, mass defect is a measure of the binding energy of an atom/nucleus.

Example 1: Calculate the mass defect of deuterium ${}^2_1\text{H}$, which is an isotope of hydrogen known as heavy hydrogen

Solution: Let us would expect that its mass should be equal to the mass of one neutron in its nucleus. Thus, one atom, i.e.

$$m_n + M_H = 1.008665 + 1.007825u$$

Therefore, the expected mass of deuterium is $2.016490u$.

However, the measured mass of deuterium, is found to be $2.014102u$.

Therefore, for deuterium,

the mass defect (Δm) = $2.016490 - 2.014102u$

$$\Delta m = 0.002388u.$$

2.3.2 Binding Energy per Nucleon

According to the proton-neutron model of the nucleus, it is obvious that the particles that make up the stable nucleus are kept together by potent attraction forces, and that effort is required to break them apart. In other words, the nucleus needs energy to be given in order to be divided into its component parts. We use the well-known Einstein mass-energy connection $E=mc^2$, where E and m are the energy and mass of the particle and c is the speed of light in a vacuum, to investigate what shape this energy can take. Simply said, this shows that energy and mass are different representations of the same entity.. Therefore, we should anticipate that the mass of the nucleus as a whole will be less than the sum of the masses of its constituent parts. In fact, a significant deal of experimentation has led to this observation.

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The difference between the energy of the constituent particles and the energy of the entire nucleus is thus used to determine the binding energy of the nucleus. Take the example of a nucleus ${}_Z M^A$, where the binding energy is given by

$$B = [ZM_p + NM_N - zM^A]c^2 \dots\dots\dots(3)$$

where M_p = Mass of the Proton,

Z = Number of Protons,

M_N = Mass of the Neutron,

N = Number of Neutrons = $(A - Z)$

zM^A = Measured mass of the neutral atom, [also written as $M(Z, A)$]

The above expression is, now-a-days, generally expressed as

$$B = [ZM_H + NM_N - zM^1]c^2$$

where M_H represents the mass of the neutral hydrogen atom. Since there are A nucleons in the nucleus, the binding energy per nucleon is also given as

$$\frac{B_e}{A} = \frac{c^2}{A} [ZM_H + NM_N - zM^1] \dots\dots\dots(4)$$

When binding energy fraction B/A is plotted against A , the curve similar to Fig.1 is obtained.

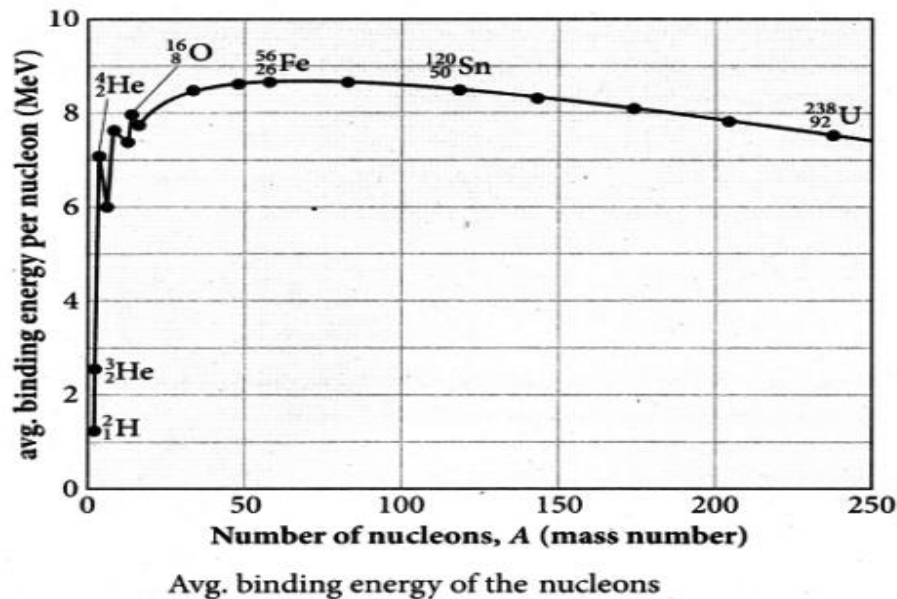
We find from this curve that B/A almost remains constant between $A = 30$ and $A = 100$ and decreases for small and large values of A .

The coulomb repulsion between the protons, which obviously makes the nuclei less and less stable, is what causes the reduction for large A . Because only a small number of other nucleons are attracted to each particular nucleon in light nuclei, their separation distances are greater, which again affects stability. The drop in B/A for small A is a surface effect. It is obvious that

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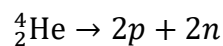
the particles at the surface are less tightly bonded than those inside. stronger ties than those in the interior across. The fraction of constituents at the nuclear surface increases with the size of the nucleus. This effect reads B/A for a low A .



The nuclides with an even number of protons and neutrons have greater B/A values than the nearby odd mass nuclides, according to an analysis of the aforementioned curve. Since there are no remaining unpaired neutrons in such nuclides, the even number of protons can couple off all the nucleons, explaining the even-even nucleon's better stability and natural abundance.. This low mass number nuclides, the stability rule is $N = Z$, H^3 , Li_2BH^2 , N^{14} are examples of odd-odd examples of odd-odd nuclides which are most stable.

Example 1: Masses of helium nucleus, proton and neutron are 4.0026u, 1.007895u and 1.008665u. Find the energy required to knock out nucleons from the helium nucleus.

Solution: We have the equation



Mass of two protons and 2 neutrons is

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$$= 2 \times (1.007895 + 1.008665) \\ = 4.03312u$$

Therefore, mass defect is given as

$$\Delta m = 4.03312 - 4.0026 \\ = 0.03052u$$

And the equivalent energy is

$$= (0.03052u) \times 931.49 \\ = 28.42 \text{ MeV}$$

2.3.3 Packing Fraction

We have seen that atomic masses are not whole numbers. This divergence of the masses of the nuclides from whole number was studied by a number of workers like Aston and is expressed in terms of packing fraction. Packing fraction is defined as

$$f = \frac{\text{Atomic mass} - \text{Mass number}}{\text{Mass number}} \\ = \frac{zM^A - A}{A} \dots \dots \dots (5)$$

where M^A is the actual mass of a nuclide on the physical atomic mass scale and A mass number.

The quantity $zM^A - A$ is known as 'Mass-defect'. If packing fraction is represented as f , then

$$zM^A = A(1 + f) \dots \dots \dots (6)$$

When f is plotted against A , we get the following curve of the shape as shown in fig.2

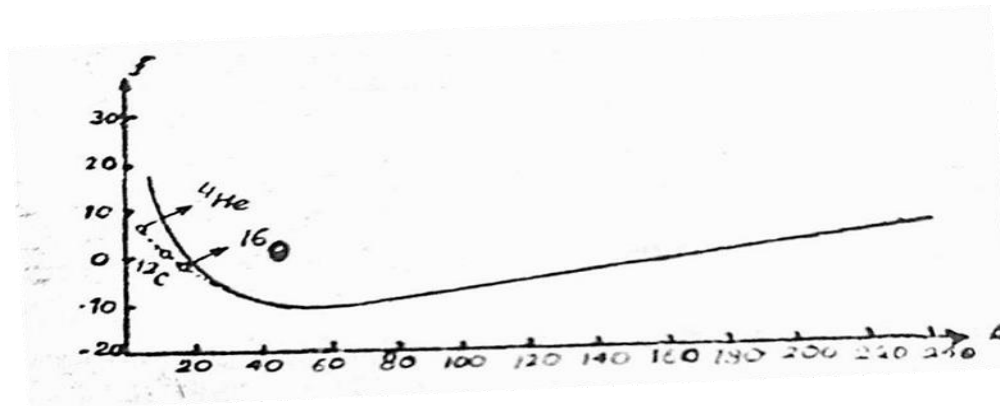


Fig 2: Plot of packing fraction(f) versus mass number(A)

The packing fraction for all the elements fall on this curve except He^4 , C^{14} , and O^{16} . For O^{16} the value of packing fraction is zero. As A increases, the falls of the packing fraction becomes negative, passes through a flat minimum rises gradually becoming positive again at values of A above 180 . A positive packing fraction has an atomic mass M greater than its mass nude while This means that the loss of mass due to binding energy requirements, whera, nucleons combine to form a nucleus, is less than that for oxygen. The packing fractions for light elements like oxygen, carbon, helium which do not lie on the frac. have positive values but much smaller ($\approx 5 \times 10^{-1}$ or less) than other light curve like nitrogen, lithium and hydrogen ($\approx 80 \times 10^{-4}$ for hydrogen) light elements of the former type (O^{16} , C^{12} , He^4) have more stable nuclei's that latter.

Example 1:

The value of f_B can be estimated as follows:

For deuteron (H^2), since $Z=1$, $N=1$,

$$\begin{aligned} B.E(H^2) &= M_H + M_n - M_d \\ &= (1.007825 + 1.008665 - 2.0414102) \times 931.5 \\ &= 2.224 \text{ MeV} \end{aligned}$$

$$f_B(H^2) = \frac{2.224}{2} = 1.112 \text{ MeV per nucleon}$$

For alpha-particle (He^4), $Z=2$, $N=2$,

$$\begin{aligned} B.E(He^4) &= (2 \times 1.007825 + 2 \times 1.008665 - 4.002603) \times 931.5 \\ &= 28.3 \text{ MeV} \end{aligned}$$

$$f_B(He^4) = \frac{28.3}{4} = 7.075 \text{ MeV per nucleon}$$

For (O^{16}), since $Z=8$, $N=8$,

$$\begin{aligned} B.E(O^{16}) &= (8 \times 1.007825 + 8 \times 1.008665 - 15.994915) \times 931.5 \\ &= 127.62 \text{ MeV} \end{aligned}$$

$$f_B = \frac{127.62}{16} = 7.98 \text{ MeV per nucleon}$$

The binding energy fractions of the different nuclei represent the relative strengths of their binding. Thus H^2 is very weakly bound, compared to He^4 or O^{16} .

2.4 NUCLEAR REACTION

Typically, an equation representing a nuclear reaction may be written as

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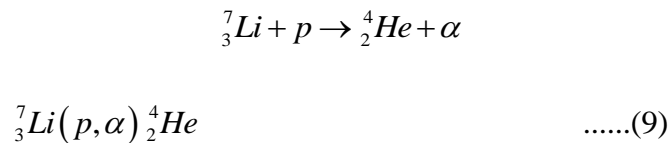
$$x + X = Y + y \quad \text{.....(7)}$$

In words this would read like: when an incident projectile x hits the target nucleus X , a nuclear reaction takes place and as a result there is a new nucleus Y and an outgoing particle. The above reaction can also be written in short form as $X(x, y)Y$.

In 1919, Lord Rutherford discovered that protons are created when nitrogen is attacked with polonium particles, and that these protons are capable of piercing 28 cm of air. Rutherford deserves credit for piercing the "inaccessible armour" of typical non-radioactive nuclei and causing a transmutation because this was the first nuclear reaction to be set off in a laboratory. You might think of the nuclear reaction as



In 1930 Cockcroft and Walton used artificially accelerated protons and produced the following nuclear reaction.



Thousands of nuclear reactions have previously been examined by various laboratories throughout the world, and the number is constantly increasing. In this chapter, we'll look at the scenario when the target nucleus is at rest before to the collision and all of the particles can be considered non-relativistically. Finding out when a specific nuclear reaction becomes energetically feasible is what we're interested in learning. By using the equations of momentum and energy conservation, we may determine this.

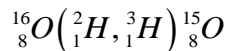
2.5 TYPES OF NUCLEAR REACTION

Nuclear reactions are classified on the basis of the projectile used, the particle detected and the residual nucleus.

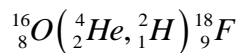
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- i. *Scattering*: In the scattering reaction, the projectile and the detected (outgoing) particle are the same. The scattering is elastic when the residual nucleus is left in the ground state. When the residual nucleus is left in an excited state, the scattering is called inelastic.
- ii. *Pickup reactions*: When the projectile gains nucleons from the target, the nuclear reaction is referred to as pickup reaction, e.g.



- iii. *Stripping reactions*: In this type, the projectile loses nucleons to the target nucleus, e.g.

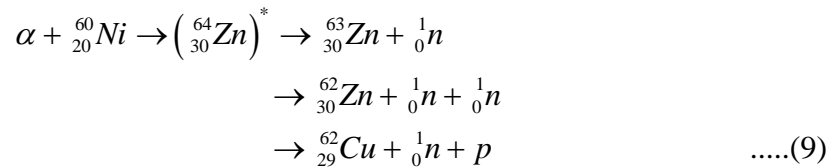
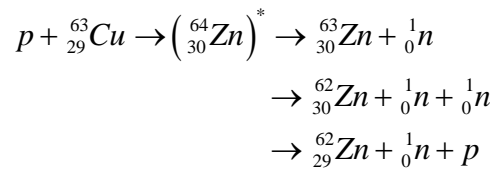


In a pickup or stripping reaction it is assumed that the nucleon involved in the process enters or leaves (the shell of) the target nucleus without disturbing the other nucleons. These reactions are therefore referred to as direct reactions. In contrast, we have the following type compound nuclear reactions. Of

- iv. *Compound nuclear reactions*: Here the projectile and target form a compound nucleus which has a typical life span of $\sim 10^{-16}$ sec. When this time is compared with a typical nuclear time, i.e., the time taken by the projectile to traverse the target nucleus ($\sim 10^{-22}$ sec.) as in the case of direct reactions, we can conclude that the decay compound nucleus does not depend on the way it was formed. of A

This situation is often described as: the compound nucleus does not "remember" how it was formed. Usually, the same compound nucleus is given rise to by a number of nuclear reactions. This compound nucleus can decay in a number of ways or channels. This is illustrated by taking for example, the compound nucleus ${}^{64}_{30}\text{Zn}$ formed in an excited state

$({}^{64}_{30}\text{Zn})^*$ by two different methods and then decays as given below:



These reactions were experimentally studied by S.N. Ghoshal [Phys. Rev. 80939 (1950)]

Figure 4 depicts how a compound nucleus in an excited state can have different modes of decay, giving rise to different residual nuclei and detected particles.

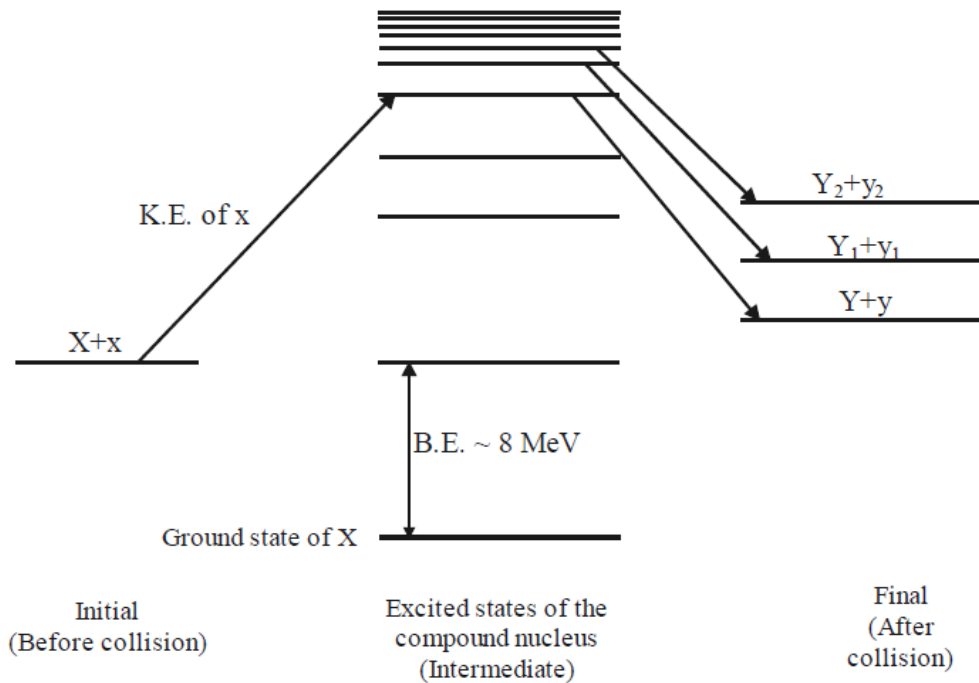


Fig 4: Modes of decay in a compound nucleus in an excited state

The compound nucleus model has been successfully applied for targets of $A > 10$ and projectile energies up to about 15 MeV. For projectile energies between 15 MeV and 50 MeV the direct reaction model and the optical model are found to be successful. In this unit, our main emphasis is on understanding energies of nuclear reactions.

2.6 THE BALANCE OF MASS AND ENERGY IN NUCLEAR REACTIONS

Consider the nuclear reaction,

$$x + X = Y + y$$

Since the total mass and energy is conserved, we have

$$(E_x + m_x c^2) + M_x c^2 = (E_Y + M_Y c^2) + (E_y + m_y c^2) \quad \text{.....(10)}$$

Where,

E_x = kinetic energy of the projectile

$m_x c^2$ = rest energy of the projectile

And similarly, E_Y and $M_Y c^2$, E_y , $m_y c^2$ and $M_x c^2$

The target nucleus X is assumed to be at rest.

The Q value is expressed as,

$$Q = E_Y + E_y - E_x \quad \text{.....(11)}$$

i.e., it is the change in the total kinetic energy.

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This change in the total kinetic energy in a nuclear reaction is clearly the nuclear disintegration energy.

From Eq. (10) and Eq. (11) becomes,

$$Q = E_Y + E_y - E_x \\ = \left[(M_X + m_x) - (M_Y + m_y) \right] c^2 \quad \text{.....(12)}$$

Thus, we see that Q is also change in the total rest mass. We have an exoergic nuclear reaction if Q is positive. The nuclear reaction is an endoergic reaction if the Q value is negative.

From Eq. (12) it is clear that to determine the Q value or nuclear disintegration energy for a nuclear reaction, we must know the kinetic energy of the particles.

E_y , the kinetic energy of the residual nucleus, is usually small and hard to measure.

In the following section, we will see how E_y can be eliminated and how the Q equation can be set up.

2.7 THE Q EQUATION

The analytical relationship between the kinetic energy of the projectile and outgoing particle and the nuclear disintegration energy Q is called as the Q equation.

To understand the dynamics of two body nuclear reactions in laboratory coordinate system, refer to Fig.5

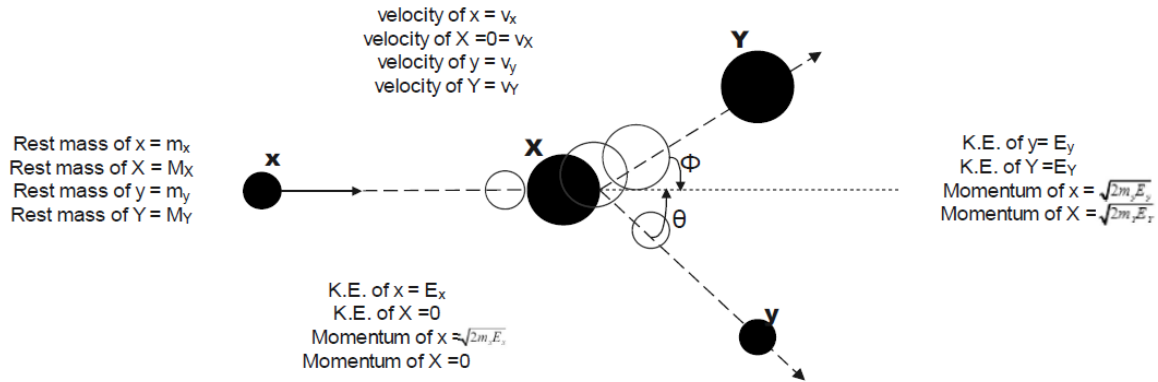


Fig. 5: Two body nuclear reaction in lab system

Conservation of mass-energy gives

$$Q = E_Y + E_y - E_x$$

As remarked before, E_y is small and hard to measure and is therefore eliminated.

Conservation of linear momentum along the direction of projectile x gives,

$$\sqrt{2m_x E_x} = \sqrt{2m_y E_y} \cos \theta + \sqrt{2M_Y E_Y} \cos \varphi \quad \dots\dots(13)$$

Angles θ and φ are coplanar as linear momentum perpendicular to the θ plane is equal to zero.

Therefore, conservation of linear momentum normal to projectile direction (in the θ, φ plane)

gives,

$$0 = \sqrt{2M_Y E_Y} \sin \varphi - \sqrt{2m_y E_y} \sin \theta \quad \dots\dots\dots(14)$$

From these equations E_y and φ are eliminated. Squaring and adding Eqs. (13) and (14),

$$M_Y E_Y = m_x E_x + m_y E_y - 2\sqrt{m_x m_y E_x E_y} \cos \theta$$

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But

$$E_Y = Q - E_y + E_x$$

$$\begin{aligned} Q &= E_y - E_x + \frac{m_x}{M_Y} E_x + \frac{m_y}{M_Y} E_y - \frac{2\sqrt{m_x m_y E_x E_y}}{M_Y} \cos \theta \\ &= E_y \left(1 + \frac{m_y}{M_Y} \right) - E_x \left(1 - \frac{m_x}{M_Y} \right) - \frac{2\sqrt{m_x m_y E_x E_y}}{M_Y} \cos \theta \end{aligned} \quad \text{.....(15)}$$

This is the standard form of the Q equation.

It is interesting to note that one can also obtain Eq.15 by solving the momentum triangle and substituting for E_y . This is indicated below:

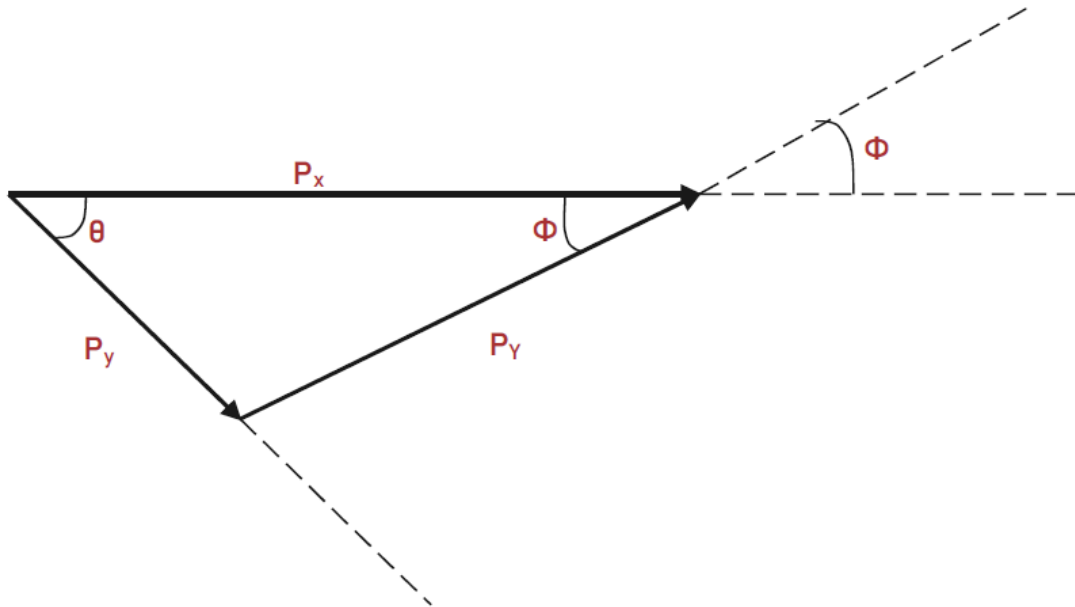


Fig 6: Momentum triangle, using the notation of fig. 05

We see that,

$$P_Y^2 = p_x^2 + p_y^2 - 2p_x p_y \cos \theta \quad \text{.....(16)}$$

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$$2M_Y E_Y = 2m_x E_x + 2m_y E_y - 2.2\sqrt{m_x m_y E_x E_y} \cos \theta$$

giving,

$$E_Y = \frac{m_x}{M_Y} E_x + \frac{m_y}{M_Y} E_y - \frac{2\sqrt{m_x m_y E_x E_y}}{M_Y} \cos \theta \quad \text{.....(17)}$$

From Eq. (6)

$$E_Y = Q - E_y - E_x$$

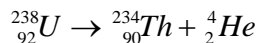
Substituting this in Eq (12) we get,

$$Q = E_y \left(1 + \frac{m_y}{M_Y} \right) - E_x \left(1 - \frac{m_x}{M_Y} \right) - \frac{2\sqrt{m_x m_y E_x E_y}}{M_Y} \cos \theta$$

Which is the Q Equation (10).

The kinetic energy E_x , E_y and θ all are measured in laboratory system. Since the Q equation is based on mass-energy conservation in a nuclear reaction, it holds for all types of reactions. The exact masses can be replaced by the corresponding mass numbers, in many applications without significant error. For very accurate calculations, the neutral atomic masses are used. Let us see how we can use the isotopic masses (neutral atomic masses) to obtain Q value in alpha and beta decay reactions. For convenience, let the element symbol represent the isotopic mass. We recall that $Q = (m_x + M_x) - (m_y + M_y)$ where the masses are the nuclear masses and Q is in mass units.

i. Consider the reaction



$$Q = (U - 92e) - [(Th - 90e) + (He - 2e)]$$

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Where e stands for electronic mass.

i.e.,

$$Q = (U - 92e) - [(Th + He - 92e)]$$

$$= [U - (Th + He)]$$

∴ Isotopic masses can be used to evaluate Q.

ii. Consider the reaction,

$${}^{14}_6\text{C} \rightarrow {}^{14}_7\text{N} + {}^0_{-1}\text{e} + {}^0_0\bar{\nu}$$

$$Q = (C - 6e) - [(N - 7e) + e]$$

$$= (C - 6e) - (N - 6e)$$

$$= C - N$$

iii. Consider the reaction,

$${}^{63}_{30}\text{Zn} \rightarrow {}^{63}_{29}\text{Cu} + {}^0_{+1}\text{e} + {}^0_0\nu$$

$$Q = (Zn - 30e) - [(Cu - 29e) + e]$$

$$= (Zn - 30e) - (Cu - 30e + 2e)$$

$$= Zn - (Cu + 2e)$$

Thus, we see that the isotopic masses can be used to compute Q value in a positron decay reaction, provided two electron masses are included with that of the product particle.

2.8 SOLUTION OF THE Q EQUATION

We are often interested in E_y , the energy of the detected particle and its variation with E_x , the energy of the bombarding projectile, for a fixed Q. For this purpose, the Q equation (Eq. 10) can be regarded as a quadratic in $\sqrt{E_y}$. Then its general solution can be conveniently put in the form,

$$\sqrt{E_y} = v \pm \sqrt{v^2 + w} \quad \text{.....(18)}$$

Where,

$$v = \frac{\sqrt{m_x m_y E_x}}{m_y + M_Y} \cos \theta \quad \text{.....(19)}$$

And,

$$w = \frac{M_Y Q + E_x (M_Y - m_x)}{m_y + M_Y} \quad \text{.....(20)}$$

When $\sqrt{E_y}$ is real and positive the reaction is energetically possible.

Emission of m_y becomes energetically impossible when Q value is negative, $(M_Y - m_x)$ is negative (i.e., a heavy projectile) and a large angle of observation θ , making $\cos \theta$ negative.

Equation (13), for a various energy E_x of the following particle, tells us about the types of nuclear reactions which can occur.

Let us now consider energies of exoergic and endoergic reactions.

2.8.1 Exoergic Reactions: Here $Q > 0$.

- a. Very low energy projectiles: e.g., thermal neutrons.

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We can take $E_x \cong 0$

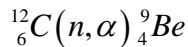
$$v = 0 \text{ and } w = \frac{M_Y Q}{m_y + M_Y}$$

And so,

$$E_y = Q \frac{M_Y}{m_y + M_Y} \quad \text{.....(21)}$$

i.e., E_y is same for all angles θ

An example of such reaction is,



b. *Finite energy projectiles*: For most of the reactions, $m_x \ll M_Y$ and so, $w > 0$ for all E_x .

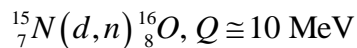
Thus, it can be seen from Eq. (13) that E_y is single valued for all E_x , and is given by,

$$\sqrt{E_y} = v + \sqrt{v^2 + w} \quad \text{.....(22)}$$

An example of such a reaction is,



c. *Double values of E_y* : In a number of reactions E_y is no single valued. For example, in the reaction,



If the observed particle m_y is chosen to be the residual nucleus ${}^{16}\text{O}$, then, $m_y \sim$ mass number, $A_y = 16$ and $M_Y - A_y = 16$ (for neutron).

Equation (15) gives,

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$$w = \frac{Q - E_x}{17}$$

Which is negative for all deuteron bombarding energies greater than,

$$E_x = Q = 10 \text{ MeV}$$

Thus, for $E_x > 10 \text{ MeV}$, we have two real positive values of $\sqrt{E_y}$ at $\theta = 0$.

That is, in the forward direction, there are two monoenergetic groups of ^{16}O nuclei. Physically this implies that in the center of mass system, these two groups are projected forward and backward.

2.8.2 Endoergic Reactions: $Q < 0$.

For every nuclear reaction with a positive Q value, the inverse reaction has a negative Q value of the same magnitude. Thus, for example, $^{16}_8\text{O}(n, d)^{15}_7\text{N}$ has $Q = -10 \text{ MeV}$, or $^{13}_6\text{C}(p, \alpha)^{10}_5\text{B}$ has $Q = -4 \text{ MeV}$

a) *Very low energy projectiles:*

$$E_x \cong 0, Q < 0$$

$$v^2 + w^2 < 0 \text{ and so...}$$

$\sqrt{E_y}$ is imaginary, implying that no reaction occurs. (E_x is insufficient to start the reaction.)

b) *Threshold energy* $(E_x)_{\text{thresh}}$: In an endoergic reaction, the energy $-Q$ is needed to excite the compound nucleus sufficiently so that it will break up. The bombarding particle must supply this energy in the form of kinetic energy. However, not all of that kinetic energy is available for excitation because some is used to give momentum to the compound nucleus which, is distributed among the products of the reaction. Thus, the bombarding particle (projectile) must supply some energy in addition energy in addition to $-Q$ so that the energy $-Q$ becomes available for the excitation of the compound nucleus.

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The *smallest value* of projectile energy (bombarding energy) at which an endoergic reaction can take place is called the threshold energy for that reaction.

From Eq. 13, it can be seen that the reaction first becomes possible when E_x is large enough to make,

$$v^2 + w^2 = 0$$

E_x has its minimum possible value at $\theta = 0$, which is the $(E_x)_{\text{thresh}}$.

This condition gives:

$$\frac{m_x m_y E_x}{(m_y + M_Y)^2} + \frac{M_Y Q + E_x (M_Y - m_x)}{m_y + M_Y} = 0$$

Which gives,

$$(E_x)_{\text{thresh}} = -Q \left(\frac{m_x + M_X}{M_X} \right) \quad \dots\dots(23)$$

In obtaining Eq. (18), we have made use of the fact that the value of $\frac{Q}{c^2}$ is much less than the masses of particles in a nuclear reaction, and so

$$m_y \cong m_x + M_X - M_Y \quad \dots\dots(24)$$

Equation (18) can be also obtained in a straightforward way by considering M_c and V_c as mass and velocity of the compound nucleus. Then we have,

$$m_x v_x = M_c V_c$$

Or

$$V_c = \frac{m_x}{M_c} v_x$$

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Now, the part of kinetic energy of the projectile needed for the excitation of the compound nucleus is,

$$\begin{aligned} Q &= \frac{1}{2} m_x v_x^2 - \frac{1}{2} M_c V_c^2 \\ &= \frac{1}{2} m_x v_x^2 - \frac{1}{2} M_c \left(\frac{m_x}{M_c} \right)^2 v_x^2 \\ &= \frac{1}{2} m_x v_x^2 \left(1 - \frac{m_x}{M_c} \right) \end{aligned}$$

But

$$M_c = m_x + M_X$$

Therefore,

$$-Q = \frac{1}{2} m_x v_x^2 \left(\frac{M_X}{m_x + M_X} \right)$$

The threshold energy is then,

$$(E_x)_{\text{thresh}} = \frac{1}{2} m_x v_x^2 = -Q \left(\frac{m_x + M_X}{M_X} \right)$$

Which is Eq.23.

2.9 GLOSSARY

Kinetic Energy - the energy possessed by a body due to its motion.

Magnetic Moment - a vector quantity that provides a measure of the torque exerted on a magnetic system (as a bar magnet or dipole) when placed in a magnetic field.

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Mass Defect - the difference between the mass of an atomic nucleus and the sum of the masses of its constituent particles.

MeV (Mega Electron Volt) – the energy possessed by a particle with one electronic charge in passing through a potential difference of one million volts.

Nuclear Binding Energy – the minimum energy required to separate an atomic nucleus into its constituent particles i.e., protons and neutrons.

Nuclear magneton - a unit of magnetic moment, used to measure proton spin and approximately equal to 1/1836 times Bohr magneton.

Q value - the Q value for a reaction is the amount of energy absorbed or released during the nuclear reaction.

Exoergic(exothermic)reaction - a nuclear reaction that releases energy and have positive Q-value.

Endoergic (or endothermic)reaction – a nuclear reaction that requires an input of energy to take place with negative Q-value.

2.10 SUMMARY

In this unit, we have explained various important terms like binding energy, mass defect, binding energy per nucleon and packing fraction with suitable examples. In this consequence we have defined the nuclear reaction and also its various types has also been explained. In this unit we have also learnt that to determine the Q value or nuclear disintegration energy for a nuclear reaction, we must know the kinetic energy of the particles. We have an exoergic nuclear reaction if Q is positive. The nuclear reaction is an endoergic reaction if the Q value is negative. The analytical relationship between the kinetic energy of the projectile and outgoing particle and the nuclear disintegration energy Q is called as the Q equation has also been explained

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mathematically. In addition to this the solution of the Q equation has also been established in this unit for better understanding of the topic.

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2.12 TERMINAL QUESTIONS

1. What do you meant by nuclear binding energy.
2. Explain the terms mass defect and packing fraction.

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3. Plot the general shape of the binding energy curve (Binding energy per nucleon versus mass number A. Explain fission and fusion on the basis of this plot.
4. What do you mean by Q value of a nuclear reaction?
5. Write down the expression for Q value in the class of α decay.
6. Find the Binding Energy per nucleon for ^{56}Fe . (Given that mass of Fe atom = 55.93493 amu).
7. Find the B/A for deuterium atom. Mass of deuterium atom (${}^2_1\text{H}$) is 2.01402 amu.
8. Find the B/A for ${}^7\text{Li}$. Mass of lithium atom is 7.016004 amu.
9. The B/A for Deuterium and Helium are 1.1 MeV and 7 MeV respectively. What would be the energy released when two deuterons are fused together to form a He nucleus.

UNIT 3

NUCLEAR FORCES I

Structure of the Unit

- 3.1 Introduction
- 3.2 Objectives
- 3.3 Basic Understanding of Nuclear Forces
- 3.4 Basic properties of deuteron
 - 3.4.1 Binding Energy
 - 3.4.2 Size
 - 3.4.3 Spin of Deuteron
 - 3.4.4 Magnetic Dipole Moment
 - 3.4.5 Quadrupole Moments
- 3.5 Existence of excited states of deuteron
- 3.6 n-p scattering at low energies with specific square well potential
 - 3.6.1 What is Scattering
 - 3.6.2 Neutron – Proton Scattering at Low Energies
- 3.7 Results of Low Energy n- p and p-p Scattering
- 3.8 Glossary
- 3.9 Summary
- 3.10 References
- 3.11 Suggested Readings
- 3.12 Terminal Questions

3.1 INTRODUCTION

In the previous unit we have studied the basic information about binding energy which refers to the stability of nucleons. Now in this unit we shall study about the forces that binds the nucleon together i.e. nuclear force. In this consequence we know that large number of nuclei, available in nature, are stable. Now the natural question is what bounds the nucleons together in the nucleus to make it stable? It is further known that most of the nuclei consist of more than one proton – the positive charged particle; it implies that there exists in the nucleus forces which are strong enough to overpower the coulomb repulsion and hold the nucleons together. The forces which hold the nucleons together are commonly called nuclear forces and are short range forces as it is evident from the fact that binding energy is proportional to the number of constituent nucleons.

3.2 OBJECTIVES

The main aim of studying this unit is to understand the nuclear forces which is responsible for binding the nucleons. For understanding the nuclear force, we will study the Basic properties of deuteron viz, its binding energy, its size, spin, magnetic and quadrupole moments etc. After going through this unit learners should be able to:

- Understand the various properties of deuteron.
- Analyze the existence of excited states of deuteron with the solution of spherically symmetric square well potential for higher angular momentum states.
- Learn n-p scattering at low energies with specific square well potential.
- Comparatively study the results of low energy n-p and p-p scattering.

3.3 BASIC UNDERSTANDING OF NUCLEAR FORCES

The nuclear forces cannot be of electromagnetic origin because nuclear forces involve uncharged particle while electromagnetic forces do not. The purely magnetic forces cannot

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account the binding energy of about 8 *MeV* per nucleon (1.1 *MeV* per nucleon in deuteron) in nuclei in general and it is clear that magnetic forces are some hundred times smaller than nuclear forces.

The gravitational forces too cannot explain the existence of such huge forces because they are about 10^{36} times smaller. It implies that nuclear forces cannot be of any of the type discussed so far. If we compare the stable nucleus with the atom then we find that stability of the atom is governed by predominant central particle but in the case of nucleus there is no predominant central particle. The forces which hold together the different nucleons should have mutual forces between the individual nucleons in the ensemble. It turns out that the nuclear forces are strange and of intriguing nature.

Now let us turn to the nuclear interaction which according to Yukawa's theory, may be conceived as due to the exchange of a relatively massive particle - the π - meson or pion with a mass approximately 270 times that of an electron. Thus, nuclear interaction is about 10^{38} times stronger than the gravitational interaction and about 1000 times than the electromagnetic interaction and so comes under what are called 'strong interaction'.

- We then infer that none of the only two interactions, encountered previously, is able to account for the existence of nuclei.
- The only way out then is to recognize the existence of another fundamental interaction – the nuclear interaction (force).
- Since nuclei are composed of protons and neutrons only which are packed very densely within the small volume of the nucleus, the heavier nuclei will be subjected to very strong Columbian repulsive force – the one acting between the positively charged protons, which tends to tear the nucleus apart.
- The fact that nuclei stay as bound systems even in the take of these strong repulsive.
- Columbian forces, is a sufficient proof of the great strength of the nuclear forces and that at distances of the order of nuclear dimensions, it should be attractive in nature .

As a general rule, the wave nature of matter (quantum mechanical principles) is relevant where the de Broglie wavelength of the particles is of the order of the size of the system to be studied.

So let us compare nuclear size with the wavelength of a nucleon of energy 10 MeV.

$$\lambda = \frac{h}{M_N v} = \frac{h}{\sqrt{(2M_N E)}} = \frac{6.6256 \times 10^{-27}}{\left[2 \times 1.67 \times 10^{-24} \times 10 \times 1.6 \times 10^{-19}\right]^{1/2}} \dots\dots\dots(1)$$

$$= 9.3 \times 10^{-23} \text{ cms.} = 9.3 \text{ Fermi} \left(1 \text{ Fermi} = 10^{-13} \text{ cms.}\right)$$

Which is obviously of the size of the nuclei and hence quantum mechanical considerations are indeed relevant to the study of nuclei.

Having ascertained that nuclei are quantum mechanical systems composed of nucleons, it is quite plausible to study the nuclear forces under the simplest possible conditions.

- The simplest case in which the nuclear force is effective is when there are only two experimentally achievable situations:

1- When the two nucleons are bound together

Of the three possible bound states of a two – nucleon system, di-neutron (*nn*), di-proton (*pp*) and deuteron (*np*), nature has provided us with only the deuteron and the other two are unstable.

2- When the two nucleons are in free state and one is made to impinge on the other, i.e. The scattering processes.

In practice, it is not possible to make a neutron target and therefore scattering experiments are limited only to neutron proton (*np*) scattering and proton-proton (*pp*) scattering.

3.4 BASIC PROPERTIES OF DEUTRON

The deuteron consists of a neutron and proton, having charge equal to proton +e, mass 2.014735 atomic mass units and it obeys the Bose-Einstein statistics.

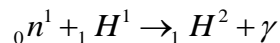
The experimentally measured properties of deuteron are:

3.4.1 Binding energy

Binding energy . (Experimental) = 2.225 ± 0.003 Mev.

The binding energy of deuteron can be determined from a number of experiments.

The easiest one comprise of allowing slow neutrons to be captured by protons in a material containing hydrogen *i.e.* hydrogenous substances such a paraffin, plastic etc. and measuring the energy of the emerging γ rays. The reaction is called (*n-p*) capture reaction and may be written as



Because the neutron carries no charge, the nuclear force binding the deuteron cannot be electrical. It can also not be to $M_N = 1.67 \times 10^{-24}$ gm. to provide a 2.225 MeV binding energy. Therefore the binding force is of nuclear origin .

3.4.2 Size:

Deuteron radius: The root- mean – square value of deuteron radius is 2.1 Fermi.

3.4.3 Spin:

Spin. 1 (in units of h).

3.4.4 Magnetic dipole moment:

The magnetic dipole moment of deuteron is $\mu_d = 0.85735 \pm 00003$ nuclear magneton.

In a structure made up of particles, one expects the total magnetic moment to be the vector sum of the magnetic moments due to the orbital motion of the charged particles.

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Applying quantum mechanics to describe deuteron we may reasonably assume the ground state of deuteron to be an S state for which the angular momentum $L = 0$. With $L=0$, the wave function ψ is spherically symmetrical and for the S state the angular momentum quantum number $l=0$, no contribution from orbital motion is expected to spin.

- The nucleons are half spin particles and deuteron is known to have a spin equal to unity which implies that the proton and neutron spins are parallel.
- In such a case the proton and neutron magnetic moments should also add up.
- Experimental measurements for nucleon magnetic moments give the following values:
 - Magnetic moment of proton $\mu_p = 2.79281 \pm 0.00004$ nm.
 - Magnetic moment of neutron $\mu_n = -1.913148 \pm 0.000066$ nm.
 - Sum of the two moment $(\mu_p + \mu_n) = 0.879662 \pm 0.00005$ nm.

Thus deuteron is expected to have a magnetic moment of 0.8797 ± 0.00015 nm. However from experimental measurements deuteron magnetic moment is found to have a value 0.85733 ± 0.0002 nm between the expected and the measured values which is difficult to explain.

The simplest interpretation being the at deuteron possesses some orbital motion and that our previous assumption of $l=0$ in the ground state is not correct.

Even this approximate agreement is valid only for the S state . Others give values very much different from the measured value.

Thus magnetic moment measurements of deuteron establish the following important conclusions.

- (1) In the ground state of deuteron, the proton and neutron spins are parallel (triple state 3S_1 .)
- (2) Neutron is a half spin particle.

In the ground state of deuteron, the orbital angular momentum is zero

($l=0, S=1$ state).

3.4.5 Quadrupole moments:

The electric quadrupole moment of deuteron as measured by Rabi *et al* in a radio frequency molecular beam method is

$$Q_d = 2.82 \times 10^{-27} \text{ cm}^2 \text{ or } 0.00282 \text{ barn},$$

Which although small but is not zero.

Alternately it can be put as to give

average z^2 for proton

$$\frac{1}{3}(1.14) = \frac{\overline{z^2}}{r^2} \dots\dots\dots(2)$$

average r^2 for proton

It implies that charge distribution in the ground state is not spherically symmetric because a spherical charge distribution needs a value for $Q_d=0$

Or for the $\frac{\overline{z^2}}{r^2} = \frac{1}{3}$.

The result also indicates that charge distribution is of prolate shape, *i.e.*, elongated along the z axis. The electric quadrupole moment and the magnetic moment discrepancy cannot be explained by assuming the state to have some other value of l .

- This suggests that the wave function contains a mixture of l values. Since total angular momentum is equal to the vector sum of the orbital and spin angular moments *i.e.*

$$J = L + S \dots\dots\dots(3)$$

In a system like deuteron which consists of one proton and one neutron, each having a spin $\frac{1}{2}$, the spin quantum number S be given by :

$$S = \left| \frac{1}{2} \pm \frac{1}{2} \right| = 1 \text{ or } 0 \dots \dots \dots (4)$$

For $L=1$ and a maximum value of $S=1$. from equation l can have only the values 0, 1 and 2 . But the conservation of parity demands that even and odd values of l should not be simultaneously present in the same wave function and therefore with $l=0$, only $l=2$ can be present.

The wave function then may be written as:

$$\psi = a_0\psi_{1s} + a_2\psi_{1d} \dots \dots \dots (5)$$

This means that the system spends a fraction $|a_0|^2$ of its time in $l=0$ state and $|a_2|^2$ of its time in an $l=2$ state. Therefore the ground state may be taken to be a mixture of 3S_1 and 3D_1 states.

The magnetic moment and electric quadrupole moment discrepancies can be fully accounted for with $|a_2|^2 = 0.96$ and $|a_0|^2 = 0.04$. This means that deuteron spends 96% of the time in an $l = 0$ state and only 4% of the time in an $l = 2$ state.

We therefore infer that the *deuteron is not in a purely spherically symmetric state*. However, we shall assume that the ground state function is spherically symmetric one .

3.5 EXISTENCE OF EXCITED STATE OF DEUTRON

Extending the calculations of the bound state to cases where the orbital angular quantum number l is greater than zero leads to a result that deuteron cannot exist in these states.

For the extreme case, binding energy $E_B \approx 0$, kr_0 is still only slightly greater than $\pi/2$, since the binding energy E_B of the ground state has already been found negligible compared to the potential well depth V_0 . For the first excited state kr_0 would have to be greater than $3\pi/2$, since

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the wave function $u(r)$ would have to have a radial node inside the well. But from equation: kr_0 must certainly be less than π for all positive values of binding energy.

We shall here prove that for ($l \neq 0$) no bound state exists. It shall be assumed that the potential is central and of square well type in this case.

The differential equation used for this case ($l \neq 0$), is which through the substitution $u(r) = r\psi(r)$ takes the form as shown below.

$$\frac{d^2u(r)}{dr^2} + \frac{M}{\hbar^2} \left[E - V(r) - \frac{l(l+1)\hbar^2}{Mr^2} \right] u(r) = 0. \quad \dots\dots\dots(6)$$

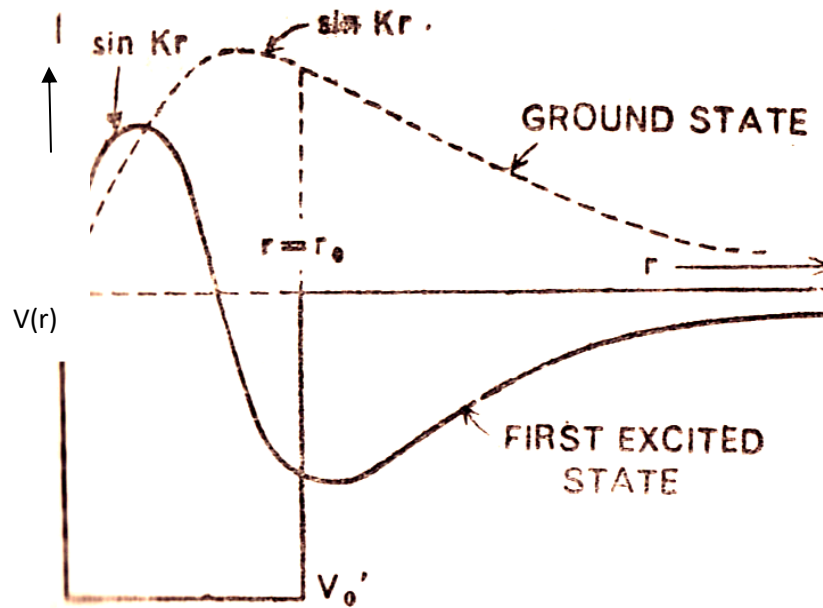


Fig. 1. Deuteron wave function of the first excited states

When we compare these equations, we find that it is analogous to an S-wave radial equation with potential form

$$V_{eff}(r) = V(r) + \frac{l(l+1)\hbar^2}{Mr^2} \quad \dots\dots\dots(7)$$

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Here the second term on R.H.S. is called the centrifugal potential as its space derivative gives the classical centrifugal force. This potential is repulsive, there forces 'l' increases, the binding energy of the lowest bound state decreases.

Coming back to equation(6) and setting $l=1$, the next acceptable value of l after 0, we have ,

$$\frac{d^2u(r)}{dr^2} + \frac{M}{\hbar^2} \left[E - V(r) - \frac{2\hbar^2}{Mr^2} \right] u(r) = 0 \quad \dots\dots\dots(8)$$

Now $E=-E_B'$ the binding energy of deuteron in the P - state ($l=1$) and using a square well potential $V(r) = V_0'$ for $r < r_0$, for the p - state , equation may be written as

$$\frac{d^2u(r)}{dr^2} + \frac{M}{\hbar^2} \left[V_0' - E_B' - \frac{2\hbar^2}{Mr^2} \right] u(r) = 0 \quad \text{for } r < r_0 \quad \dots\dots\dots(9)$$

and
$$\frac{d^2u(r)}{dr^2} + \frac{M}{\hbar^2} \left[E_B' + \frac{2\hbar^2}{Mr^2} \right] u(r) = 0 \quad \text{for } r > r_0 \quad \dots\dots\dots(10)$$

Now assuming

$$k' = \sqrt{\left[\frac{M}{\hbar^2} (V_0' - E_B') \right]} \quad \text{and} \quad \gamma' = \sqrt{\left(\frac{ME_B'}{\hbar^2} \right)}. \quad \dots\dots\dots(11)$$

The above equation (11) may be written as

$$\frac{d^2u(r)}{dr^2} + \left[k'^2 - \frac{2}{r^3} \right] u(r) = 0 \quad \text{for } r < r_0 \quad \dots\dots\dots(12a)$$

and
$$\frac{d^2u(r)}{dr^2} - \left[\gamma'^2 + \frac{2}{r^3} \right] u(r) = 0 \quad \text{for } r > r_0 \quad \dots\dots\dots(12b)$$

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The least well depth, just repaired to produce this bound state, is the one for which the binding energy E_B is just equal to zero, *i.e.*, when $\gamma' = 0$ and $k' = \sqrt{(MV_0'/h^2)} = k_0$.

If we put $k_0 r = x$, the wave equation reduces to

$$\frac{d^2 u(r)}{dx^2} + u(r) - \frac{2u(r)}{x^2} = 0 \quad \text{for } x < k_0 r_0 \dots\dots\dots(13a)$$

and
$$\frac{d^2 u(r)}{dx^2} - \frac{2u(r)}{x^2} = 0 \quad \text{for } x > k_0 r_0 \dots\dots\dots(13b)$$

The solution of equation(13b) with the correct boundary condition becomes

$$u(r) = Ax^{-1} \quad \text{for } x > k_0 r_0, \quad \dots\dots\dots(14)$$

To solve equation(13a), we make the substitution $v = xu(r)$, so that

$$\frac{dv}{dx} = x \frac{du(r)}{dx} + u(r)$$

and
$$\frac{d^2 v}{dx^2} = x \frac{d^2 u(r)}{dx^2} + 2 \frac{du(r)}{dx}$$

and equation (13a) can then be re-written as follows

$$\frac{d^2 v}{dx^2} - \frac{2}{x} \frac{dv}{dx} + v = 0 \quad \text{for } x < k_0 r_0.$$

Differentiating this equation with respect to x, we get

$$\frac{d^3 v}{dx^3} - \frac{2}{x} \frac{d^2 v}{dx^2} + \frac{2}{x^2} \frac{dv}{dx} + \frac{dv}{dx} = 0 \text{ for } x > k_0 r_0.$$

Dividing this equation by x throughout , we get

$$\frac{1}{x} \frac{d^3 v}{dx^3} - \frac{2}{x^2} \frac{d^2 v}{dx^2} + \frac{2}{x^3} \frac{dv}{dx} + \frac{1}{x} \frac{dv}{dx} = 0 \text{ for } x < k_0 r_0. \quad \dots\dots\dots(15)$$

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Now since
$$\frac{d^2}{dx^2} \left(\frac{1}{x} \frac{dv}{dx} \right) = \frac{1}{x} \frac{d^3v}{dx^3} - \frac{2}{x^2} \frac{d^2v}{dx^2} + \frac{2}{x^3} \frac{dv}{dx}$$

then the equation(15) may be re-written as

$$\frac{d^2}{dx^2} \left(\frac{1}{x} \frac{dv}{dx} \right) + \frac{1}{x} \frac{dv}{dx} = 0 \quad \text{for } x < k_0 r_0. \quad \dots\dots\dots(16)$$

Now since $u(r)=vx^{-l}$, must vanish for $x = 0$, the solution of above equation is found to be

$$\frac{1}{x} \frac{dv}{dx} = A_1 \sin x \quad \text{for } x < k_0 r_0.$$

Integrating it, we get

$$v = xu(r) = A_1 (\sin x - x \cos x) \quad \text{for } x < k_0 r_0, \quad \dots\dots(17)$$

Equation (14) and (17) respectively provides the solution for outside and inside the well. To satisfy continuity condition at the boundary ($r=r_0$ or $x=k_0 r_0$), these solutions should be matched at the boundary. Which yields,

$$\frac{d}{dx} (\sin x - x \cos x) = 0 \quad \text{at } x = k_0 r_0$$

Or
$$x \sin x = 0 \text{ at } x = k_0 r_0$$

Or
$$k_0 r_0 \sin k_0 r_0 = 0 \text{ at } x = k_0 r_0 \quad \dots\dots\dots(18)$$

The smallest positive root of this equation is $k_0 r_0 = \pi$. Hence a bound state of the deuteron for $l \neq 0$ can exist only if $k_0 r_0 < \pi$ and this contradicts the previous statement that $k_0 r_0 < \pi$. Therefore we conclude that no bound states exist for deuteron when $l \neq 0$, i.e., deuteron does not possess any excited state.

3.6 n-p SCATTERING AT LOW ENERGIES WITH SPECIFIC SQUARE WELL POTENTIAL

3.6.1 What is Scattering

When an intense and collimated beam of nucleons is bombarded on target nuclei the interactions between incident nucleus and target nuclei takes place.

As a result we may observe the following two possibilities:

CASE I :-

- (i) The interactions does not change the incident particles, *i.e.*, incoming and outgoing particles are the same.
- (ii) The change is in the path of incoming nucleons, *i.e.*, they are deviated from their original path. This process is known as scattering,
- (iii) In scattering processes the outgoing particles may have same energy as that of incident particles or may have the changed energy value. The former is known as elastic scattering and latter is known as inelastic scattering.

CASE II:-

The second possibility is that the outgoing particles are different from the incident particles. Then the interaction process is known as nuclear reaction.

In nuclear reaction we have two alternatives:

- (a) The incident material particles are fully captured by the target and instead γ -radiations (γ -photons) are emitted. The situation is termed as radiative capture (e.g., n - γ -reaction).

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(b) In the second alternative the outgoing particles are either charged particles or some other material particles which are the product of the process itself, then the process is known as nuclear reaction.

It should be remembered that any of the above alternative may occur, either alone or with other competing processes. Among the nucleon-nucleon scattering, neutron proton ($n-p$) scattering is the simplest one, because here the complication due to coulomb forces are not present.

- In ($n-p$) scattering neutron proton system is analyzed in the state of positive energy, *i.e.*, in a situation when they are free.
- In the experiment, a beam of neutrons from an accelerator is allowed to impinge on a target containing many essentially free protons.
- The simplest substance is hydrogen gas but in some cases other substances like thin nylon sheet and paraffin are used. Hence, it is natural to think that in target protons are not free but are bound in molecules.
- The molecular binding energy is so small about $1eV$, therefore, for the impinging neutrons of energy greater than $1eV$, protons are treated as free.
- The presence of electrons also do not affect the process because they are too light to cause any appreciable trouble to incoming neutrons.
- When neutrons impinge on protons, some of them are captured to form deuteron and balance of energy is radiated in the form of γ rays; but the great majority of neutrons undergo elastic scattering.
- In the process, the interactions between two nucleons is of such a order that the neutrons changed their velocities in magnitude as well as in direction.

The proton - proton ($p-p$) scattering is due to the presence of coulomb repulsion between two protons.

- The presence of coulomb repulsion increases the change of direction the account of which is made in estimating the nuclear forces.

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- It appears that nuclear force between the protons is not sufficiently strong to bind the protons against coulomb repulsion. (It is supported by the fact that no bound state with two protons He^3 exists).
- It will be seen later that $p-p$ system remains unbound even in the absence of coulomb repulsion.

The neutron neutron ($n-n$) scattering is not practically possible because of the non availability of neutron target (because neutron decays into proton in a few minutes).

However, there are evidences to support if $n-n$ forces are similar to $p-p$ forces, a bound state for two neutrons cannot exist.

3.6.2 Neutron – Proton Scattering at Low Energies

In the low energy range, most of the measurements of scattering cross section are due to Melkonian and Rainwater et.al. A beryllium target bombarded at by deuterons accelerated in a cyclotron, provided the neutron beam which was shot at a target containing free protons.

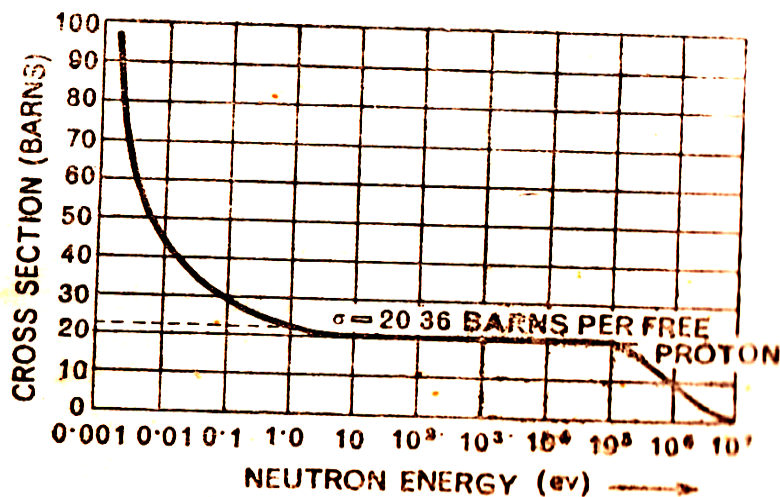


Fig. 2. n-p scattering cross section

- These results show that the scattering cross section depends very much on the energy of the incident neutrons.

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- At low energies below 10 MeV, the scattering is essentially due to neutrons having zero angular momentum ($l=0$) and hence in the Centre of mass system, the angular distribution of scattered neutrons is isotropic.
- In order to avoid complications due to Coulomb forces we shall consider the scattering of neutrons by free protons viz. those not bound to molecules.
- However in practice the protons are of course bound to molecules but the molecular binding energy is only about 0.1 eV.
- Therefore if the incident neutrons have an energy greater than about 1eV. The protons can be regarded as free.
- In describing elastic scattering events like the scattering of neutrons by free protons it is more convenient to use the center of mass system.

The quantum mechanical problem describing the interaction between two particles, in the center of mass system, is equivalent to the problem of interaction between a reduced mass such as the system.

Although while wording out the following theory we shall think in terms of a neutron being scattered by a proton but it applies equally well to spin less, reduced mass particle which is being scattered by a fixed force center.

Let us suppose that the neutron and the proton interact via a spherically symmetric force field whose potential function is $V(r)$, where r is the distance between the particles .

The Schrodinger equation for a central potential $V(r)$ in the center of mass system , for the $n-p$ system is

$$\left[\nabla^2 + \frac{M}{\hbar^2} \{E - V(r)\} \right] \psi = 0 \quad \dots\dots\dots (19)$$

Where M is the reduce mass of the $n-p$ system .

To analyze the scattering event, we have to solve this equation under proper boundary conditions.

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- The action in the immediate region of the scattering centre will be violent and difficult to describe.
- Things will be easier, though, if the experimenter waits for the scattered particles at a sufficient distance from the scattering centre.
- For scattering, the boundary condition states that the wave should have two components when it is far from the scattering centre.
 - (i) an incident plane wave that describes the unscattered particles and superimposed upon it,
 - (ii) an outgoing scattered spherical wave which emanates from the scattering center.

To solve in asymptotic form,

$$\psi = \psi_{inc} + \psi_{sc} \dots\dots\dots(20)\dots$$

The wave function that describes an incident plane wave (a beam of particles) moving in the positive z-direction is

$$\psi_{inc} = e^{ikz} = e^{ikz} \cos \theta, \dots\dots\dots(21)$$

$$\text{Where } k = \sqrt{\left(\frac{ME}{h^2}\right)}$$

Which is a solution of the wave equation with $V(r)$ set equation zero,

$$\left[\nabla^2 + \frac{ME}{h^2} \right] \psi_{inc} = 0, \dots\dots\dots(22)$$

eliminate scattering so that the total wave function ψ becomes identical with the incident wave function ψ_{inc} .

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- The wave function represents one particle per unit volume since the square of the wave function is equal to unity.
- Having known the form of the incident wave function, the next problem is to devise a suitable form for the scattered wave function .

This obviously is

$$\psi_{sc} = f(\theta) \frac{e^{ikr}}{r}, \dots\dots\dots(23)$$

- For large 'r' $f(\theta)$ in this expression indicate amplitude of the scattered wave in the direction θ . This wave function is a necessary consequence of the assumption that the scatterer simply scatters the particles and does not absorb them at all. The probability density and hence the number of scattered particles per unit volume shall be proportional to $|\psi_{sc}|^2$.
- If scattering is considered to be isotropic, the density (number per unit volume) of scattered particles through a large spherical shell of radius r is inversely proportional to r^2 since the volume of the spherical shell, being given by $4\pi r^2 dr$, is proportional to r^2 and density therefore is proportional to $1/r^2$ which is also proportional to $|\psi_{sc}|^2$ Hence $1/r^2$ dependence of ψ_{sc} .

Therefore the wave function ψ , in a form we are actually interested viz. asymptotic, may be written as.

$$\psi = \psi_{inc} + \psi_{sc} = e^{iks} + f(\theta) \frac{e^{ikr}}{r}. \dots\dots\dots(24)$$

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Now, in Fourier analysis we often expand an arbitrary function into a series of harmonic functions of various frequencies.

So we expand the incident plane wave function e^{ikz} in terms of Legendre Polynomials $P_l(\cos \theta)$ and write.

$$\psi_{inc} = e^{ikrcos\theta} = \sum_{l=0}^{\infty} B_l(r)P_l(\cos \theta) \dots \dots \dots (25)$$

Where l is an integer representing the various partial waves. This particular way of writing the wave function is termed as the partial wave expansion.

The radial functions $B_l(r)$ in the above equation are given by

$$B_l(r) = i^l(2l + 1)j_l(kr), \dots \dots \dots (26)$$

Where $J_l(kr)$ is the Spherical Bessel function which is related to the ordinary Bessel function through the formula

$$j_l(kr) = \left(\frac{\pi}{2kr}\right)^{1/2} J_{l+1/2}(kr) \dots \dots \dots (27)$$

and can be represented as

$$j_l(kr) = (-kr)^l \left[\frac{1}{kr} \frac{d}{d(kr)} \right]^l \left(\frac{\sin kr}{kr} \right) \dots \dots \dots (28)$$

Whence asymptotically

$$j_l(kr)_{r \rightarrow \infty} \rightarrow \frac{\sin\left(kr - \frac{l\pi}{2}\right)}{kr} \dots \dots \dots (29)$$

Asymptotically, $B_l(r)$ from is given by

$$B_l(r)_{r \rightarrow \infty} \rightarrow i^l(2l + 1) \frac{\sin\left(kr - \frac{l\pi}{2}\right)}{kr}$$

$$\cong \frac{1}{2ikr} i^l (2l + 1) \cdot [e^{i(kr - l\pi/2)} - e^{-i(kr - l\pi/2)}] \dots\dots\dots(30)$$

The Spherical Bessel function $J_l(kr)$ for various values of l are

$$j_2(kr) = \left[\frac{3}{(kr)^3} - \frac{1}{kr} \right] \sin(kr) - \frac{3 \cos(kr)}{(kr)^2} \dots\dots\dots(31)$$

These functions are plotted in the Fig 3.

Similarly $f(\theta)$ can also be expanded in terms of the Legendre Polynomials as follows

$$f(\theta) = \frac{i}{2k} \sum_{l=0}^{\infty} f_l (2l + 1) P_l(\cos \theta) \dots\dots\dots(32)$$

Substituting the values in equation(24) we have

$$\psi = \psi_{inc} + \psi_{sc} \approx \sum_{l=0}^{\infty} [i^l (2l + 1) j_l(kr) + f_l \frac{e^{ikr}}{r}] P_l(\cos \theta) \dots\dots\dots(33)$$

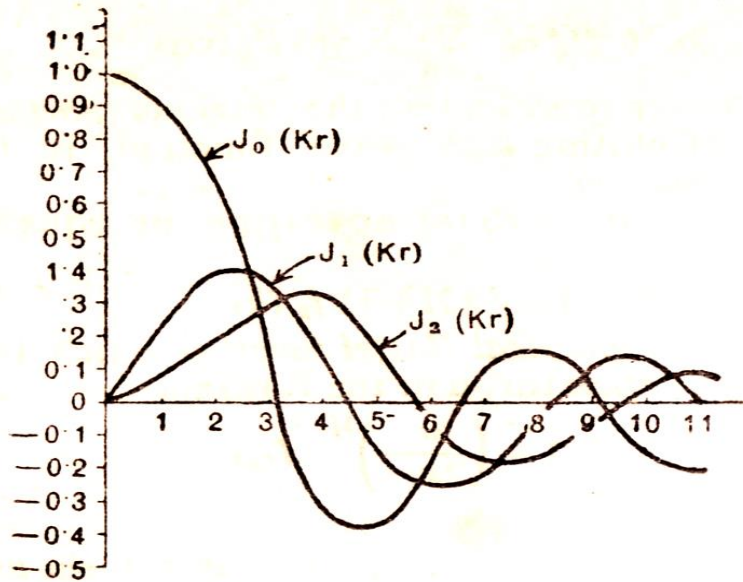


Fig.3. Variation of Bessel function with orbital angular momentum quantum number.

Due to the fact that each term in the sum [equation 33] with a particular value of the orbital angular momentum quantum number 'l' reflects a solution of the wave equation in spherical polar coordinates for constant potential energy. Because the majority of the scattering occurs at energies below 10 MeV and the number of partial waves is severely constrained in this situation, it is sufficient to study the scattering only for l=0 particles, or S-waves, the expansion classifies the particles in the beam according to their angular momenta.

From equation (30) by putting l=0, we have

$$B_0(r) = \frac{\sin(kr)}{kr} \approx 1 - \frac{(kr)^2}{6} + \dots \dots \dots (34)$$

And for l=1

$$B_1(r) = 3i \left[\frac{\sin(kr)}{kr} - \frac{\cos(kr)}{kr} \right]$$

$$\cong 3i \left[\frac{kr}{3} - \frac{(kr)^3}{30} + \dots \dots \dots \right].$$

$$\therefore \left| \frac{B_1(r)}{B_0(r)} \right| \cong (kr)^2.$$

Since $B_l^2(r)$ determines the probability density, we instead determined the ratio of the square (r) instead of just $\frac{B_1(r)}{B_0(r)}$. Consider a neutron with energy 1 MeV in the L-system compared to 0.5

MeV in the C-M system to get a sense of the magnitude of this ratio. then, neutron momentum is

$$p = (2ME)^{\frac{1}{2}} = \left[\frac{2 \times 1.67 \times 10^{-24} \times 1.6 \times 10^{-6}}{2} \right] \dots \dots \dots (35)$$

$$= 1.63 \times 10^{-15} \text{ gm.cm./sec.}$$

Now the wave number number is given by

$$k = \frac{p}{h} = \frac{1.63 \times 10^{-15}}{1.0545 \times 10^{-27}} = 1.55 \times 10^{12} \text{ cm.}^{-1}$$

If we assume the nuclear forces to have a range $r = 2$ Fermi, then

$$\left| \frac{B_1(r)}{B_0(r)} \right|^2 \cong (kr)^2 = (1.55 \times 10^{12} \times 2 \times 10^{-13})^2 = (0.31)^2,$$

$$= 0.0961,$$

i.e., only 9% of the scattering at 1 Mev is caused by neutrons with $l=1$. Similar calculations increase this amount to roughly 49% for a neutron with an energy of 10 MeV. In the energy range below 10 Mev, thus. The major scattering type is S-wave ($l=0$).

3.7 RESULTS OF LOW ENERGY n-p and p-p SCATTERING

The theory for the scattering cross-section that was constructed in the previous section is actually a theory for the phase shift, which in turn depends on the assumptions about the characteristics of the scattering potential $V. (r)$. To provide an example, we now carry out the computations using the same rectangular potential well that was presupposed in earlier sections for the deuteron ground state.

Equation inside and outside the nuclear square potential well, which is the radial Schrodinger equation for $l=0$, can be represented as

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$$\frac{d^2u(r)}{dr^2} + \frac{M}{\hbar^2}(E + V_0)u(r) = 0 \quad \text{for } r < r_0,$$

$$\frac{d^2u(r)}{dr^2} + \frac{M}{\hbar^2}Eu(r) = 0 \quad \text{for } r > r_0 \quad (36)$$

Since the negative binding energy is substituted in the current case of n-p scattering by a low positive energy E that is substantially smaller than the well-depth V₀. These calculations can be expressed as

$$\frac{d^2u_i}{dr^2} + K^2u_i = 0 \quad \text{for } r < r_0, \dots\dots\dots(37a)$$

$$\frac{d^2u_0}{dr^2} + k^2u_0 = 0 \quad \text{for } r > r_0, \dots\dots\dots(37b)$$

Where u_i is the wave function inside the well and u₀ that outside the well and

$$K^2 = \frac{M(E+V_0)}{\hbar^2}, \quad k^2 = \frac{ME}{\hbar^2}. \quad \dots\dots\dots(37c)$$

Equation(37a) has the solution

$$u_i = A \sin Kr \quad \dots\dots\dots(38)$$

Equation(37b) has the solution

$$u_0 = C \sin kr + D \cos kr, \quad \dots\dots\dots(39)$$

Which may be written as

$$u_0 = B \sin(kr + \delta_0). \quad \dots\dots\dots(40)$$

To understand the significance of the phase shift δ_0 , the Schrodinger equation would V(r) set equal to zero, the solution of which is of the form

$$u(r) = \sin kr. \quad \dots\dots\dots (41)$$

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Because it must disappear when $r = 0$, the only viable solution outside of the well. This is how turning on the scattering potential causes the phase shift at great distances.

We now require that the solution (38) and (40) join smoothly at $r = r_0$ *i.e.* the logarithmic derivative must be continuous at $r=r_0$ *viz*,

$$\left. \frac{1}{u_i} \frac{du_i}{dr} \right|_{r=r_0} = \left. \frac{1}{u_0} \frac{du_0}{dr} \right|_{r=r_0}$$

This condition, with equation (38) and (40) gives

$$K \cot Kr_0 = k \cot(kr_0 + \delta_0) \quad \dots\dots\dots (42)$$

By comparing this result with the continuity condition equation for the ground state of the deuteron *viz*.

$$\frac{\sqrt{\{M(V_0 - E_B)\}}}{\hbar} \cot \left(\left[\frac{\sqrt{\{M(V_0 - E_B)\}}}{\hbar} r_0 \right] \right) = -\gamma$$

We assume that inside the well, the scattering wave function is not significantly different from the deuteron wave function in order to simplify the matching condition in the case of n-p scattering. This seems quite acceptable given that the sole difference between the two scenarios is that while the total energy E in this case is little, it is positive, whereas the deuteron binding energy E_B is also small, but it is negative. We therefore suppose that the value of the logarithmic derivative of the ground state wave function of the deuteron, $-\gamma$, might be used to estimate the logarithmic derivative $K \cot Kr_0$ of the inside wave function for scattering. Hence from (42)

$$k \cot(kr_0 + \delta_0) = -\gamma \quad \dots\dots\dots(43a)$$

At this point we introduce another approximation that r_0 is very small (possibly zero) compared to $k = \sqrt{(ME/\hbar)}$ so that kr_0 may be neglected in the above equation and then

$$k \cot \delta_0 = -y \text{ or } \cot \delta_0 = -y/k \quad \dots\dots\dots(43b)$$

Now the total scattering cross-section for $l=0$ from equation is given by

$$\begin{aligned} \sigma_{sc,0} &= \frac{4\pi}{k^2} \sin^2 \delta_0 = \frac{4\pi}{k^2} \cdot \frac{1}{(1 + \cot^2 \delta_0)} = \frac{4\pi}{k^2} \cdot \frac{1}{(1 + y^2/k^2)} \\ &= \frac{4\pi}{k^2 + y^2} = \frac{4\pi h^2}{M(E + E_B)} \quad \dots\dots\dots(44) \end{aligned}$$

Where we have substituted the values of k^2 and y^2 from equations (37). The relation was first arrived at by E.P. winger which although agrees with experimental results at high energies but fails miserably at low energies.

3.8 GLOSSARY

Nucleon: The proton and the neutron, constituting atomic nuclei.

Bound state: A quantum state of a particle subject to a potential such that the particle has a tendency to remain localized in one or more regions of space.

Binding Energy: Amount of energy required to separate a particle from a system of particles or to disperse all the particles of the system.

Magnetic moment: Magnetic Moment is defined as magnetic strength and orientation of a magnet or other object that produces a magnetic field.

Electric Quadrupole Moment: A parameter which describes the effective shape of the ellipsoid of nuclear charge distribution.

Scattering : In physics, a change in the direction of motion of a particle because of a collision with another particle

3.9 SUMMARY

After going through this unit, you would be able to achieve the following objectives. Now we recall what we have discussed so far.

- We have learnt the basic properties of deuteron, its charge (+e), mass (~ 2.014 amu), its radius (2.1 fermi), its binding energy ($= 2.225 \pm .003$ Mev), Spin ($1 \hbar$) and statistics (Bose-Einstein) and the electric quadrupole moment $Q_d = 0.00282$ barn .
- The study of deuteron problem, although hopelessly limited in as much as deuteron possesses only the ground state and no-excited states exist for the bound neutron-proton system, gives invaluable clues about the nature of the nuclear force.
- We learnt that neutron and proton can form stable combination (deuteron) only in the triplet state means when the n & p spins are parallel. The singlet state, i.e. a state of anti parallel n-p spins being unbound.
- The existence of non-zero magnetic moment and electric quadrupole moment for deuteron suggests that at least a part of the neutron proton force acting in deuteron is non-central.
- The nuclear forces are spin dependent i.e., nuclear forces not only depend upon the separation distance but also upon the spin orientations of two nucleons. They are independent of the shape of nuclear potential.

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3. Lectures on Nuclear Theory (translated from the Russian) by Landau, Plenum Press, New York.

3.12 TERMINAL QUESTIONS

1. Discuss the n-p and p-p scattering at low energies.
2. Solve the Schrodinger equation for the deuteron in a S-state under the assumption of square well potential.
3. Show that deuteron has no excited state.
4. Write short note on scattering length.
5. Explain clearly how the properties of the deuteron indicate the presence of spin dependent force and tensor force between two nucleons.
6. Briefly explain the properties of nuclear forces.
7. If the binding energy of the deuteron were 12 MeV , what would be roughly the depth of the potential well, assuming it to be square?

UNIT 4

NUCLEAR FORCES II

Structure of the Unit

4.1 Introduction

4.2 Objectives

4.3 Spin Dependence

4.4 Scattering Length

4.5 Pauli Principle and Antisymmetrisation

4.6 Isospin

4.7 Two Nucleon system

4.8 Generalized Pauli Exclusion Principle

4.9 Meson Theory of Nuclear Forces:(Yukawa Theory)

4.9.1 Estimation of mass of meson using uncertainty principle

4.9.2 Yukawa Potential

4.10 Summary

4.11 References

4.12 Suggested Readings

4.13 Terminal Questions

4.1 INTRODUCTION

The nucleons that make up the nucleus have two important properties. Firstly, you cannot distinguish one proton/neutron from another proton/neutron. All protons are indistinguishable and all neutrons are indistinguishable. We can extend this to also say that, as far as strong or nuclear interaction properties are concerned, no nucleon is distinguishable from any other nucleon. This charge independence symmetry is at the root of the concept of isospin. This is a seemingly obscure quantity but it allows simplicity and transparency when specifying wavefunctions and has some useful properties giving rise to useful selection rules

4.2 OBJECTIVES

After studying the unit learners will be able to

- Know the spin dependence and scattering length.
- Explain meson theory of exchange forces.
- Understand Pauli exclusion principle
- Differentiate between symmetric and antisymmetric wave functions
- Describe Isospin
- Solve questions based on Isospin
- Explain the Yukawa potential

4.3 SPIN DEPENDENCE

E. P. Wigner postulated that spin effects the internucleon forces. Since the neutron and proton are both spin particles, their spins can either be parallel or antiparallel during n-p scattering. The neutron and proton spins are parallel in the bound state of the n-p system in deuterium, which has a binding energy of E_B , suggesting that this equation may remain true in the parallel spin case. The triple state of parallel spins has a statistical weight of three, which corresponds to the three

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permitted orientations of the angular momentum vector in the presence of an outside magnetic field. Because a vector with zero length cannot be oriented, the state of antiparallel spins is a singlet state with a statistical weight.

1- The singlet and triplet states of the n-p system will occur in scattering experiments in proportion to the statistical weight factors for these states, which are 1 and 3 respectively. This is because neutron and proton spins are generally randomly oriented in scattering experiments, as are the spins of neutrons in the incident beam. As a result, the overall scattering cross-section will consist of two components: the scattering cross-section for the triplet state and the scattering cross-section for the singlet state, as shown below.

$$\sigma_0 = \frac{3}{4}\sigma_t + \frac{1}{4}\sigma_s \quad \text{.....(45)}$$

From a naive perspective, the two spins are equally likely to be parallel and antiparallel in a random distribution of spins, such as in n-p scattering, giving the two states identical statistical weight. The phrase "spin pointing up" merely indicates that the spin vector is pointing somewhere in a cone that is centered on the vertical direction because, according to quantum mechanics, the spin direction cannot be defined as uniquely as a vector in space. The four equally likely configurations for the respective spins of the two particles are schematically shown in the following figure.

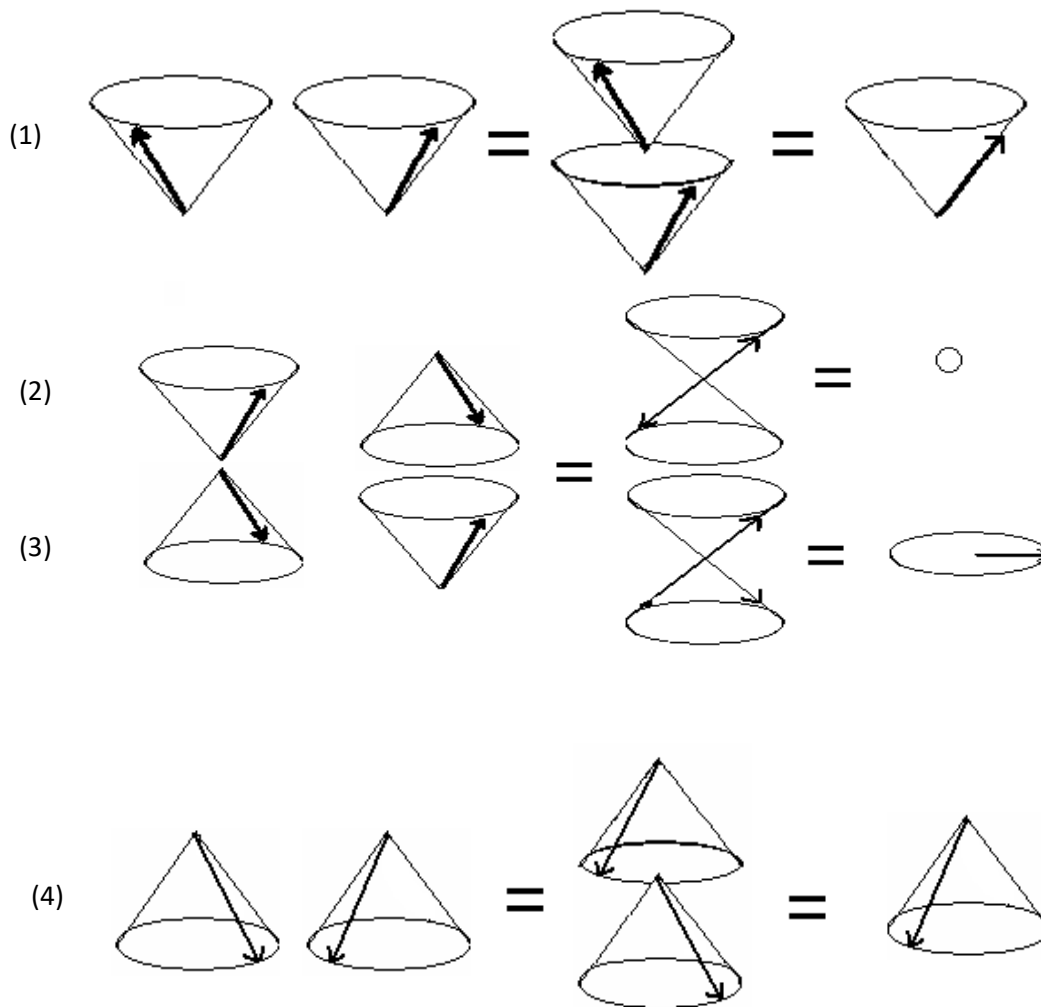


Fig. 5 Spin dependence

The total spin unity shown in Figures (1) and (4) corresponds to the magnetic quantum number values +1 and -1, respectively. Since the spins are not aligned along the z-axis, the z-components in examples (2) and (3) may add up to zero, resulting in a singlet state, or they may add up to a total spin perpendicular to the z-axis, resulting in a triplet state.

4.4 SCATTERING LENGTH

Fermi and Marshall introduced a very useful concept the 'scattering length a ' for the discussion of nuclear scattering at very low incident neutron energy.

$$[i.e. E \rightarrow 0 \quad \text{and hence} \quad k = \sqrt{\left\{ \left(\frac{ME}{h^2} \right) \right\}} \rightarrow 0 \quad \dots\dots\dots(46)$$

Which may be defined as follows:

$$a = \lim_{k \rightarrow 0} \left(-\frac{\sin \delta_0}{k} \right); \quad \dots\dots\dots(47)$$

By this definition, equation which gives the total scattering cross section for S-wave ($l=0$) can be written for very low incident neutron energy as

$$\lim_{k \rightarrow 0} (\sigma_{sc}, 0) = \lim_{k \rightarrow 0} \left(\frac{4\pi \sin^2 \delta_0}{k^2} \right) = 4\pi a^2 \quad \dots\dots\dots(48)$$

Equation therefore shows that "a" has the dimensions of length, hence the name scattering length, and has the geometric significance of being the radius of a hard sphere around the scattering center from which neutrons are scattered.

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Now, it should be noticed from equation that as the energy E of the incident neutron approaches 0, it must also approach either 0 or else the cross-section at zero neutron energy would become infinite, which is physically absurd. Therefore, at very low incident neutron energies ($E \rightarrow 0$), equation reduces to

$$a = -\frac{\delta_0}{k} \dots\dots\dots(49)$$

Then at very low incident neutron energies, the wave function outside the range of nuclear force as expressed by equation may be written as

$$\lim_{k \rightarrow 0} U(r) = \lim_{k \rightarrow 0} (r\psi_0) = \lim_{k \rightarrow 0} \left[e^{i\delta_0} \frac{\sin(kr + \delta_0)}{k} \right]$$

The scattering length is then simply represented graphically by the equation. The scattering length 'a' is the intercept on the r-axis and this equation depicts a straight line for $U(r)$. Fig. 6 illustrates this.

The relevance of positive or negative scattering length is that it informs us what is the significance of attaching a positive or negative sign with at the scattering length, an inquisitive reader may inquire quite naturally after we have defined the scattering length using equations. whether a bound or unbound state exists in the system.

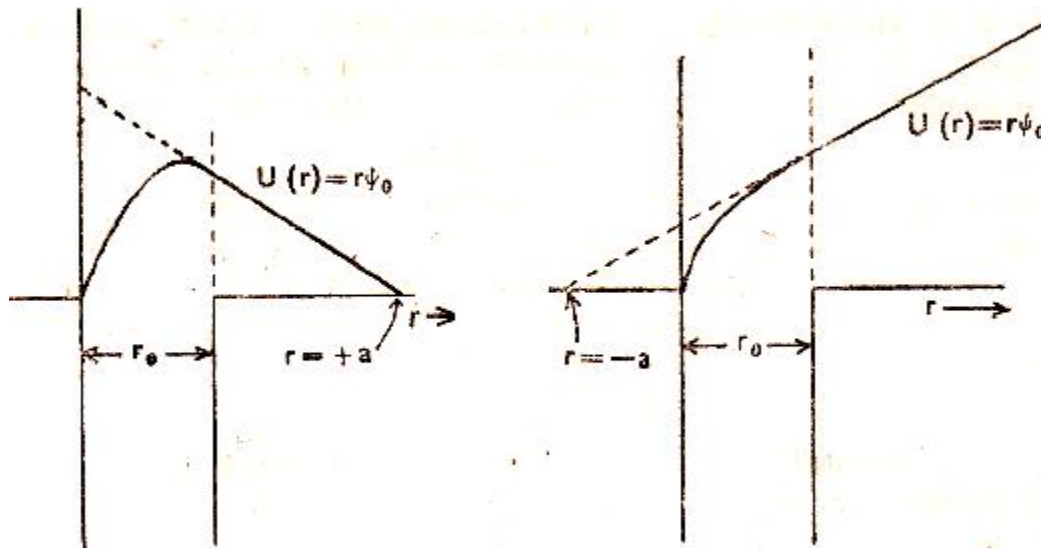


Fig. 6. Graphical interpretation of scattering length

It is evident from Fig. 6 that a positive scattering length denotes a bound state while a negative scattering length denotes an unbound or virtual state.

Since the wave function for the bound state of the n-p system, known as the deuteron wave function, must curve towards the r-axis in order to match the exponentially decaying solution (c.f. equation), $r > r_0$ will result in a positive intercept on the r-axis, indicating a positive scattering length. When extrapolating $U(r)$, the wave function for an unbound state must match an increasing solution outside of the range r_0 , and this must result in a negative intercept on the r-axis, signifying a negative scattering length

4.5 PAULI PRINCIPLE AND ANTISYMMETRISATION

The nucleons that make up the nucleus have two important properties. Firstly, you cannot distinguish one proton/neutron from another proton/neutron. All protons are indistinguishable and all neutrons are indistinguishable. We can extend this to also say that, as far as strong or nuclear interaction properties are concerned, no nucleon is distinguishable from any other nucleon. This charge independence symmetry is at the root of the concept of isospin.

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Nucleons are fermions and therefore should obey the Pauli principle. We can guarantee this if we construct wavefunctions which are antisymmetric with respect to particle exchange.

Consider for now a simple two-particle system with no interactions between the two identical particles. Under these conditions the Hamiltonian is simply the sum of the Hamiltonians for the two particles alone:

$$\hat{H} = \hat{H}_1 + \hat{H}_2$$

You can easily show by substitution that a simple product wavefunction, $\Psi = \varphi_a(r_1)\varphi_b(r_2)$, satisfies the Schrodinger equation, $\hat{H}\psi = E\psi$, if $E = E_1 + E_2$. Here the a and b represent the quantum numbers of the two single-particle states, and r_1 and r_2 are the spatial coordinates of the two nucleons.

There are problems with writing the wavefunction as $\psi = \varphi_a(r_1)\varphi_b(r_2)$. It suggests that you can label the first nucleon and distinguish it from the second; if you exchange the first for the second nucleon you get a different state $\psi = \varphi_a(r_2)\varphi_b(r_1)$. Also if the two single-particle states are the same, $a = b$, the wavefunction becomes $\psi = \varphi_a(r_2)\varphi_a(r_1)$, rather than zero as expected by the Pauli principle.

We can construct something which obeys Pauli by forming the wavefunction:

$$\psi = \frac{1}{\sqrt{2}}[\varphi_a(r_1)\varphi_b(r_2) - \varphi_a(r_2)\varphi_b(r_1)]$$

By substitution, show that this wavefunction satisfies the Schrodinger equation for the two-particle system and that it is properly normalised.

It is clear that in this particular case if $a = b$, the wavefunction is zero and therefore the Pauli principle is obeyed. If you perform a particle exchange operation on this wavefunction, where you swap the first for the second ($1 \rightarrow 2$) nucleon and the second for the first ($2 \rightarrow 1$) nucleon, you end up with the final answer $-\Psi$. This wavefunction is therefore antisymmetric under

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particle exchange. Generally, wavefunctions must be antisymmetric to be consistent with the Pauli principle.

In general, the Hamiltonian depends on both spatial and spin components. Consider a simple separation of spin and spatial coordinates can be performed such that the wavefunction is written as a product of a spatial and a spin part, ΨX . In order to be consistent with the Pauli principle, it is the overall wavefunction that must be antisymmetric. This means that either the spatial part is antisymmetric combined with a symmetric spin part, or the spatial part is symmetric and the spin part is antisymmetric.

Consider the possible spin wavefunctions of two identical independent nucleons. They both have spin-1/2 and so the system can couple to give an overall spin of $S = 0$ or 1 .

Let's take the $S = 1$ possibilities first.

To have $S = 1$, we must have wavefunctions with $m_S = +1, 0$ and -1 .

$$M_S = m_{s1} + m_{s2}.$$

For $M_S = 1$ we have m_{s1} and $m_{s2} = +1/2$.

For $M_S = -1$ we require m_{s1} and $m_{s2} = -1/2$.

If we interchange 1 for 2 and visa versa for these wavefunctions we would see that they are symmetric under particle exchange since each nucleon has the same quantum numbers. The $M_S = 0$ wavefunction could be made from either $m_{s1} = 1/2$ and $m_{s2} = -1/2$, or $m_{s2} = +1/2$ and $m_{s1} = -1/2$.

4.6 ISOSPIN or ISOTOPIC SPIN

.Protons and neutrons are identical as far as the operation of the strong nuclear force is concerned. Here are a few observations that indicate that we could start to treat them as two different types of the same particle:

- (a) The proton-neutron mass difference is very small.
- (b) In low-energy ($<5\text{MeV}$) np and pp scattering, the cross section for the ^1S channel is equal to within a few percentage if Coulomb effects are corrected.
- (c) If we exchange all the protons in a nucleus to neutrons, and all the neutrons to protons, and generate its mirror system. Apart from small Coulomb effects and the n-p mass difference, the energy levels are remarkably similar. The nuclear interactions are therefore unchanged if n - n forces are exchanged for p p forces and the nuclear force must be **CHARGE SYMMETRIC**.
- (d) If we exchange protons, one by one for neutrons gradually and generate a sequence of nuclei of the same mass but different numbers of protons and neutrons. States in nuclei in this isobaric sequence have analogues in the other members of the sequence. The nuclear interactions between n-p and p-p and n-n are therefore the same and the nuclear force must be **CHARGE INDEPENDENT**.

An example was discussed before in ^{30}Si , ^{30}P and ^{30}S where analogue states were seen in all three. Some states though were unique to ^{30}P and we will see that these are states which are allowed in an np system but forbidden in the pp or nn cases by Pauli.

This is the origin of the name nucleon. Heisenberg was the first to treat protons and neutrons as two substates of the same particle, the nucleon back in 1932. This is a two substate system, rather like intrinsic spin- $1/2$ which can have substates $m_s=+1/2$ or $-1/2$, hence the name of this new characteristic, isospin.

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A nucleon has isospin $I=1/2$; a proton has $I_z=-1/2$ and a neutron has $I_z=+1/2$. This z doesn't refer to real space anymore; isospin operates in a fictitious new space called isospin space. All the quantum mechanical mathematics of spin operators, Pauli spin matrices, angular momentum coupling and raising/lowering operators can be applied to isospin in a completely analogous way.

4.7 TWO-NUCLEON SYSTEM

Now we take the two nucleons and couple their isospin together. Each nucleon carries $I=1/2$, so we can couple them together to make $I=0$ or 1 . The third components couple according to $I_z=I_{z1}+I_{z2}$. A system of states is then generated completely analogous to the spin components discussed above. The $T=0$ state is a singlet with $I_z=0$, and it is antisymmetric under particle exchange. The $I=1$ coupling is a triplet of states with $I_z=+1, 0$, and -1 and is symmetric under particle exchange. What does all this mean?

We have two nucleons so we have generated nuclei with $A=2$. The third component tells us something important. Protons individually have $I_z=-1/2$ and neutrons $I_z=+1/2$, so the third component is all wrapped up in what electrical charge the system has. The quantity, $-(I_z-1/2)$, tells us the charge on a nucleon. In a multinucleon system, the overall charge can be found by summing $-(I_z-1/2)$ over all the nucleons. Doing this gives you, $Z=-(I_z-A/2)$. For the $A=2$ nuclei, $I_z=+1, 0$, and -1 must therefore correspond to $Z=0, 1$ and 2 , or equivalently two neutrons (a dineutron), the deuteron and two protons (a diproton). Notice that in the dineutron and diproton, there is no $I=0$ state since it is prevented by the Pauli principle. But in the deuteron, both $I=0$ and $I=1$ states are available.

In addition, for the $A=2$ case we would expect that the spatial wavefunction is going to be $s(1/2)$ for both nucleons. The spatial wavefunction in this case cannot be antisymmetric, and only a symmetric version is non-zero. Prove this by looking at the spatial two-particle wavefunctions above and substitute in the same single-particle states. To get an overall antisymmetric wavefunction, if the spin part is symmetric, the isospin part must be antisymmetric, or visa versa.

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So the $I=1$ (symmetric) state must have $J=0$ (antisymmetric). The $T=0$ state must have $J=1$ for the same reason.

Combining this with the knowledge of the deuteron we have from before, namely the ground state is $J=1$ and all other states are unbound and at higher energies, we can draw a diagram summarising all of this.

We've used charge independence to tell us that, apart from Coulomb effects and pn mass differences, the $I = 1$ states should be at the same energy.

We would expect that the diproton and the dineutron are not bound nuclei. Two identical nucleons cannot stick together under nuclear forces.

Notice that for the deuteron two isospin couplings are possible, $I = 0$ and $I = 1$. We see experimentally that the $I = 0$ coupling is lowest. It turns out that for most nuclei, many isospin couplings are available, but usually the one that is lowest in energy is $I = I_z$.

Take a nucleus composed of A nucleons, with Z protons and N neutrons. The I_z of the nucleus is $(N - Z)/2$. Many values of the isospin are available by coupling together the isospin of each nucleon i.e. couple $\frac{1}{2}$, A times. The maximum value of the isospin is the $A/2$. I cannot be lower than I_z .

For example, ^{30}S has $A = 30, Z = 14$ and $N = 16$

$$I_z = 1/2(N - Z) = +1$$

Maximum $I = 15$

Minimum $I = 1$

Ground-state $I = 1$

4.8 GENERALIZED PAULI EXCLUSION PRINCIPLE

The Pauli exclusion principle says that no two identical fermions can simultaneously occupy the same quantum state. Generalized Pauli Exclusion Principle All fermions and particles derived from fermions, such as protons and neutrons, obey Fermi-Dirac statistics; this includes obeying the Pauli exclusion principle. Quarks (up and down) and leptons (electrons, electron neutrinos,

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muons, muon neutrinos, taus, and tau neutrinos) are all fermions. Particles obeying the exclusion principle have a characteristic value of spin, or intrinsic angular momentum; their spin is always some odd whole-number multiple of one-half.

The Pauli exclusion principle does not apply to bosons: these are particles that obey Bose-Einstein statistics; they all have integer values of spin. Photons, gluons, gravitons, and the W, Z and Higgs bosons are all bosons

Quantum numbers should be considered as address of each electron within an atom, each address has four components, and no two electrons can have the exact same address. The Pauli exclusion principle does not apply to bosons: these are particles that obey Bose-Einstein statistics; they all have integer values of spin. Photons, gluons, gravitons, and the W, Z and Higgs bosons are all bosons. This Pauli Exclusion principle states that no two electrons in an atom can have the same four quantum numbers. If two electrons occupy the same orbital, they must have different spins. Pauli's Exclusion Principle limits the numbers of electrons that a shell or a subshell may contain.

4.9 MESON THEORY OF NUCLEAR FORCES:(YUKAWA THEORY)

The range and power of the nuclear force were topics covered in the previous section. The experimental understanding of the deuteron served as the foundation for this information. Instead of providing us with a theoretical representation of the nuclear force, it informs us about some of its characteristics. Any theory regarding it hinges on whether the deuteron system's spin is 0 or 1 and whether the orbital momentum is even or odd. Why should the nuclear force be dependent on these? The well-known Yukawa theory of the exchange of mesons by nucleons offers the solution. This meson exchange produces exchange forces. Let's examine the fundamental aspects of Yukawa's concept.

- (1) Nuclear force originates from exchange of a particle with a non -zero rest mass called meson.
- (2) Each nucleon is surrounded by a meson field as a result it continuously emits and absorbs mesons. This exchange of meson gives rise to the nuclear force and exchange of meson leads to

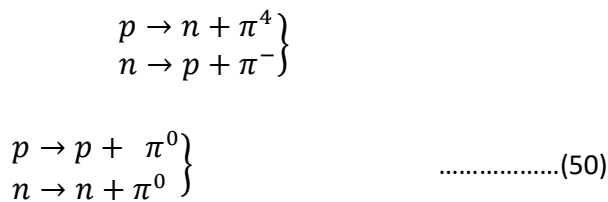
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constant transfer of momentum from one nucleon to other and hence the force is exerted between them.

(3) The emission of meson ought to reduce the mass of nucleon. However this is not observed. Thus, exchange of meson must takes place in such a short time that the uncertainty in the energy is in consistence with the Heisenberg's uncertainty principle $\Delta E \cdot \Delta t \approx \hbar$

This makes the detection of meson exchange impossible and so the meson in this exchange process is referred to as virtual mesons.

(4) The meson responsible for stop g nuclear forces are π -mesons (π^+, π^0, π^-). The force between p n is due to π^+ and π^-

**4.9.1 Estimation of mass of meson using uncertainty principle**

If a nucleon emits a virtual meson π of rest mass M_π , it will loose an amount of energy $M_\pi c^2$

$$\Delta E = M_\pi c^2 \quad p \rightarrow p + \pi^0 \quad \dots\dots\dots(51)$$

Now the proton mass in above transition must decrease which is not observed in the experiment This violation of conservation of energy is understood if the meson is absorbed again in a short time Δt such that $\Delta E \cdot \Delta t \approx \hbar$. So, the complete reaction is given as

$$p \rightarrow p + \pi^0 \rightarrow p, \quad \dots\dots\dots(52)$$

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Thus, the emission and absorption of a particle with rest energy $M_{\pi}c^2$ is consistent with uncertainty principle. If the particle encounters the same or another nucleon in time Δt given by $(\Delta t \approx \frac{\hbar}{M_{\pi}c^2})$. Even if we assume that meson travels at the maximum possible speed that is the speed of light (c), then the maximum distance it can cover in this time is $c.\Delta t$.

$$R = c.\Delta t = \frac{\hbar}{M_{\pi}C} \dots\dots\dots (53)$$

Mass of meson is

$$M_{\pi} = \frac{\hbar}{RC}$$

$$M_{\pi} = \frac{6.64 \times 10^{-34}/2\pi}{2 \pi \times 1.2 \times 10^{-15} \times 3 \times 10^8} \approx 270 m_e \dots\dots\dots(54)$$

Table 01: Comparison of masses, spin and magnetic moment for pions

	π^+	π^0	π^-
M_{π}	$273.2 m_e$	$264.4 m_e$	$273.2 m_e$
Spin	0	0	0
Magnetic Moment	0	0	0

From the equation (54), a mass of about $270m_0$ (where m_0 is the rest mass of an electron) for the meson mass M_{π} . One can estimate the experimentally observed value of R by putting rest mass M_{π} of the meson.

$$R \sim \frac{\hbar}{2 \times (270m_0)c} \sim 10^{-15} \text{ m} = \text{one Fermi}$$

So, we have estimated roughly that the nuclear force is due to the process of meson exchange then its range is of the order of a few fermis, which agrees well with the experiments.

4.9.2 Yukawa Potential

The discovery of π -meson predicted by Yukawa was a triumph of the meson theory and an important landmark in our quest to understand the nuclear force. Yukawa was also able to write an approximate expression for the potential energy of interaction between two nucleons:

$$V = -C \frac{e^{-r/R}}{r} \dots\dots\dots(55)$$

where C is a constant, r is the internucleon distance and R is the range of force.

The Yukawa potential has been found to be successful in the discussion of deuteron problem and also in understanding the low energy nucleon scattering data. The constant C occupies a place in the Yukawa meson theory, similar to that of charge e in the electromagnetic theory.

In other words, the electromagnetic potential energy $V = e\phi$ and in an analogous manner the Yukawa potential energy is $V = \sqrt{C}\phi$. We can also show the meson field equation which can be written in the same way as electromagnetic field equations are

$$\nabla^2\phi - \frac{1}{R^2}\phi = 0 \dots\dots\dots(56)$$

With the solution

$$\phi = \sqrt{C} \frac{e^{-r/R}}{r} \dots\dots\dots(57)$$

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Which have the potential

$$\phi = \sqrt{C} \frac{e^{-r/R}}{r} = C \frac{e^{-r/R}}{r} \dots\dots\dots(58)$$

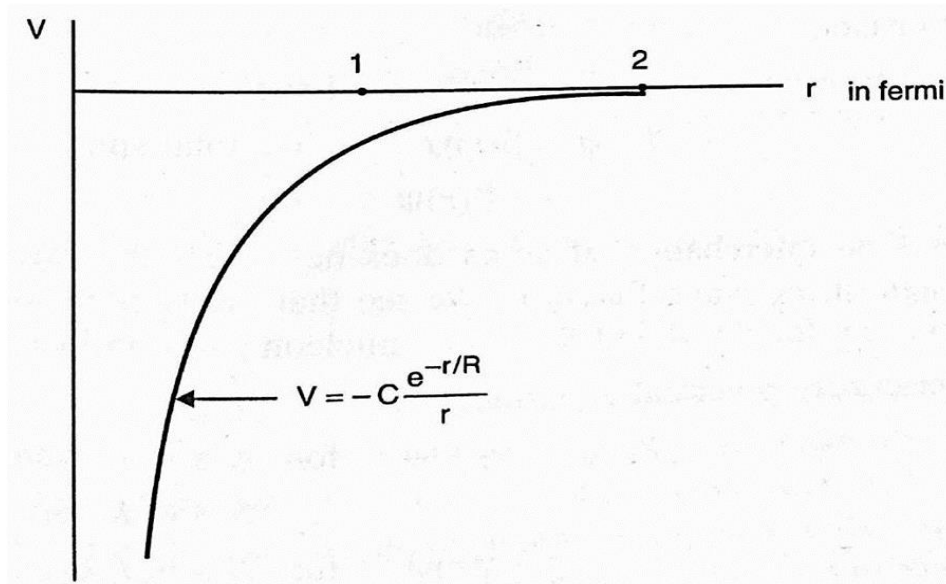


Fig.7: Yukawa Potential.

The Yukawa mesons were predicted to interact with nuclei quite strongly. It took around 12 years for physicists at Berkeley, University of California, to make the discovery of the mesons responsible for the strong nuclear interaction. These are referred to mesons (pi mesons). It has been discovered that there are three different kinds of mesons or pions

Salient Features of Nuclear Forces:

- (1) It is short range attractive force.
- (2) It is in general a non-central force.

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(3) They have property of saturation only i.e each nucleon interacts only with its nearest neighbors and not with all the constituents in the nucleus.

(4) They are charge independent i.e. $n - n, n - p, p - p$ have same nuclear forces.

(5) They are spin dependent.

(6) They are exchange forces proposed by Yukawa.

4.10 SUMMARY

After going through this unit, you would be able to achieve the following objectives

- Know the spin dependence and scattering length.
- Understand Pauli exclusion principle
- Differentiate between symmetric and antisymmetric wave functions
- Describe Isospin
- Solve questions based on Isospin
- Explain the Yukawa potential .
- Studied the meson theory of nuclear forces as proposed by Yukawa with experimental verified results predicted for nuclear forces.

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4.12 SUGGESTED READINGS

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4.13 TERMINAL QUESTIONS

1. Explain meson theory of exchange forces in detail..
2. What do you understand by Pauli exclusion principle?
3. Differentiate between symmetric and antisymmetric wave functions with examples.
4. Describe Isospin and discuss its utility.
5. Explain the Yukawa potential
6. Show that deuteron has no excited state.
7. Write short note on scattering length.
8. Explain the nature of Yukawa Potential.

UNIT 5**NUCLEAR MODELS**

Structure of the Unit

5.1 Introduction

5.2 Objectives

5.3 Liquid drop model

5.4 Semi Empirical Mass Formula

5.4.1 Formula for the total binding energy of a nucleus

5.4.2 Weizsacker's Semi Empirical formula

5.5 Nuclear Shell Model

5.5.1 Experimental evidences of Shell Model

5.5.2 Predictions about Shell Model

5.5.3 Spin-Orbit Coupling

5.5.4 Achievements and Applications of Shell model

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5.1 INTRODUCTION

When we use any nuclear model, there are numerous significant facts that need to be explained. How well a model's predictions are supported by experiments determines how valuable it is. Understanding the nature of the internucleon reaction is essential for comprehending the observable features of the nucleus. According to Yukawa's hypothesis, there is a highly powerful short-range force that acts between nucleons when the distance between the points is less than the interaction's range. This force is based on the exchange of pions between two nucleons. It should be noted that even if the precise nature of the inter-nucleon interaction had been understood, it would have been extremely challenging to construct a suitable explanation of the structure of the nucleus because the Schrodinger equation cannot be precisely solved for a system with that many bodies. Different models for the nucleus have been developed, each of which can explain some of the attributes of various categories of nuclei, due to the aforementioned challenges in constructing a satisfactory theory of the nucleus' structure.

The liquid drop model and collective model are based on the similarity to a drop of liquid, and the Fermi gas model and shell model are based on the similarity to a weakly interacting gas.

Some limiting characteristics of the nucleus can be explained by the various nuclear models that have been periodically offered. The nucleons are thought to influence and interact significantly only with their close neighbors in the liquid drop model of the nucleus.

Thus, the nuclear binding energy found to vary with mass number and heavy nuclei fission may be explained by the liquid drop model. The vast spacing of low-lying excited states in nuclei, however, could not be accounted for by the liquid drop model. This and other characteristics of the nucleus would force us to think about how individual nucleons move inside a potential well, leading to the formation of a nuclear shell structure like to the electronic shells in an atom.

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It is very likely that the nucleons in the nuclei are grouped in distinct shells, just as it is the case with the binding of the electrons in the atoms.

The nuclear shell model, developed in 1949 by Maria Mayer and separately by Haxel, O. Jensen, J.H.D., and Suess, H.E., was able to explain all the magic numbers under the assumption that individual nucleons had strong spin orbit forces. The development of the j-j coupling scheme was a crucial step that led to a series of independent particle states that matched the empirically discovered magic numbers.

The shell model is quite effective at predicting the excited states of nuclei and explaining the extent to which many nuclei are non-spherical, in addition to providing proof of the magic numbers. This suggests that, despite the shell model's reasonable representation of a nucleus, it is still a simplified model. A nuclear model, however, becomes an effective plan when it can explain certain experimental data.

5.2 OBJECTIVES

After studying this unit, you will be able to

- Understand liquid drop model
- derive the semi empirical formula
- Find the stable nucleus
- discuss the role of shell model
- understand magic number s
- Solve numerical based on shell model

5.3 LIQUID DROP MODEL

The liquid drop model of the nucleus was put forth by Neils Bohr and Wheeler in their theory of nuclear fission, which is based on Gamow's idea of a potential barrier. According to this model, an electrically charged, incompressible liquid drop with a mass that varies but has a very high

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density of 10^7 kg/m^3 can be used to represent an element's nucleus. Additionally, it is assumed that the nucleons in the nucleus are free to interact with their nearest neighbors, similar to how liquid molecules are free to move about a given intermolecular distance from their nearest molecules.

The nuclear forces have a number of characteristics, including a short range and a tendency to saturate. These forces, which resemble the characteristics of the forces that keep the liquid drop together, were inferred from the linear relationship between the binding energy and the volume of the number of particles in the nucleus. This theory has been used to support the analogies between the molecules in the liquid drop and the nucleons in the nucleus, which have been shown to be comparable.

These are the following assumptions on which liquid drop model is based upon

- (i) surface tension force of a liquid is analogous to the nuclear forces.
- (ii) the constant binding energy per nucleon is analogous to the latent heat of vaporization.
- (iii) the disintegration of nuclei by the emission of particles is analogous to the evaporation of the molecules from the surface of liquid.
- (iv) the energy of nuclei corresponds to internal thermal vibrations of liquid drop molecules and
- (v) the formation of compound nucleus and absorption of bombarding particles correspond to the condensation of liquid drops.

5.4 SEMI EMPIRICAL MASS FORMULA (SEMF)

The mass and binding energy of the nucleus have received a considerable deal of attention, thus a formula that would enable the computation of these quantities would be extremely useful and informative. Weizsacker devised such a formula, which is also known as the semi-empirical mass formula or the semi-empirical binding energy formula. Bohr and Wheeler were able to establish the stability limit against spontaneous fission using the Weizsacker formula.

5.4.1 Total binding energy formula of the nucleus

The total binding energy of a nucleus can be constructed by the following five energy terms

- (i) Volume energy
- (ii) Surface energy
- (iii) Coulomb energy
- (iv) Asymmetry energy and
- (v) Pairing and shell energy.

Various contributions to the binding energy of a nucleus are shown below.

(i) Volume energy

This first term that contributes mainly to the binding energy of the nucleus comes from a term which is proportional to the mass A , which can be shown as

$$E_V = a_V A \quad \dots\dots\dots(1)$$

Since the volume of the nucleus is also proportional to A , may be regarded as volume energy. In Eqn.(1) a_V is a constant called volume co-efficient.

(ii) Surface Energy

The nucleons on the surface of the drop interact only with half as many other particles as that done by the particles in the interior of the nucleus. So the binding energy is reduced because this nuclear surface. In Eqn.(1) it has been assumed that all nucleons are being attracted uniformly to all sides and hence it is necessary to subtract a term proportional to the surface area of the nucleus.

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The surface energy is proportional to the surface area of the nucleus, which is assumed to be spherical shape i.e.,

$$E_S \propto 4\pi R^2 \quad \dots\dots(2)$$

where R is the nuclear radius.

$$R = 1.07A^{1/3} \text{ fermi}$$

$$1 \text{ fermi} = 10^{-15} \text{ m}$$

By substituting the above values the expression for the surface energy of the nucleus

$$E_S = -a_s A^{2/3} \quad (3)$$

Where, a_s is called the surface co-efficient and can be evaluated using empirical data.

(iii) Coulomb Energy

The binding energy tends to decrease due to the Coulomb energy between the protons, and this tendency is represented by a term with a minus sign. Numbers represent the amount of work required to construct a nucleus against Coulomb repulsion. Considering the nucleus to be a uniformly charge spherical shell with a constant charge density $\rho = \left[\frac{Ze}{4/3\pi r^3} \right]$ upto their radius r.

The work dW which is required to bring the spherical shell up to its radius r is given by

$$dW = \frac{1}{4\pi\epsilon_0 r} \left[\frac{4}{3\pi r^3} \times 4\pi r^2 dr \rho \right] \quad \dots\dots\dots(4)$$

Now the Coulomb repulsion energy E_c can be calculated by integrating the Eqn.(4) between the limits $r=0$ and $r = R$. Thus Eqn.(4) can be written after integrating and substituting for the value of ρ . We have

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$$W = E_c = \frac{3/5Z^2e^2}{4\pi\epsilon_0R} \quad \text{.....(5)}$$

the total Coulomb energy of a nucleus of charge Z is given by

$$E_c = \frac{-3/5Z(Z-1)e^2}{4\pi\epsilon_0R} \quad \text{.....(6)}$$

The negative sign here shows the disruptive nature of this term Eqn.(6) now takes the form

$$E_c = -4a_c \frac{(Z-1)}{A^{1/3}}$$

$$\text{where, } a_c = \frac{3e^2}{5(4\pi\epsilon_0)R_0}$$

where ϵ_0 represent the permittivity of free space.

(iv) Asymmetry energy

This term arises because in heavy nuclei, the number of neutrons (N) are greater as compared to the number of protons (Z). On the other hand in light nuclei $N = Z$ and these nuclei are highly stable.

Without the Coulomb effect, a deviation from $Z = A/2$ would typically result in instability and a lower binding energy value.

As the number of neutrons rises, the nucleus becomes more asymmetrical, which brings about a force that lowers the volume energy.

The size of this effect might be expressed as a term proportional to the square of the neutron excess $(A-2Z)$ over protons. The symmetry effect is also inversely proportional to A, as established by a detailed analysis of it. Now

$$E_A \propto \frac{(N-2Z)^2}{A} \quad \text{.....(7)}$$

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Or

$$E_A = \frac{-a_A(N - 2Z)^2}{A} \quad \dots\dots(8)$$

where a_A . called the asymmetry co-efficient.

(v) Pairing Energy

Nucleons have a tendency to exist in pairs. Hence the nuclei with even number of protons and even number of neutrons are the most stable and the most abundant in nature. Nuclei with odd numbers of both protons and neutrons are the least stable, while nuclei for which either the proton or neutron number is even are not so much stable.

This pairing effect can be shown by

$$E_p = -a_p A^{-3/4} \quad \dots\dots(9)$$

It was empirically determined by Fermi. Eqn.(9) is called the pairing coefficient.

5.4.2 Weizsacker's Semi-Empirical Mass formula

Total binding energy (B.E) can be expressed by combining all the terms which are contributing towards the binding energy of the nucleus

$$\begin{aligned} \text{B.E.} &= E_V + E_S + E_C + E_A + E_P \\ \text{B.E.} &= a_V A - a_S A^{2/3} - \frac{a_C Z(Z-1)}{A^{1/3}} - \frac{a_A (A - 2Z)^2}{A} - a_P A^{-3/4} \quad \dots\dots(10) \end{aligned}$$

Above equation can also be expressed in terms of the nuclear (or atomic) mass, since the mass and binding energy are related by the relation

$$B.E. = (ZM_p + NM_n - M)c^2$$

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or

$$M = ZM_p + (A - Z)M_n - \frac{B.E.}{c^2} \quad \dots\dots(11)$$

where M_p , M_n and M are the masses of proton, neutron and nucleus respectively. By substituting Eqn.(11) in Eqn.(10), we get

$$M = ZM_p + (A - Z)M_n - \frac{a_v A}{c^2} + \left(\frac{4a_c}{c^2}\right) \times \frac{Z(Z - 1)}{A^{1/3}} + \left(\frac{a_s}{c^2}\right) A^{2/3} + \left(\frac{a_a}{c^2}\right) \times \frac{(A - 2Z)^2}{A} + a_p A^{-3/4} \quad \dots\dots(12)$$

Eqn.(12) is known as Weizsacker's semi-empirical mass formula.

Fig.1 summarizes the effect of different energy terms in deciding B.E/A of a nucleus.

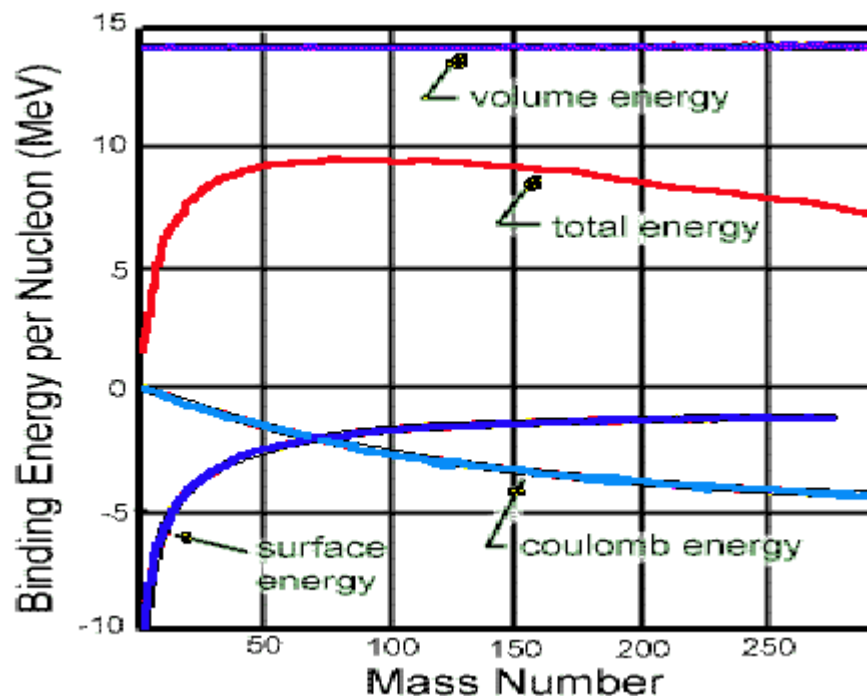


Fig. 1: Schematic representation of the various terms in the mass formula as a function of A

(source: <https://astarmathsandphysics.com>)

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The values of the co-efficient can be determined by a combination of theoretical observed calculations and adjustments to fit experimental values of the masses.

Typical values of the co-efficients are:

$$a_v = 14MeV$$

$$a_s = 13MeV$$

$$a_c = 0.60MeV$$

$$a_a = 19MeV$$

$$a_p = -33.5MeV \text{ for even-even nuclei}$$

$$= 0 \text{ for even-odd nuclei}$$

$$= +33.5MeV \text{ for odd-odd nuclei}$$

Examples 1: By Using the semi empirical binding energy formula calculate binding energy of ${}_{20}\text{Ca}^{40}$.

Solution : The semi empirical formula for binding energy is

$$B.E. = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(A - 2Z)^2}{A} + a_p A^{-3/4}$$

where, $a_v = 14MeV$; $a_s = 13MeV$; $a_c = 0.60MeV$

$$a_a = 19MeV \text{ and } a_p = -34MeV$$

as A is even = 40 and Z is even = 20.

$$a_v A = 14 \times 40 = 560MeV$$

$$a_s A^{2/3} = 13 \times 40^{2/3} = 13 \times 11.696 = 152MeV$$

$$a_c \frac{Z^2}{A^{1/3}} = 0.60 \times \frac{400}{40^{2/3}} = 0.60 \times 11.696 = 70MeV$$

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$$a_a \frac{(A-2Z)^2}{A} = a_a \times 0 = 0$$

$$a_p A^{3/4} = -34 \times 40^{-3/4} = -34 \times 0.063 = 2.14 \text{ MeV}$$

$$B.E. = 560 - [152 + 70 + 2.14] = 335.86 \text{ MeV}$$

Example 2. Calculate the atomic number of the most stable nucleus for a given mass number based on the liquid drop model. Hence explain why out of ${}^2\text{He}^6$, ${}^4\text{Be}^6$ and ${}^3\text{Li}^6$ only the last one is stable.

Stable nucleus. According to the liquid drop model the binding energy B.E. is given by

$$B.E. = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} + a_p A^{3/4}$$

The most stable nucleus for a given mass number A is that which has the maximum value of binding energy i.e., for which

$$\frac{(\partial B.E.)}{\partial Z} = -2a_c A^{-2/3} Z + 4a_a (A-2Z) A^{-1} = 0$$

Or

$$4a_a - 8a_a A^{-1} Z = 2a_c A^{-1/3} Z$$

Or

$$Z(4a_a + a_c A^{2/3}) = 2a_a A$$

$$Z = \frac{A}{2 + \frac{a_c}{2a_a} A^{2/3}}$$

Substituting the value of $a_c = 0.5053$ and $a_a = 23.702$ MeV, we get

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$$Z = \frac{A}{2 + 0.015A^{2/3}}$$

For light nuclei having small A the term $0.015A^{2/3}$ can be neglected as it is very small for stability $Z = A/2$ this result is confirmed experimentally.

Out of ${}^2\text{He}^6$, ${}^4\text{Be}^6$ and ${}^3\text{Li}^6$ it is only ${}^3\text{Li}^6$ for which $A=6$ and $Z=3$

i.e $Z=A/2$

Therefore it is the only stable nucleus out of the given three.

Example 3: For "mirror" nuclei which have N and Z differing by one unit, determine the mass difference. Consider A to be odd.

Solution : Mirror nuclei to be considered have the same odd value of A but the values of N and Z are interchanged such that they differ by one unit.

The $(A-2Z)$ value in the asymmetry term in mass formula can be written as,

$$A - 2Z = N + Z - 2Z = N - Z \text{ so that,}$$

If N and Z differ by one unit,

$$N - Z = \pm 1$$

$$\begin{aligned} M_{z+1} - M_z &= (M_p - M_n) [(Z+1) - Z] + \frac{a_c}{A^{1/3}} [(Z+1)^2] \\ &= M_p - M_n + a_c \frac{(2Z+1)}{A^{1/3}} \end{aligned}$$

$$M_{Z+1} - M_Z = M_p - M_n + a_c A^{2/3}$$

Since $A = 2Z+1$ for these mirror nuclei

Example 4: The masses of N^{15} and O^{15} are 15.000108u and 15.003070u respectively. Using this data, determine the Coulomb coefficient a_c in the semi-empirical mass formula.

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Solution : N^{15} and O^{15} are mirror nuclei, Therefore

$$M_{Z+1} - M_Z = M_p - M_n + a_c A^{2/3}$$

Using the respective data

$$(2.96 \times 10^{-3})u = (-0.000844) + a_c (15)^{2/3}$$

$$a_c = 3.542/6.08 \text{ MeV}$$

$$a_c = 0.58 \text{ MeV}$$

5.5 NUCLEAR SHELL MODEL

This model is similar to the Bohr model for electrons in the extra nuclear space. By analogy with the closed sub-shells and shells in the case of atoms it is assumed that nucleons also form similar closed sub-shells and shells within the nucleus.

5.5.1 EVIDENCES THAT LED TO SHELL MODEL

The following are the facts, that are favoring the shell model

1. Pairs of nucleons frequently form inside. A single unpaired nucleon can be taken out of the nucleus more easily than a paired one. Two protons and two neutrons combine to form a very stable nucleus. The fact that a particle has a significant binding energy of approximately 28.3 MeV lends support to this.
2. The graph between binding energy per nucleon and atomic number exhibits numerous kinks, one of which is illustrated as an example in the range $A = 126$ to 150 in Fig.2. These kinks are associated with a sharp increase in the binding energy per nucleon.

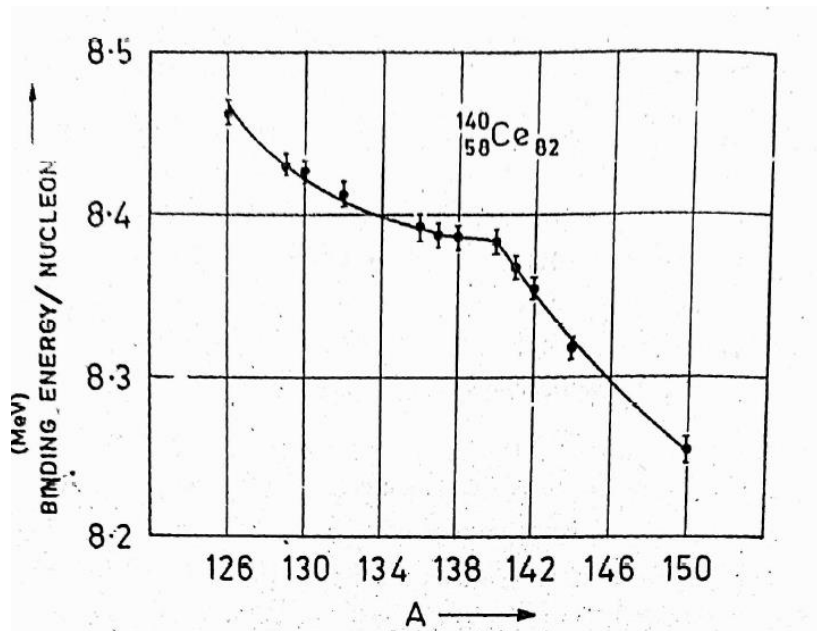


Fig. 2: Graph between binding energy per nucleon vs atomic number

These kinks or discontinuities have been found to occur whenever either the Neutron number or the proton number or both take the values 2,8,20,50,82 and 126. Nuclei containing 2,8,20,50,82 and 126 nucleons of the same kind known as Magic numbers, have a very high stability. For example, ${}^2_2\text{He}^4$ with $Z=N=2$ and ${}^8_8\text{O}^{16}$ with $Z = N = 8$ are highly stable. Similarly, the nuclei with 14,28 and 40 nucleons (semi-magic numbers) are slightly less stable but are more stable than the rest.

3. The most numerous nuclei are those with even numbers of both protons and neutrons; the least abundant are those with odd numbers of both protons and neutrons; and the intermediate types are those with odd numbers of one type and even numbers of the other type. High natural abundance is, of course, related to stability. Brown provided information regarding the relative abundances of nuclei in 1949 using information about the elements that make up the sun, the earth, and the stars.

The relative abundances of naturally occurring isotopes with nuclei that contain magic numbers of neutrons or protons are typically larger than 60%. For instance, the relative

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abundances of the isotopes ^{88}Sr ($N = 50$), ^{138}Ba ($N = 82$), and ^{140}Ce ($N = 82$) are 82.56%, 71.66%, and 88.48%, respectively. The stable end product of the natural radioactive series, Lead ^{208}Pb has underline $Z = 82$ and $N = 126$ both magic numbers.

4. In contrast to other elements, an element with a magic number of protons typically has a higher number of stable isotopes. For instance, whereas argon ($Z = 18$) and titanium ($Z = 22$) have 3 and 5 stable isotopes, respectively, calcium ($Z = 20$) has 6 stable isotopes. Again, the greatest number of stable isotopes is found in tin with $Z = 50$. This value is 10, as opposed to 8 for tellurium ($Z = 52$) and cadmium ($Z = 48$).
5. Compared to the nearby isotones, the number of naturally occurring isotones with the magic numbers of neutrons is typically high. For instance, at $N = 82$ there are seven stable isotones as opposed to three and two at $N = 80$ and two at $N = 84$, respectively. Similar circumstances exist at $N = 20, 28$, and 50 , which have 5, 5, and 6 isotones, respectively. These numbers are larger than in the cases of the nearby isotones.
6. The nuclei with the magic numbers of neutrons typically have low neutron capture cross-sections. The probability of these nuclei acquiring an extra neutron is low because their neutron shells are already full, as seen in Fig. 3. Similar to this, the cross sections for proton capture are small for nuclei with the magic proton numbers.

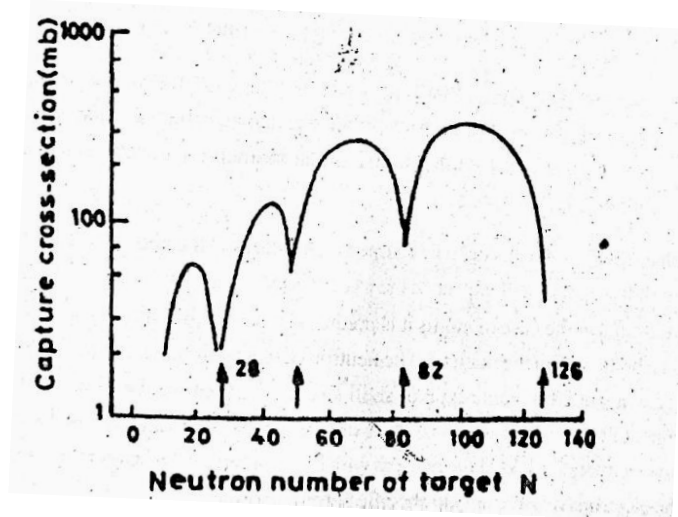


Fig.3: Neutron capture

7. If the heavy nuclei's disintegration energies are plotted as functions of mass number A for a given Z , a regular fluctuation is typically seen up until the magic neutron number $N = 126$, at which point there is a sharp discontinuity. This demonstrates the neutron number 126's magical properties.

8. β -emitters exhibit discontinuities at the magic proton or neutron values

9. The three lead isotopes, which all have the same magic number $Z = 82$ of protons in their nuclei, are the stable end products of all three naturally occurring radioactive series.

10. The earliest excited states of nuclei with magic numbers of neutrons or protons occur at energies higher than those of the nearby nuclei.

5.5.2 PREDICTIONS OF SHELL MODEL

Shell model predictions are given as:

1. Within the nucleus, nucleons arrange themselves into closed sub shells in a manner akin to that of atoms' electrons. Thus, shell model is the name given to it.
2. There is some sort of shell structure to the nucleons that make up the nucleus. Protons and neutrons in the proper quantity are used to complete the shells.
3. The extra nuclear electrons spin around in the nucleus' Coulomb electric field, which is meant to be heavy and far away. Just a few allowed orbits are used by electrons to rotate. Similar to this, each nucleon in the nucleus moves freely in a defined orbit under

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the influence of the central potential created by the average interaction amongst the remaining (A - 1) nucleons in the nucleus.

4. Each nucleon is predicted to have a spin angular = $\sqrt{s(s+1)}\hbar$ where s is the spin quantum number = 1/2 and an orbital angular momentum = $\sqrt{l(l+1)}\hbar$ where l is the orbital angular momentum quantum number having values 0, 1, 2, 3,... etc.

These facts have served as the foundation for the development of a nuclear shell model. The theory is comparable to the Bohr model for additional nuclear space electrons. It is expected that nucleons will likewise create similar closed sub shells and shells within the nucleus by similarity with the closed sub shells and shells in the case of atoms.

The protons and neutrons that make up the nucleus are meant to be arranged in a shell structure, and these shells are supposed to close with the right number of protons and neutrons. The excess nuclear electrons are predicted to spin under the nucleus' heavy and far-reaching Coulomb electric field. The electrons only rotate in certain specified orbits.

The theoretical understanding of the origin of the nuclear shell model is based on the assumption of the existence of a dominant spherically symmetric central field potential force governing the motion of the individual nucleons in the nuclei. From the compelling similarity of stability between the magic nuclei and the inert gases, it is predicted in the shell model that each nucleon moves independently inside the nucleus in a fixed orbit under the influence of a central field of force or a central potential V(r) produced by the average interaction between all the remaining (A - 1) nucleons in it.

It is assumed that the nucleons move in an average harmonic oscillator potential given by

$$V(r) = \frac{1}{2}kr^2 = \frac{1}{2}m\omega^2r^2 \quad \dots(13)$$

where m is the mass of the nucleon and ω the oscillator frequency. The 3-dimensional Schrodinger's wave equation for the harmonic oscillator can be solved using rectangular

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Cartesian co-ordinates by a generalization of the method. However, for the present purpose it is more appropriate to use spherical polar co-ordinates.

If the potential given by Eqn. 13 is substituted in the 3-dimensional Schrodinger equation, then the following radial equation can be obtained by solving it through the method of variables

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dR_t}{dr} \right) + \frac{2m}{\hbar^2} \left\{ E - V(r) - \frac{l(l+1)\hbar^2}{2Mr^2} \right\} R_t = 0 \quad \text{.....(14)}$$

where $R_l(r)$ is the radial function. The term $l(l+1)$ is the centrifugal potential. The angular part of the wave function is the spherical harmonic $Y_l^m(\theta, \phi)$ so that the total wave function is

$$\psi = R_{nl}(r) Y_l^m(\theta, \phi) \quad \text{.....(15)}$$

By employing the quantum mechanical approach, the 3-D harmonic oscillator problem can be solved. Thus, from the solution for the Eqn. 14 we find that the various energy levels are given by

$$E = (\lambda + 3/2)\hbar\omega \quad \text{.....(16)}$$

Further it can be shown that angular part of the wave function ψ requires that the oscillator quantum number λ is related to the orbital quantum number l and the radial part of the quantum number (similar to the principal quantum number of the electronic orbit), referred to as the radial quantum number n , by the relation

$$\lambda = 2(n-1) + l = 2n + l - 1 \quad \text{.....(17)}$$

The levels of different azimuthal quantum numbers l are designed by the usual symbols used in atomic spectroscopy, as given below:

1 : 0 1 2 3 4 5 6

Spectroscopic notation: s p d f g h i

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The nucleons are designed by marking their n values followed by spectroscopic notation giving the l value. The lowest energy level of the harmonic oscillator is the 1s level with n = 1, l = 0 its energy being $\frac{3}{2}\hbar\omega$ which is the zero-point energy. The level structures for the neutrons and the protons are similar.

5.5.3 Spin Orbit Coupling

In order to explain the disagreement at the higher magic numbers, Mayer and independently Haxel and Jensen suggested that a spin-orbit interaction term should be added to the central potential V(r) in Eqn.13.

Taking the spin-orbit interaction into account, the sequence of the energy levels is designated as follows:

$$1s_{1/2}; 1p_{3/2}, 1p_{1/2}; 1d_{5/2}, 1d_{3/2}; \dots 2s_{1/2}; 2p_{3/2}, 2p_{1/2}; \dots 2f_{7/2}, 2f_{5/2}; \text{etc.}$$

In accordance with the Pauli exclusion principle, each sub level of a given j can accommodate a maximum of (2j + 1) nucleons of either kind for which the magnetic quantum numbers m, are different. The possible values are m = j, j-1, ..., -j when a sub level of given j is completely filled up with (2j + 1) nucleons of particular kind, the extra nucleons of the same kind must go the next higher state of different j,

The group of sub levels (n,l,J) having energy values close to one another now constitute a shell. The number of nucleons required to fill up the shell is the sum of the nucleon numbers (2j + 1).

Using this scheme, the number of nucleons required to complete a sub shell can be calculated as follows:

The lowest level $\lambda=0$, according to this scheme is $1s_{1/2}$ with j = 1/2 which contains $(2 \times \frac{1}{2} + 1)$ or 2 nucleons.

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The next higher level with $\lambda = 1$ is a combination of two sub levels $1p_{3/2}$ and $1p_{1/2}$. The maximum number of nucleons which can occupy these sub-levels are 4 i.e. $(2 \times \frac{3}{2} + 1)$ and 2 i.e. $(2 \times \frac{1}{2} + 1)$ respectively, so that the total number of nucleons in the group of sub-levels is (4+2) or 6. So the shell closure takes place in this case with (2+6) or 8 nucleons.

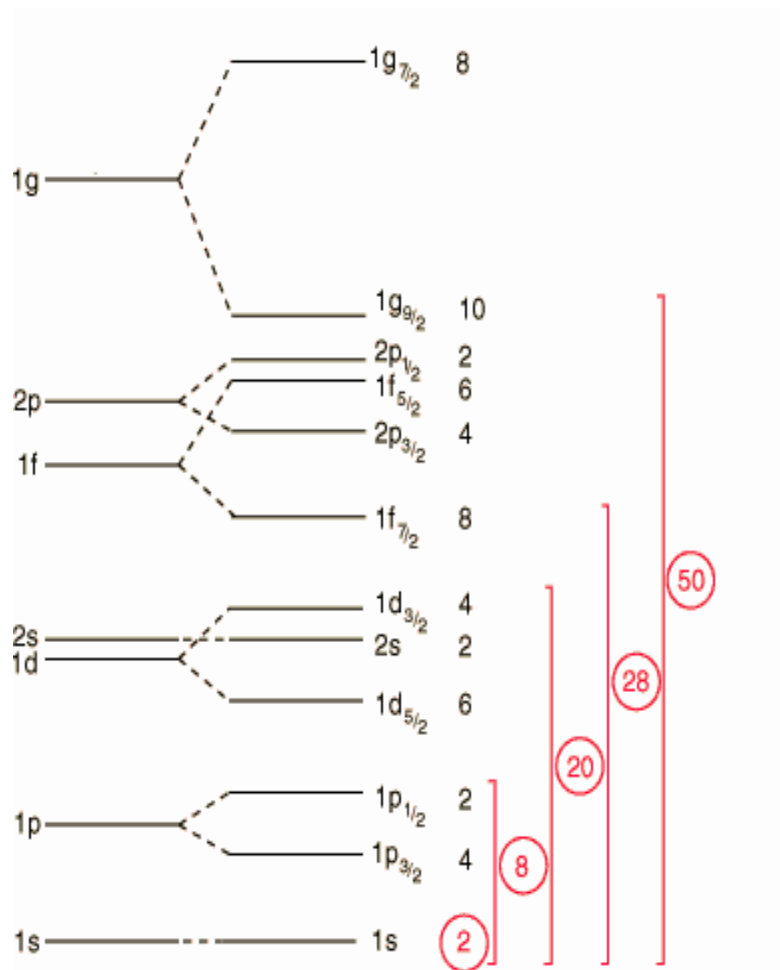


Fig. 04 : Representation of energy levels in shell model with magic numbers (<https://nuclear-power.com>)

5.5.4 Achievements of the Shell Model

The shell model in atomic physics can be used to predict the order and relative positions of excited states in addition to providing a good description of the atomic ground state. Using the nuclear shell-model, we may try to forecast the locations, angular moments, and parity of nuclear energy levels.

Magic numbers are explained by the shell model. By referring to Table 10.1 and adding the nucleons from below, it can be determined that the magic numbers 2, 8, 20, 50, 82, and 126 correspond to neutrons or protons when the shell ends.

The magnetic moments and ground state spins of the nuclei are successfully explained by the shell model. In order for the mechanical and magnetic moments of the neutrons and protons with opposing spin to cancel, they pair off. Thus, the odd or left proton or neutron that could not pair with one another contributes to the overall nucleus's spin and magnetic moment.

The j -values of the individual nucleons in it are added to get the total angular momentum quantum number l , also known as the spin quantum number, under the typical quantum conditions.

For example, ${}_1\text{H}^2$ has $Z=1$ i.e, one proton and $N = A - Z = 1$ neutron. Each nucleus will, therefore, be in the ground state $l = 0$ and will have a j -value $\frac{1}{2}$ for either. The j 's of the proton and the neutron are oriented parallel to each other so that $l = 1$ which agrees with the experimentally observed value of spin.

For ${}_1\text{H}^3$ and ${}_2\text{He}^3$ nuclei we have two nucleon of the same kind which are paired off and one nucleon of the other kind with j value $\frac{1}{2}$. Thus, the l value of each of these two nuclei is $1/2$.

The j values of the odd nucleon in the nuclei are used to explain the observed nuclear magnetic moments. Due to spin, the odd neutron has a negative magnetic moment, whereas the odd proton has a positive magnetic moment. Because the two extra neutrons are paired off and have no influence on the single odd nucleon that contributes to the nucleus' spin and magnetic moment,

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the spins and magnetic moments of isotopic nuclei with odd A that differ in their A values by two are the same. The nuclear spin and magnetic moment of ${}_{47}\text{Ag}^{107,109}$ and ${}_{55}\text{Cs}^{133,135,137}$ isotopes, for instance, are equal.

On the basis of closed shells, it is possible to explain the extremely high stability and low binding energy of nuclei. For instance, the first shell of ${}^2\text{He}_4$ is completely filled with protons and neutrons (2 of each), resulting in $l = 0$. Additionally, the closure of the shell gives it an extremely high binding energy of 28.3 MeV. The α particle ${}^2\text{He}_4$ nucleus has an extremely stable structure as a result. The case of ${}^8\text{O}_{16}$ is also comparable, with the first two shells of neutrons and protons completely occupied with 2 and 6 nucleons, for a total of 8 nucleons, respectively.

For maximum stability, the proton and neutron levels should be filled equally when adding nucleons to the nucleus. As a result, the number of neutrons and protons is almost equal for low values of mass number A . The most stable nuclides will once more develop as the number of nucleons increases when the lowest energy levels accessible to protons and neutrons are full. The number of neutrons will exceed the number of protons as the space between proton levels increases.

Due to the large variation in the nuclear spins of the isomeric states of these nuclei and the fact that their A values are close to the magic numbers needed to complete the various nuclear shells, the phenomenon of nuclear isomerism, or the existence of isobaric, isotopic nuclei in different energy states, has been explained on the basis of the shell model.

5.6 THE COLLECTIVE MODEL OF A NUCLEUS

Let's examine some of this model's features. Collective motions of all nucleons are taken into consideration in addition to the shell model characteristics. In essence, this combines elements of both the liquid drop model and the shell model. Consequently, a substantially greater amount of

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experimental data may be explained by the collective model. Nuclear deformation resulting in collective modes of excitation, such as collective oscillations and rotations, are two forms of collective effects that stand out in particular. Centrifugal and Coriolis forces are responsible for the less evident impacts.

Large quadrupole moments, which are seen in many nuclei, cannot be explained by the shell model, which necessitates large nuclear deformations. The nuclear quadrupole moment, which we defined before, is a measurement of the nuclear shape's departure from a sphere.

As a result, the experimentally observed quadrupole moment suggests the existence of a distorted, non-spherical nucleus. It was brought up during the debate of the shell model that the quadrupole moment of the nuclei corresponding to magic numbers is almost zero. In the region between closed shells, nuclei have significant quadrupole moments. They cannot be explained by a single particle shell model where the electromagnetic moments are determined by the final odd nucleon. Due to its spherical nature, the core does not contribute to the quadrupole moment.

$$Q = Z \int (3z^2 - r^2) \rho(\vec{r}) d^3r \quad \dots\dots\dots(18)$$

The quadrupole moment Q has dimensions of area.

If a homogeneously charged ellipsoid with charge Ze and semi-axes a and b is considered (b along z-axis), then,

$$Q = \frac{2}{5} Z (b^2 - a^2) \quad \dots\dots\dots(19)$$

For small deviation from spherical symmetry, we can talk about an average radius, $R_0 \approx \frac{a+b}{2}$

and $\Delta R = b - a$. Defining a deformation parameter $\delta = \frac{\Delta R}{R_0}$, the quadrupole moment becomes,

$$Q = \frac{4}{5} Z R_0^2 \delta \quad \dots\dots\dots(20)$$

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It is common to introduce a reduced quadrupole moment

$$Q_{reduced} = \frac{Q}{ZR_0^2} \dots\dots\dots(21)$$

For a uniformly charged ellipsoid, from

$$Q_{reduced} = \frac{4}{5} \delta \approx \text{deformation parameter } \delta.$$

In wave mechanics, $\rho(\vec{r})$ the probability density in Eq.18 has to be replaced by $\psi_j^* \psi_j$ where, j is the spin quantum number. The nuclear spin is taken along the z -direction so that $j=m$, the magnetic quantum number

Thus,

$$Q = Z \int \psi^* (3z^2 - r^2) \psi d^3r \dots\dots\dots(22)$$

To calculate Q within the framework of the single particle shell model, the wave function ψ is taken to be a single particle wave function $\psi_{n/m}$. It can be for example, the wave function of a particle in a harmonic oscillator potential well. It can be shown that,

$$Q = -\frac{2j-1}{2(j+1)} \langle r^2 \rangle \dots\dots\dots(23)$$

Where j =angular momentum quantum number of the single particle ($j \neq 1/2$).

$\langle r^2 \rangle$ = expectation value of the square of the radius of the single particle orbit.

For a single proton, $j = 3/2, 5/2, 7/2$, etc. and so, for order of magnitude calculation, for higher values of j , $\frac{2j-1}{2(j+1)} \approx 1$. Also, if we take, $\langle r^2 \rangle = R_0^2$, then,

$$Q \approx -R_0^2$$

$$Q_{reduced} (\text{single-particle}) = \frac{Q}{ZR_0^2} \dots\dots\dots(24)$$

$$\approx -\frac{1}{Z}$$

Now, experimentally the measured quadrupole moment of middle of the shell nuclei like ${}^{175}_{71}\text{Lu}$ is

$$Q_{reduced} ({}^{175}_{71}\text{Lu}) = +0.25$$

On the other hand, the single-particle estimate is (from Eq. 24).

$$Q_{reduced} (\text{single-particle}) ({}^{175}_{71}\text{Lu}) = -0.014$$

Giving,

$$\frac{Q_{reduced} (\text{Expt.})}{Q_{reduced} (\text{cal. single-particle})} \approx -18$$

The single-particle estimate of quadrupole moment is not only wrong in magnitude but also wrong in sign. On the other hand, for a doubly magic plus one proton nucleus like

$${}^{209}_{83}\text{Bi} \text{ (83 protons and 126 neutrons),}$$

$$Q_{reduced} (\text{Expt.}) \approx -0.014$$

And

$$Q_{reduced} (\text{cal. single-particle}) \approx -\frac{1}{Z} = -0.012 \dots\dots\dots(25)$$

These values are in good agreement.

Obviously, the shell model is not able to explain the observed large quadrupole moments for middle of the shell nuclei. For the nuclei like ${}^{175}\text{Lu}$, the core is not spherical (as assumed by shell

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model). It is permanently deformed by the nucleons in the outermost shell. Above equation give us an estimate of the deformation that produces the large quadrupole moment.

Because orbits are oriented arbitrarily in a closed shell configuration, the deforming effects of multiple nucleons balance out. The nucleus often changes its surface so that it aligns with the density distribution of the nucleons in the unfilled shell. The nucleus takes on a spheroidal equilibrium shape when there are enough nucleons outside the closed shells. Contradictory forces resulted in this equilibrium shape. The individual nucleons outside the closed shells that make up the core, on the other hand, attempt to change this core, which enables them to move in a deformed nuclear potential. The pairing forces, on the other hand, couple two identical nucleons in order to decrease their net angular momentum to zero.

This second effect has the tendency to yield a spherical symmetrical condition. More nucleons outside the closed shell core certainly has a positive impact on the first effect. In other words, the deformation of a nucleus rises as we move away from closed shells, leading to the observation of large quadrupole moments. Do other experimentally observed phenomena also exhibit nuclear deformation? Yes, it is clearly noticeable in the nuclei's rotational and vibrational spectra.

5.7 GLOSSARY

Nucleon: The proton and the neutron, constituting atomic nuclei.

Binding Energy: Amount of energy required to separate a particle from a system of particles or to disperse all the particles of the system.

Magnetic moment: Magnetic Moment is defined as magnetic strength and orientation of a magnet or other object that produces a magnetic field.

Electric Quadrupole Moment: A parameter which describes the effective shape of the ellipsoid of nuclear charge distribution.

5.8 SUMMARY

In this unit liquid drop model of the nucleus explained. Semi empirical mass formula has also been explained by using binding energy formula. Along with it some numerical problems has also been solved for the convenience of learners. Shell model and its role in significance of magic numbers is explained by considering magic numbers the basis of spin orbit interaction theory. Some achievements of the shell-model is discussed in this unit as well. The collective model which is the combined form of the liquid drop model which is totally built on the foundation that the collective nucleon motion dominates over individual nucleon motion with we have shell model where nucleons are assumed to move completely independently. These models based on apparently opposite assumptions, are found to be very useful in understanding what actually goes on with in the nucleus. The evidence of the existing magic numbers is beyond of the range of the liquid drop model of nucleus.

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5.10 SUGGESTED READINGS

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5.11 TERMINAL QUESTIONS

1. Derive an expression for the semi-empirical formula of Weizsacker.
- 2 Draw the graph and show that the binding energy per nucleon is the sum of volume, surface, Coulomb and asymmetry energies.
3. Calculate the binding energies and atomic mass of the nuclides of Ca^{40} , Sn^{120} and Pb^{208}
4. Calculate the binding energies for the mirror nuclei K^{39} and Ca^{39} from the semi empirical binding energy equation. What do you infer from the energies about the relative stabilities?
5. Give an account of the assumptions of the Liquid drop model proposed by N.Bohr and Wheeler.
- 6.Obtain an expression for the total binding energy of a nucleus based on the liquid drop model.
7. Calculate the binding energies of the nuclei - Be^9 , Al^{27} , Cu^{63} , Mo^{98} , Xe^{130} , W^{184} , U^{238} , from the semi empirical binding energy formula and compare the results with the experimental values.

UNIT 6**RADIOACTIVITY**

Structure of the Unit

6.1 Introduction

6.2 Objectives

6.3 Properties of Radioactive Rays

6.4 The Law of Radioactive Decay

6.4.1 Unit of Activity

6.5 Radioactive Growth and Decay

6.6 Ideal Equilibrium

6.7 Transient Equilibrium and Secular Equilibrium

6.8 Radioactive Series

6.9 Determination of the Age of the Earth

6.10 Carbon Dating-Archaeological Time Scale

6.11 Summary

6.12 References

6.13 Suggested Readings

6.14 Terminal Question

6.1 INTRODUCTION

In this unit learners will study about the basic properties of Radioactivity. As we know that information about atomic nuclei started as early as 1896 with the discovery of radioactivity.

In 1895, Roentgen discovered the X-rays. The French scientist Becquerel got interested in Roentgen's work. Becquerel was seized of the fact that the production of X-rays was always accompanied by fluorescence from the material of the X-ray tube (glass). He thought that X-rays existed whenever there was fluorescence. To investigate this problem, Becquerel took uranium sulphate, which fluoresces under the action of sunlight. He found that fluorescent uranium sulphate did give out rays, which could affect a photographic plate even when wrapped in thick black paper. Becquerel argued that the fluorescent salt had given rise to X-rays, which had penetrated the black paper and affected the photographic plate.

But he soon saw that he was mistaken. During one such experiment the sky happened to be overcast and the uranium salt was hardly fluorescent. On developing the photographic plate, Becquerel was surprised to see a dark spot on it, as before. He had obviously stumbled on some new kind of rays (1896) which could penetrate the thick wrapper and affect the photographic plate. It was soon established that any salt of uranium emits Becquerel rays. Unlike the X-rays, which appear in an X-ray tube only under special conditions, the Becquerel rays are emitted in a spontaneous manner.

Marie Curie found that pitchblende, the ore from which uranium is extracted, emits Becquerel rays with a much stronger intensity than what its uranium content would. After a long and laborious process of chemical separation, Marie Curie and her husband Pierre Curie discovered two new elements, polonium and radium, which emitted Becquerel rays.

They gave the name 'radioactive' to all substances capable of emitting, Becquerel rays and phenomenon itself came to be known as 'radioactivity'. the phenomenon itself came to be known as 'radioactivity'.

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The discovery of radium was a great event, as it was found to be about a million times more radioactive than uranium. This power of radium radiation made it possible to study radioactivity systematically.

6.2 OBJECTIVES

In this unit learners will study about

- Properties of Radioactive Rays
- The Law of Radioactive Decay
- Radioactive Growth and Decay
- Ideal Equilibrium
- Transient Equilibrium and Secular Equilibrium
- Determination of the Age of the Earth
- Carbon Dating-Archaeological Time Scale

6.3 PROPERTIES OF RADIOACTIVE RAYS

(1) By a calorimetric experiment, Curie estimated that one gm of radium liberates 140 calories in one hour. Though small, this energy is released continuously over a very long period of time.

(2) Radioactive rays ionize the surrounding air and affect photographic plates.

(3) Radioactive rays act differently on different cells and tissues. Cells that multiply are most readily destroyed by these rays. This outstanding discovery made radium an invaluable aid to physicians to fight tumours, particularly the discovery made radium an

(4) Fluorescence is produced in substances like zinc sulphide particularly cancerous growths. Adding minute quantities of radium to, say, zinc sulphide, we can get a compound, by continuously

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luminous in the dark. This can be used to produce get a compound that is gun sights and instrument pointers (coating) for those to produce luminous watch dials, be read in the dark.

(5) Rutherford found that a beam of radioactive rays from radium sample splits into three components in a strong magnetic or electric field. See Fig radium sample splits into three

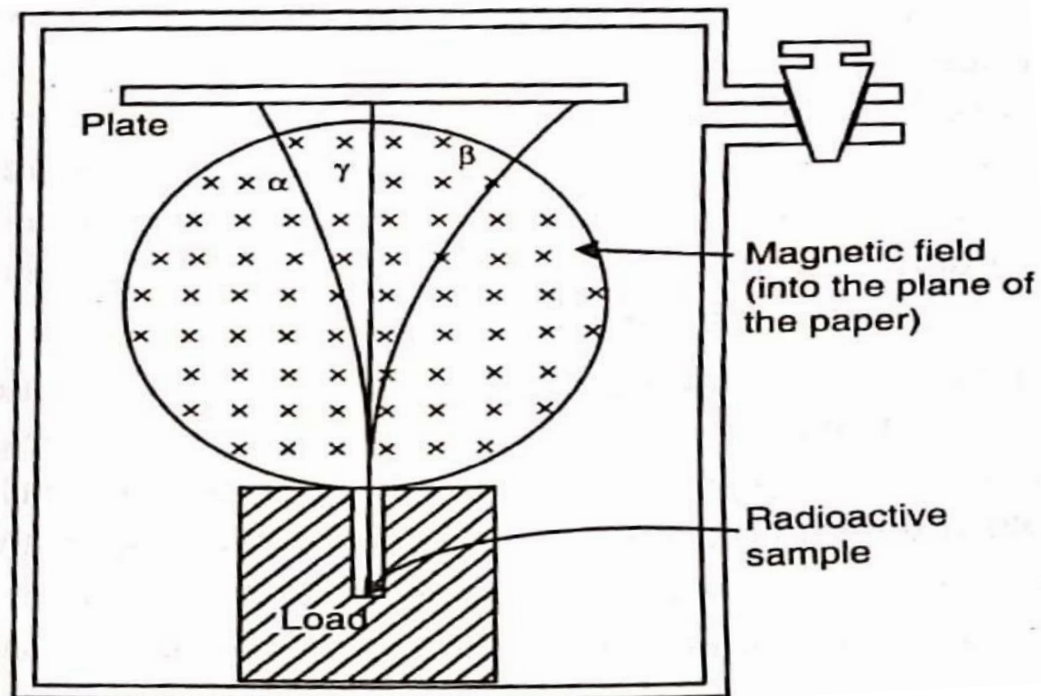


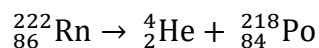
Fig. 1. Deflection of α, β, γ rays

(a) The α – rays (particles) are the nuclei of helium atoms. This identification was made by Rutherford and Royds in 1909. By the spectroscopic method, they found traces of helium in an originally pure sample of radon gas, which is an alpha emitter. When an electric discharge was passed through a tube containing pure radon gas, initially only characteristic radon lines appeared.

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After a day Rutherford found that, Radon lines became somewhat weaker and new lines started appearing. These new lines were identified with helium spectrum. As the days passed, the radon spectrum began weakening while the helium lines grew brighter. Thus, for the first time, people 'saw' decay of an element (radon) and 'birth' of a new element, helium. Such a transformation in which a parent element gives rise to a new element-called the daughter product-by emitting radioactive rays is called a radioactive transformation.

For the above example of radon, we can write the radioactive transformation equation as:

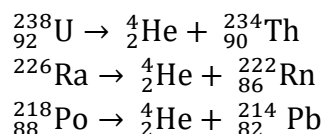


Parent element \rightarrow α -particle + daughter product

During a radioactive transformation, the mass number and total charge is conserved.

So we can say that radioactivity is a nuclear phenomenon. In other words, the radioactive rays come out of the atomic nucleus.

A few examples of alpha decay are,



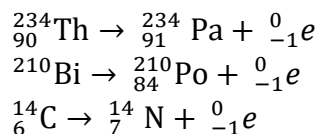
Alpha rays can be stopped by a thin sheet of paper. On the other hand, they cause intense ionization in air. Most α -particles are emitted with velocities between $\sim 1.5 \times 10^7$ m/s and $\sim 2.2 \times 10^7$ m/s. Any group of α -particles emitted from the same type of nuclei always has a definite velocity and hence a definite energy.

The alpha particles cover a definite distance in a given material, practically without any loss of intensity and then suddenly in a small distance are absorbed completely. The definite distance they travel within a given material is called their range in that material.

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(b) The β rays are identical with electrons. A β -particle therefore has a mass (1/1836) of mass of a proton.

A few examples of β decay are:



Notice that the mass number and charge are conserved and the daughter product moves one place up in the periodic table, as loss of a negative charge by a nucleus would imply gain of a positive charge. Beta rays cause much less ionization in air, but are ~ 100 times more penetrating than α -rays. They can penetrate a sheet of aluminium a few mm thick.

The velocities of β -particles emitted from various nuclei range up to $0.99c$, where $c = 3 \times 10^8$ m/s, is the velocity of light. A particular β -active element emits β -particles with energies varying between zero and a certain maximum. This maximum energy is called the end-point energy.

(c) The γ -rays are part of the electromagnetic spectrum. They have wavelengths smaller than those usually associated with the X-rays. Thus, usually γ -ray photons (energy of a photon, $E = hf = hc/\lambda$) are more energetic than the X-ray photons and are even more penetrating than the X-rays. (They are ~ 100 times more penetrating than β -rays.) The wavelength of γ -ray photons ranges between $\sim 1.7 \times 10^{-8}$ cm and $\sim 4 \times 10^{-6}$ cm.

The ionization due to γ -rays is a photoelectric effect. Owing to their large energies, the γ -ray photons can dislodge electrons not only from outer orbits (valence orbits or conduction bands) of the atoms but also from the inner orbits. Besides this photoelectric effect, γ -rays lose energy by

- (i) Compton scattering, in which the γ -photon collides with an electron and gets scattered with a shift in wavelength $[\Delta\lambda = h/m_0c(1 - \cos \alpha)]$.

- (ii) (ii) pair production in which a γ -photon is converted into a pair consisting of an electron and a positron. For this, the energy of the γ -ray has to be $> 1.02\text{MeV}$.

6.4 THE LAW OF RADIOACTIVE DECAY

When a nucleus disintegrates by emitting a particle (α or β) or a gamma ray, or by capturing an electron from the atomic shell (K -capture), the process is called radioactive decay. This decay is spontaneous.

By using a Geiger Counter, it is possible to study how radioactive decay depends on time. Let us take a radioactive sample containing N_0 nuclei at time $t = 0$, i.e., at the beginning. We wish to calculate the number N of these nuclei left after time t .

The number of nuclei of a given radioactive sample disintegrating per second is called the activity of that sample.

$$\begin{aligned} \therefore \frac{dN}{dt} &= \text{Rate of decrease of nuclei with time} \\ &= \text{Activity at time } t. \end{aligned}$$

Experimentally, it is found that the activity at any instant of time t is directly proportional to the number N of parent type nuclei present at that time.

$$\begin{aligned} -\frac{dN}{dt} &\propto N \\ -\frac{dN}{dt} &= \lambda N \end{aligned} \quad \dots\dots\dots(1)$$

where $\lambda > 0$, is the proportionality constant.

The negative sign indicates that N decreases as t increases.

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For unit time, from Eq.1, we get,

$$\lambda = \left(-\frac{dN}{N} \right)$$

i.e., λ is fractional change in N per sec. and we see that λ is not a mere proportionality constant, but it gives us the probability of decay per unit interval of time. Hence λ is called the probability constant, the decay constant or the disintegration constant.

Notice that dN is the number of parent nuclei that decay between times t and $t + dt$, and that we have taken N as a continuous variable.

From Eq.1, by integration we get,

$$\int_{N_0}^N \frac{dN}{N} = - \int_0^t \lambda dt$$

$$N = N_0 e^{-\lambda t}$$

Here, N_0 = Number of radioactive nuclei at $t = 0$.

Thus we see that the law of radioactive decay is exponential in character.

Notice that only half the amount of radon present initially remains after 3.83 days; only one-fourth of it remains after 7.66 days and only one-eighth remains after 15.32 days.

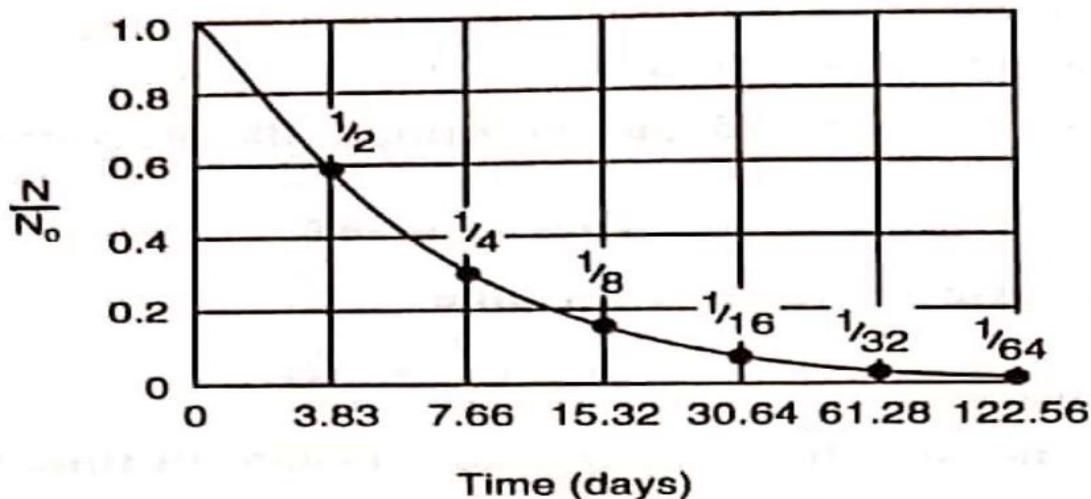


Fig. 2. Decay curve for radon, (${}^{222}_{88}\text{Rn}$).

The plot shows that in a fixed time interval (say, 3.83 days for radon) a fixed fraction (1/2 for radon) of the amount of radioactive substance at the beginning of the interval decays. This fraction is independent of the amount of radioactive substance or whether you have a freshly prepared sample or not and instead depends only on the interval of time. This is a characteristic of the exponential nature of the law of radioactive decay. Figure 2 depicts a typical experimental decay curve for radon.

The decay constant λ is a characteristic of radioactive substance and it depends in no way on the amount of the substance present.

Half Life (T) : It is convenient to define a time interval during which half of a given sample of radioactive substance decays. This interval is called the half life or half value period of that substance, denoted by T .

$$\begin{aligned}
 \frac{N}{N_0} &= \frac{1}{2} = e^{-\lambda T} \\
 e^{\lambda T} &= 2 \\
 \lambda T &= \log_e 2 = 0.693 \\
 T &= \frac{0.693}{\lambda}
 \end{aligned}
 \quad \dots\dots\dots(2)$$

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Mean Life (τ) : Individual radioactive atoms may have life spans between zero and infinity. Hence it is meaningful to talk about the average or mean life τ , defined as,

$$\tau = \frac{\text{Total life time of all nuclei in a given sample}}{\text{Total number of nuclei in that sample}} \dots\dots\dots(3)$$

To evaluate this we have from Fig.3.

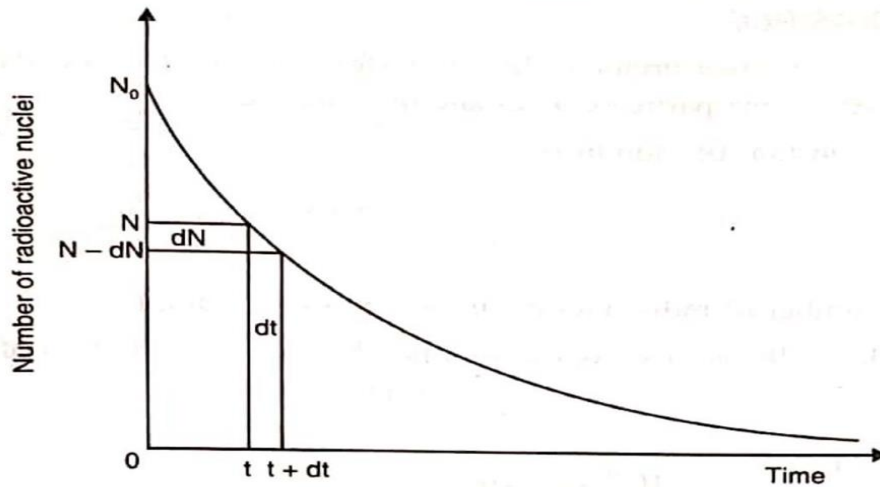


Fig.3: Curve shows how dN number of nuclei decay in time dt .

From the curve, one can see that each of the dN number of radioactive nuclei has lived a life of t seconds, i.e., the total life span of dN nuclei is $(dN \cdot t)$ seconds.

\therefore We can put Eq. 3 as,

$$\begin{aligned} \tau &= \frac{\int_{N_0}^0 t dN}{\int_{N_0}^0 dN} \\ &= \frac{-N_0 \lambda \int_0^\infty t e^{-\lambda t} dt}{-N_0}, \quad \because dN = -\lambda N dt = -\lambda N_0 e^{-\lambda t} dt \\ &= \lambda \int_0^\infty t e^{-\lambda t} dt, \end{aligned}$$

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which on integration by parts becomes,

$$\tau = \lambda \cdot \frac{1}{\lambda^2} = \frac{1}{\lambda} \quad \dots\dots\dots(4)$$

Example 01: According to measurements by Rutherford and Geiger, one gram of radium emits in one second 3.7×10^{10} alpha particles. Estimate the half life of radium.

Solution: The decay constant of radium is,

$$\lambda_{\text{Ra}} = \frac{3.7 \times 10^{10}}{2.7 \times 10^{21}} \text{sec}^{-1}$$

where 2.7×10^{21} = Number of radium atoms in one gram of radium.

(\because There are 6.02×10^{23} atoms [Avogadro's number] in one gm-atom of radium-226)

$$\therefore \lambda_{\text{Ra}} = 1.37 \times 10^{-11} \text{sec}^{-1}$$

$$\therefore \text{Half life, } T_{\text{Ra}} = \frac{0.693}{\lambda_{\text{Ra}}} \cong 5 \times 10^{10} \text{sec} \cong 1600 \text{ yrs.}$$

6.4.1 Unit of Activity

The most commonly used unit is the curie. It was originally based on the rate of decay of a gram of radium. Experiments have yielded the result that there are-about 3.7×10^{10} disintegrations per second per gram of radium. This number is taken as a standard and is called the curie. Thus by definition,

$$\text{One curie} = 1\text{Ci} = 3.7 \times 10^{10} \frac{\text{disintegrations}}{\text{sec}}$$

This is applicable to all types of nuclear disintegrations.

A Curie of activity is a very strong source of radiation.

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Thus, one has

and

$$\begin{aligned} 1 \text{ millicurie} &= 1 \text{ mCi} = 10^{-3} \text{ Ci} \\ 1 \text{ microcurie} &= 1 \mu\text{Ci} = 10^{-6} \text{ Ci}. \end{aligned}$$

Sometimes one uses another unit for activity, called the rutherford.

$$\begin{aligned} 1 \text{ rutherford} = 1 \text{ rd} &= 10^6 \frac{\text{disintegrations}}{\text{sec.}} \\ 1 \text{ mrd} &= 10^{-3} \text{ rd} \\ 1 \mu\text{rd} &= 10^{-6} \text{ rd} \end{aligned}$$

Activity can also be defined in terms of N as

$$\text{Activity} = \left| \frac{dN}{dt} \right| = \lambda N = \frac{0.693}{T} N$$

Thing to remember is that a very short-lived substance gives rise to large activity, even if it is present in minute quantities.

Example 02 : Calculate the activity of (i) One gram of radium ${}_{88}^{226}\text{Ra}$, whose half life is 1622 years and (ii) 3×10^{-9} kg of active gold, ${}_{79}^{200}\text{Au}$, whose half life is 48 mins. **Solution:**

$$\begin{aligned} \text{(i) } N &= 1 \text{ gm} \left(\frac{1 \text{ gm} - \text{mole}}{226 \text{ gm}} \right) \left(6.03 \times 10^{23} \frac{\text{atoms}}{\text{gm-mole}} \right) \\ &= 2.66 \times 10^{21} \text{ atoms} \end{aligned}$$

$$\text{and } \lambda = \frac{0.693}{T}$$

$$= \frac{0.693}{1622 \times 365 \times 24 \times 60 \times 60} \text{ sec}^{-1}$$

$$= 1.355 \times 10^{-11} \text{ sec}^{-1}$$

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$$\begin{aligned} \therefore \text{Activity} &= \lambda N = 3.604 \times 10^{10} \frac{\text{disintegrations}}{\text{sec}} \\ &= 0.974\text{Ci} \\ &\sim 1\text{Ci} \end{aligned}$$

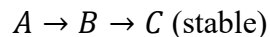
$$\begin{aligned} \text{(ii) } N &= (3 \times 10^{-6} \text{gm}) \left(\frac{1 \text{gm-mole}}{200 \text{gm}} \right) \left(6.03 \times 10^{23} \frac{\text{atoms}}{\text{gm-mole}} \right) \\ &= 9.04 \times 10^{15} \text{ atoms.} \end{aligned}$$

$$\begin{aligned} \text{and } \lambda &= \frac{0.693}{T} \\ &= \frac{0.693}{48 \times 60} \text{sec}^{-1} \\ &= 2.406 \times 10^{-4} \text{sec}^{-1} \end{aligned}$$

$$\begin{aligned} \text{Activity} &= \lambda N = 2.18 \times 10^{12} \frac{\text{disintegrations}}{\text{sec}} \\ &= 58.9\text{Ci} \end{aligned}$$

6.5 RADIOACTIVE GROWTH AND DECAY

Let us consider decay of the type,



Our aim is to find out the abundance of substance B if A decays to B and B decays to C .

Let N_A be the number of nuclei of A type at any instant. Assume that originally only A type was present and the initial number of nuclei of A type be N_0 . is zero.

Let N_B be the number of nuclei of B type at the same instant t . At $t = 0$, the initial number B is formed as a result of decay of A .

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Therefore, the number of nuclei entering the B category is,

$$-\frac{dN_A}{dt} = \lambda_A N_A \dots\dots\dots(5)$$

where λ_A is decay constant of A type.

The number of nuclei leaving the B category is,

$$= \lambda_B N_B$$

where λ_B is decay constant of B type.

\therefore The net change in number of nuclei (per sec) of the B category is,

$$\lambda_A N_A - \lambda_B N_B = \frac{dN_B}{dt} \dots\dots\dots(6)$$

Now,

$$\begin{aligned} N_A &= N_0 e^{-\lambda_A t} \\ \frac{dN_B}{dt} &= \lambda_A N_0 e^{-\lambda_A t} - \lambda_B N_B \end{aligned}$$

Multiplying by the integrating factor, $e^{\lambda_B t} dt$

$$e^{\lambda_B t} dN_B = \lambda_A N_0 e^{(\lambda_B - \lambda_A)t} dt - \lambda_B N_B e^{\lambda_B t} dt \dots\dots\dots(7)$$

L.H.S. can be now integrated by parts.

$$\int e^{\lambda_B t} dN_B = e^{\lambda_B t} N_B - \int \lambda_B N_B e^{\lambda_B t} dt \dots\dots\dots(8)$$

Integrating Eq. 7, and substituting Eq.8, we get,

$$\begin{aligned}
 e^{\lambda_B t} N_B &= \frac{\lambda_A}{\lambda_B - \lambda_A} N_0 e^{(\lambda_B - \lambda_A)t} + C \\
 N_B &= 0, t = 0 \\
 C &= - \left(\frac{\lambda_A N_0}{\lambda_B - \lambda_A} \right) \dots\dots \\
 N_B &= \frac{N_0 \lambda_A}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t}) \dots\dots\dots (9)
 \end{aligned}$$

Which is the desired result, giving the number of nuclei of B type present at any time t . The decay of A type and growth and decay of B type is shown in Fig.4.

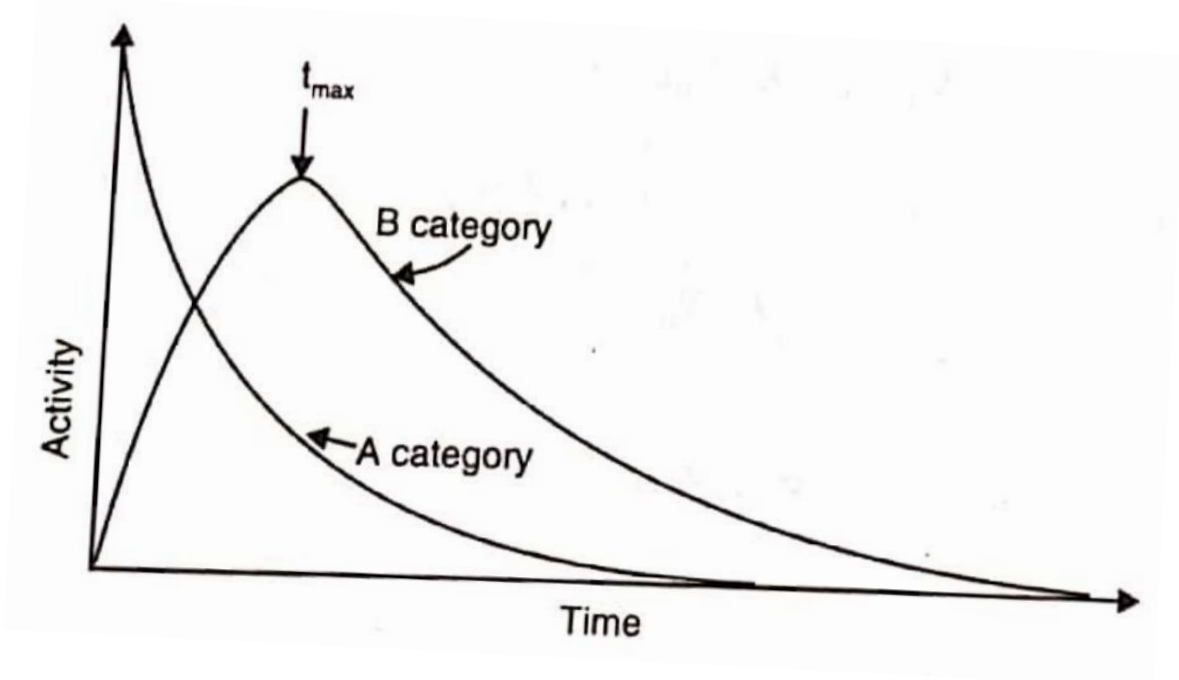


Fig.4: Decay and growth of radioactivity.

Example 3: Show that if there are initially N_0 radioactive nuclei of the parent present, the time at which the number of radioactive daughter nuclei is maximum is:

$$t_{\max} = \frac{\log (\lambda_B / \lambda_A)}{\lambda_B - \lambda_A}$$

Solution: We know that,

$$N_B = \frac{N_0 \lambda_A}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t})$$

For N_B to be maximum,

$$\frac{dN_B}{dt} = 0 = \frac{N_0 \lambda_A}{\lambda_B - \lambda_A} [-\lambda_A e^{-\lambda_A t_{\max}} + \lambda_B e^{-\lambda_B t_{\max}}]$$

$$t_{\max} = \frac{\log(\lambda_B/\lambda_A)}{\lambda_B - \lambda_A}$$

6.6 IDEAL EQUILIBRIUM

From Eq.6, at t_{\max} we have

$$\lambda_A N_A = \lambda_B N_B \quad \dots\dots\dots(10)$$

since

$$\frac{dN_B}{dt} = 0$$

Thus at time t_{\max} , (and only at t_{\max}), the activity of parent and activity of accumulated daughter are equal.

At t_{\max} (as considered in example above), we have:

$$\begin{aligned} \lambda_A N_A &= \lambda_A N_0 e^{-\lambda_A t_{\max}} \\ &= \lambda_A N_0 e^{-\lambda_A \frac{\log(\lambda_B/\lambda_A)}{\lambda_B - \lambda_A}} \\ &= \lambda_A N_0 \left(\frac{\lambda_A}{\lambda_B}\right) e^{\frac{\lambda_A}{\lambda_B - \lambda_A}} \\ &= \lambda_A N_0 \left(\frac{T_B}{T_A}\right) e^{\frac{T_B}{T_A - T_B}} \quad \dots\dots\dots(11) \end{aligned}$$

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When the activities of parent and daughter are, equal, the situation is called ideal equilibrium. Note that this situation exists only at the moment when, time = t_{\max} .

By Fig. 4 where dN_B/dt is positive between times, $t = 0$ and $t = t_{\max}$. This implies that parent activity in this time range always exceeds daughter activity. Conversely between times $t = t_{\max}$ and $t = \infty$, dN_B/dt is negative, implying that the daughter activity exceeds the activity of its parent.

6.7 TRANSIENT EQUILIBRIUM AND SECULAR EQUILIBRIUM

Let us proceed by considering, the daughter shorter-lived than the parent. i.e

$$T_A > T_B.$$

From Eq. 9 the activity of B type is,

$$\lambda_B N_B = N_0 \lambda_A \frac{\lambda_B}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t})$$

Since

$$\lambda_A N_A = \lambda_A N_0 e^{-\lambda_A t},$$

$$\lambda_B N_B = (\lambda_A N_A) \frac{\lambda_B}{\lambda_B - \lambda_A} (1 - e^{-(\lambda_B - \lambda_A)t}) \quad \dots \dots \dots (12)$$

$$\frac{\lambda_B N_B}{\lambda_A N_A} = \frac{T_A}{T_A - T_B} (1 - e^{-[(T_A - T_B)/T_A] \lambda_B t}) \quad \dots \dots \dots (13)$$

For large t , Eq.13 becomes,

$$\frac{\lambda_B N_B}{\lambda_A N_A} = \frac{T_A}{T_A - T_B} \quad \dots \dots \dots (14)$$

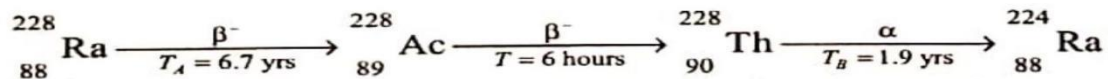
When the ratio, $\frac{T_A}{T_A - T_B}$ is greater than one, from Eq.13, it is clear that for large t ,

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Eq.14 should hold. Since we have $T_A > T_B$ such is the case. When the Eq.14 holds, we say that transient equilibrium exists between parent and the daughter. Learners can also see that according to Eq. 14 the ratio of daughter and parent activities is constant.

A prominent example of transient equilibrium is the following decay:



${}_{89}^{228}\text{Ac}$ has half life of ~ 6 hours and promptly decays into ${}_{90}^{228}\text{Th}$

Thus, we can ignore the presence of ${}_{89}^{228}\text{Ac}$ as an intermediate product. From Eq.14; the activity ratio corresponding to equilibrium is,

$$\begin{aligned} \frac{\lambda_B N_B}{\lambda_A N_A} &= \frac{\text{Activity of } {}_{90}^{228}\text{Th}}{\text{Activity of } {}_{88}^{228}\text{Ra}} \\ &= \frac{T_A}{T_A - T_B} \\ &= \frac{6.7}{6.7 - 1.9} \\ &= 1.39 \quad \dots \dots \dots (15) \end{aligned}$$

i.e., it is > 1 . of ${}_{90}^{228}\text{Th}$.

Thus it is an example of transient equilibrium. Figure 5 shows decay of ${}_{88}^{228}\text{Ra}$ and growth of ${}_{90}^{228}\text{Th}$.

Notice from the Fig. 5 that at larger values of time, the activity of the daughter approaches its transient equilibrium value of 1.39 times the remaining parent activity (as given by Eq.15).

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If the daughter is longer-lived than the parent, i.e., if $T_A < T_B$, from Eq.13 it can be seen that, the ratio $\frac{\lambda_B N_B}{\lambda_A N_A}$ continuously increases as time t increases.

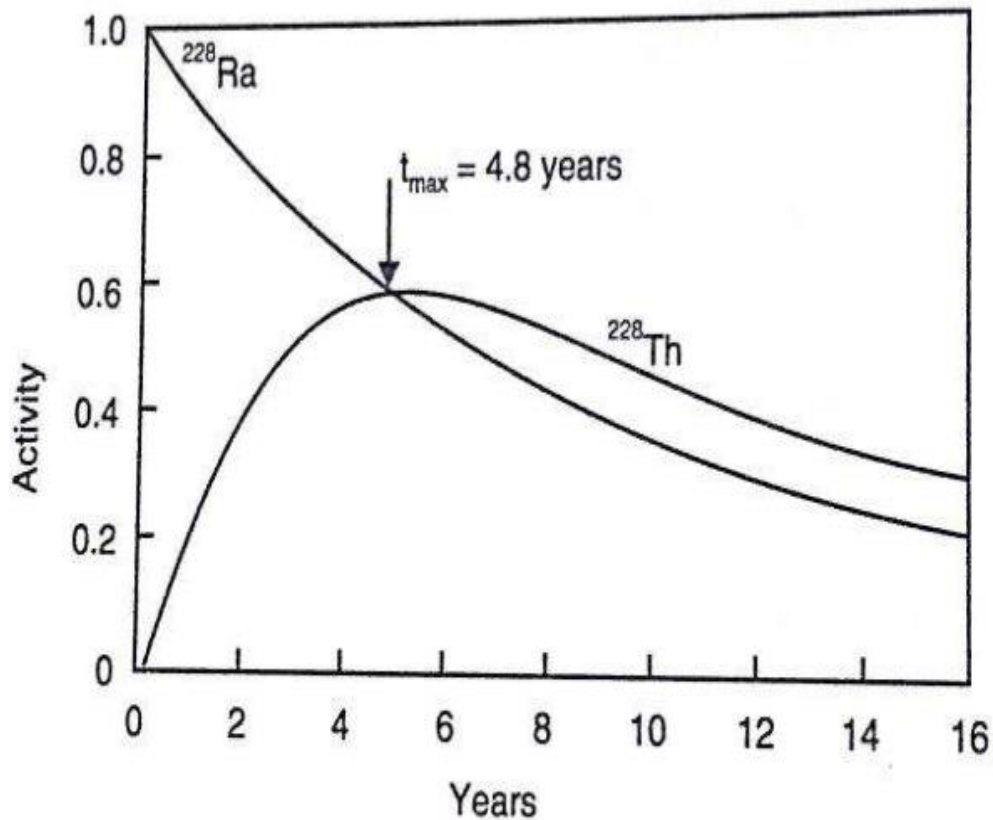
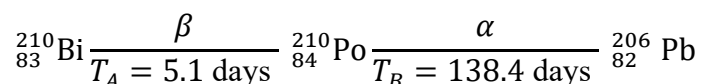


Fig.5: Transient equilibrium between ^{228}Ra parent and ^{228}Th daughter. (source is pure ^{228}Ra)

In other words, after sufficient time, the activity of the daughter becomes independent of the residual activity of the parent, and there can be no equilibrium between them. This is indicated in Fig. 6 for the case of



Let us now consider the important case of the daughter being much shorter-lived than parent. Here, and Eq. 12 becomes

$$\lambda_A \ll \lambda_B \quad (T_A \gg T_B)$$

$$\lambda_B N_B = \lambda_A N_A (1 - e^{-\lambda_B t}) \quad \dots\dots\dots(16)$$

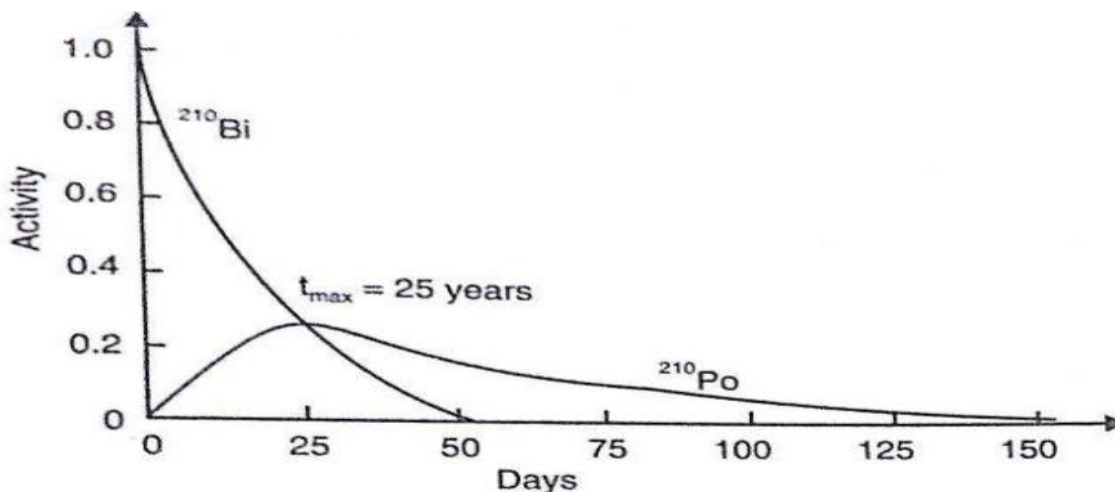


Fig.6: There is no equilibrium between ${}^{210}\text{Bi}(5.1d)$ and ${}^{210}\text{Po}(138.4d)$.

that is the daughter activity is controlled only by its own decay constant.

It can be at once seen that for,

the exponential,

$$\begin{aligned}
 t &\gg T_B, \\
 e^{-\lambda_B t} &= e^{-\frac{0.693}{T_B} t} \\
 &\cong 0
 \end{aligned}$$

and so

$$\lambda_B N_B = \lambda_A N_A \quad \dots\dots\dots(17)$$

It should be kept in mind that Eq. 17 is valid only if $T_A \gg T_B$ and $t \gg T_B$.

Thus in these cases, we have seen that when the daughter activity equals the parent activity and the equilibrium is called secular equilibrium.

For a very long-lived parent, on physical grounds, the result given by Eq. 17 is expected. In this case, the activity of the parent can be taken as almost constant and so the rate of production of the daughter too is constant. As the daughter quantity grows (accumulates), its rate of decay increases and finally catches up, after sufficient time, with the rate of its production. When the two rates become equal, we say that the daughter is in secular equilibrium with the parent, and,

giving,

$$\begin{aligned}
 \lambda_B N_B &= \lambda_A N_A \\
 \frac{\lambda_B}{\lambda_A} &= \frac{T_A}{T_B} = \frac{N_A}{N_B} \quad \dots\dots\dots(18)
 \end{aligned}$$

Therefore in the case of secular equilibrium, the ratio of the number of parent nuclei and daughter nuclei is constant, and is equal to the ratio of their half lives.

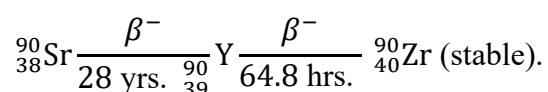
This explains why the percentage of radium contained in uranium was always experimentally found to be the same-an average of one gram of radium per 3.2 tons of pure uranium.

From Eq. 18

$$\begin{aligned}
 \frac{N_{\text{Ra}}}{N_{\text{U}}} = \frac{T_{\text{Ra}}}{T_{\text{U}}} &= \frac{1620 \text{ yrs.}}{4.5 \times 10^9 \text{ yrs.}} \quad (T_{\text{U}} \gg T_{\text{Ra}}) \\
 &= 3.6 \times 10^{-7}
 \end{aligned}$$

giving the experimentally observed ratio of 1gm to 3.2 tons. Actually, radium is not the immediate daughter of uranium and we have to consider a radioactive series. However, uranium has the longest half life in the series and hence all other products which are successively formed (except the end-product) are in secular equilibrium with uranium and their per cent content in a given sample of uranium is constant, equal to the ratio of their half lives as given by Eq. 18.

Another example of secular equilibrium, (which is not a part of a naturally occurring radioactive series) is,



6.8 RADIOACTIVE SERIES

Each series is formed by successive daughter products, all ultimately derived from a single parent. Radioactive nuclei found in nature are said to exhibit natural radioactivity. There are now a few thousand radioactive isotopes which have been produced in the laboratory, mostly by neutron bombardment. These are said to exhibit artificial radioactivity. Compared to this, the number of naturally occurring radioactive nuclides is quite small, about 70 .

When naturally occurring radioactive isotopes were studied, scientists found that they could be divided into four series (isotopes between $Z = 81$ and $Z = 92$).

The reason that there are exactly four series is a consequence of the fact that alpha decay reduces the mass number of nucleus by 4 .

Therefore, the nuclei whose mass numbers are all given by

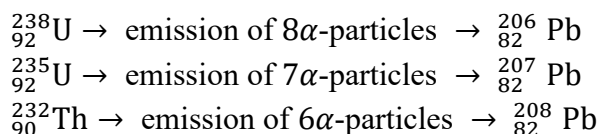
$$A = 4n.$$

where n is an integer, can decay in descending order of mass number.

6.9 DETERMINATION OF THE AGE OF THE EARTH

Radioactivity is the best clock we can employ to estimate the absolute age of the earth, as it is totally unaffected by environmental change or natural upheavals like earthquakes, storms etc. The determination of geological ages is done very often by the so-called lead methods.

It involves the following nuclear processes:



To illustrate how an estimate of time can be made, consider the ${}_{92}^{238}\text{U}$ series. The stable end-product is ${}_{82}^{206}\text{Pb}$, and so we can take $\lambda_{\text{pb}} = 0$. The half life of ${}^{238}\text{U}$ is 4.5×10^9 years. Hence after sufficient time, (say, a billion years) the only elements present in any appreciable amount will be uranium and lead. This is because, all elements in the uranium series will be in secular equilibrium with the parent ${}^{238}\text{U}$ and only ${}^{206}\text{Pb}$ will not be in equilibrium, and hence quantity of Pb will continuously go on increasing while uranium will continuously get depleted. This is indicated in Fig. 2.13 which depicts the flow of water from a reservoir into a collector, through a chain of other tanks in between. The diameters of the connecting pipes determine the 'probability' of decay (rate of flow). The situation is analogous with decay of a radioactive series, finally ending in a stable product.

Therefore, it is possible for us to apply Eq.9 not only to the first and second elements ($A \rightarrow B$) but also to the first and last. (${}^{238}\text{U}$ and ${}^{206}\text{Pb}$ in the case referred).

Equation 9 is:

$$\begin{aligned}
 N_B &= \frac{N_0 \lambda_A}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t}) \\
 \lambda_A &= \lambda_U \\
 \lambda_B &= \lambda_{Pb} = 0 \text{ as Pb is stable.} \\
 N_B &= N_{Pb} \\
 N_0 &= N_v \\
 N_{Pb} &= -N_v (e^{-\lambda_U t} - 1) \\
 &= N_v (1 - e^{-\lambda_U t}) \quad \dots \dots \dots (19)
 \end{aligned}$$

^{238}U ore always contains ^{206}Pb , which can be assumed to be of radioactive origin (^{206}Pb is the end-product).

\therefore Present number of Pb atoms + Present number of U atoms

= Number of U atoms originally present.

i.e.,

$$N_{pb} + N_U = N_v \quad \dots \dots \dots (20)$$

Eqs. 19 and 20 can be solved simultaneously to give:

$$t = \frac{1}{\lambda_U} \log \left(\frac{N_{Pb} + N_U}{N_U} \right) \dots \dots \dots (21)$$

Thus, by spectro-chemically analyzing a sample and knowing its uranium and lead content, it is possible to estimate the age of that sample. Of course, the most important factor is to ensure that no helium or uranium has escaped the rock sample during its lifetime.

The oldest surface rocks have been found to have an age of about 3×10^9 years.

When the same method is applied to determine the age of meteorites, it is found that oldest of these are about 4.5×10^9 years. This corresponds to the age of the earth and is different from the segregation of surface rocks.

Another method may be mentioned. It consists of using the $^{206}\text{Pb}/^{207}\text{Pb}$ isotopic ratio of radiogenic lead.

Using Eq. 19 it is easy to see that

$$\frac{{}^{206}\text{Pb}}{{}^{207}\text{Pb}} = \frac{{}^{238}\text{U}(1-e^{-\lambda_U t})}{{}^{235}\text{U}(1-e^{-\lambda'_U t})} \dots\dots\dots(22)$$

where

$$\begin{aligned} \lambda_U &= \lambda \text{ of } {}^{238}\text{U} \\ \lambda'_U &= \lambda \text{ of } {}^{235}\text{U}. \end{aligned}$$

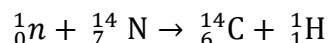
From Eq. 22, t can be evaluated.

The main disadvantage of this method is that radon which forms in the decay of ${}^{238}\text{U}$ has a half life of 3.82 days and being gaseous, might escape from the system.

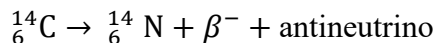
Thus for this method or the first method, the rock samples have to be taken from massive pitchblende, deeply buried. In such a case, the gas would get trapped in rock cavities and radon ($T = 3.82$ days) cannot be expected to diffuse very far from the dense rocks.

6.10 CARBON DATING-ARCHAEOLOGICAL TIME SCALE

Cosmic rays continuously form ${}^{14}_6\text{C}$ in atmosphere. The nuclear reaction is



${}^{14}_6\text{C}$ is a neutron-rich (6 protons and 8 neutrons) isotope of carbon, which is β^- active.



Half life of ${}^{14}_6\text{C}$ is 5730 years.

Thus ${}^{14}_6\text{C}$ would not have been present in the atmosphere, had it not been continuously replenished.

${}^{14}_6\text{C}$ gets combined with hydrogen and oxygen and eventually finds its way into all organic matter.

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Since ^{14}C has a half life of 5730 years it is an ideal radioactive isotope for studying the age of civilisations and is extensively used in archaeology and anthropology.

When an animal or plant dies, its intake of carbon stops and from that moment decay of ^{14}C is the only process that continues. Coal and petroleum are organic in nature, but they are so old that there is no trace of ^{14}C in them. This has been established by experiments. In a young animal or tree, ^{14}C activity is same as atmospheric carbon which is about 15 disintegrations per gm per min.

By carefully measuring the ^{14}C activity of a fossil or dead tree, it is possible to estimate its age. This is shown in the problem given below.

EXAMPLE 04: In an archaeological expedition, charcoal from an ancient fire-pit was excavated. This sample showed a ^{14}C activity of 11.3 counts per gm per min. The absolute activity of ^{14}C in a living tree is independent of species and it is ~ 15.3 counts per gm per min. Estimate the age of the charcoal sample.

Solution: We have,

$$11.3 = 15.3e^{-\lambda t}$$

where

$$\lambda = \frac{0.693}{T} = \frac{0.693}{5730 \text{ years}}$$

$$\begin{aligned} 1.354 &= e^{\lambda t} = e^{0.000121t} \text{ where } t \text{ is in year} \\ t &= \text{age of charcoal sample in years} \\ &= \frac{\log_e 1.354}{0.000121} \text{ years} \\ &= 2504.65 \text{ years.} \end{aligned}$$

6.11 SUMMARY

In this unit learners have learned about the basic Radioactive α -, β -, and γ - rays and its properties. The Law of Radioactive Decay have also been discussed along with Unit of Activity. Radioactive Growth and Decay has been graphically explained with suitable example. Ideal Equilibrium, Transient Equilibrium and Secular Equilibrium with examples has been discussed. How the Age of the Earth can be determined with the help of Radioactive phenomenon has also been discussed in this unit.

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6.13 SUGGESTED READINGS

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2. "Nuclear Two Body Problems and Elements of Nuclear Forces" Experimental Nuclear Physics by N. F. Ramsey, Wiley: New York.
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6.14 TERMINAL QUESTIONS

1. Explain the properties of the radioactive decay.
2. Discuss the law of radioactive decay.
3. Give examples to illustrate the generality of the law of exponential decay and growth.
4. Differentiate between the transient equilibrium and secular equilibrium.
5. Discuss how the age of the earth can be determined by using the phenomenon by radioactivity.

UNIT 7

ALPHA DECAY

Structure of the Unit

7.1 Introduction

7.2 Objectives

7.3 The α -Spectrum and Fine Structure

7.3.1 Long Range α - Particles

7.4 Q- value in the α -decay

7.5 Range of α particles

7.6 Range-Energy (velocity) relationship

7.7 Geiger-Nuttal law.

7.8 Gamow's theory of α -decay

7.9 Selection rules in Alpha Decay

7.10 Summary

7.11 References

7.12 Suggested Readings

7.13 Terminal Questions

7.1 INTRODUCTION

The alpha decay is one of the earliest nuclear processes and a typical type of radioactivity in heavy nuclei. A Coulomb barrier of more than 20 MeV is experienced by an alpha particle when it collides with a heavy nucleus like uranium, whereas alpha particles produced by uranium have energies of less than 5 MeV. Energy conservation laws appear to be broken in the area of the potential barrier by this. The close link between half-life and corresponding decay energy, or Q value, is another significant aspect of the α -decay. For understanding the above observed phenomena, a theory has to be developed because the classical theory was not able to explain the alpha emission process.

7.2 OBJECTIVES

After studying the unit learners will be able to:

- explain the Range of α particles
- derive Range-Energy (velocity) relationship
- describe Geiger-Nuttal law.
- Explain Gamow's theory of α -decay
- Describe α -particle energies and selection rules

7.3 THE α -SPECTRUM AND FINE STRUCTURE

When alpha particles are emitted from one distinct energy level of the parent nucleus to another distinct energy state of the product nucleus, they are said to be mono-energetic. When it comes to some nuclei, it has been noticed that the same radioactive material emits many groups of particles, each with a distinct energy. The transitions from the various discrete energy states of the parent nucleus to the various discrete energy states of the daughter nucleus make this possible. The emission of particles is caused by an energy transition between two distinct nuclear energy states; the parent nucleus' starting energy state and the daughter nucleus' final energy state are responsible,

and as a result, the emission spectrum is a line spectrum. These α particles emission give rise to the so called-'fine structure' of a spectrum. We can group such α transitions into two types. As an example, the decay of ${}_{90}^{228}\text{Th}$ as shown in Fig.1.

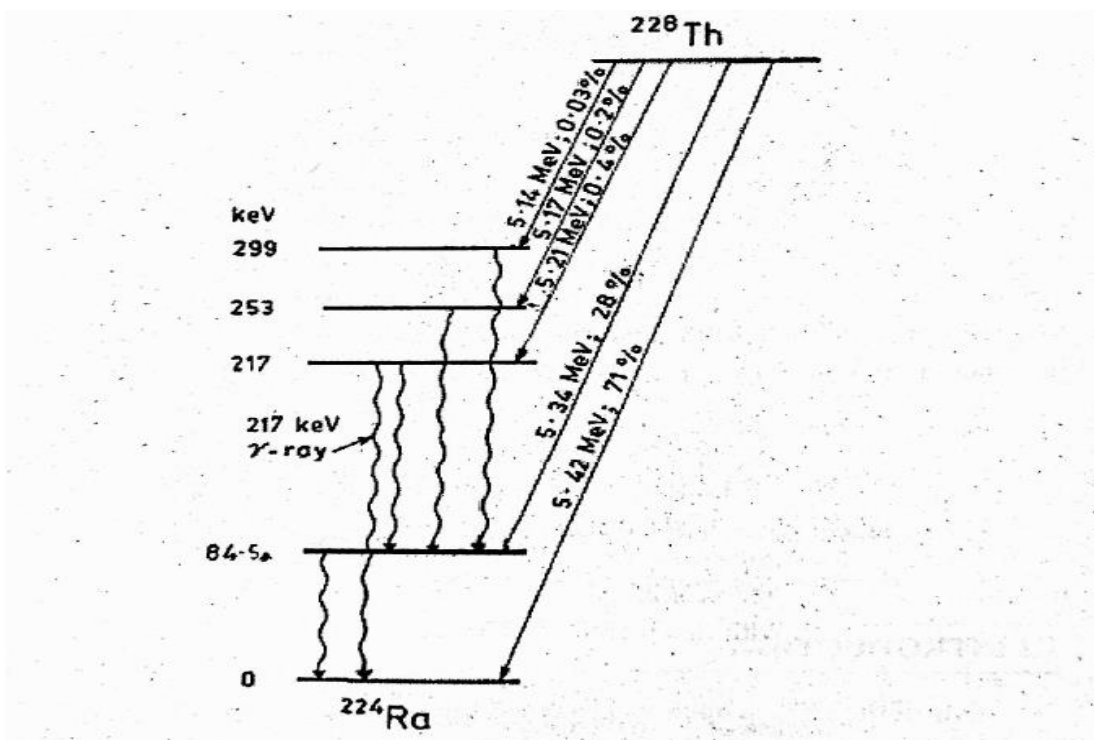


Fig. 1 Five groups of α - particles, indicated by slanting lines, emitted by ${}^{228}\text{Th}$.

The transitions in this case are from a single, clearly excited energy level of the parent nucleus (${}_Z\text{X}^A$) to many excited states of the product nucleus (${}_{Z-2}\text{X}^{A-4}$). Particles are divided into five groups according to their energy. Out of them, fifth groups of 5.42 MeV energy α -particles leave the daughter product in ground state, whereas four groups of α -particles leave the daughter nucleus in excited state. By emitting rays, which are shown by vertical lines, the daughter nucleus reaches from its excited states to the ground state.

7.3.1 Long Range α - Particles

The usual or main category of α -particles are those that are emitted by a parent nucleus in ground state and leave the daughter nucleus in ground state. Long-range α -particles are those released by an excited parent nucleus that leave the daughter product in the ground state, as in the instance of ^{212}Po 's emission, which is illustrated in Fig.

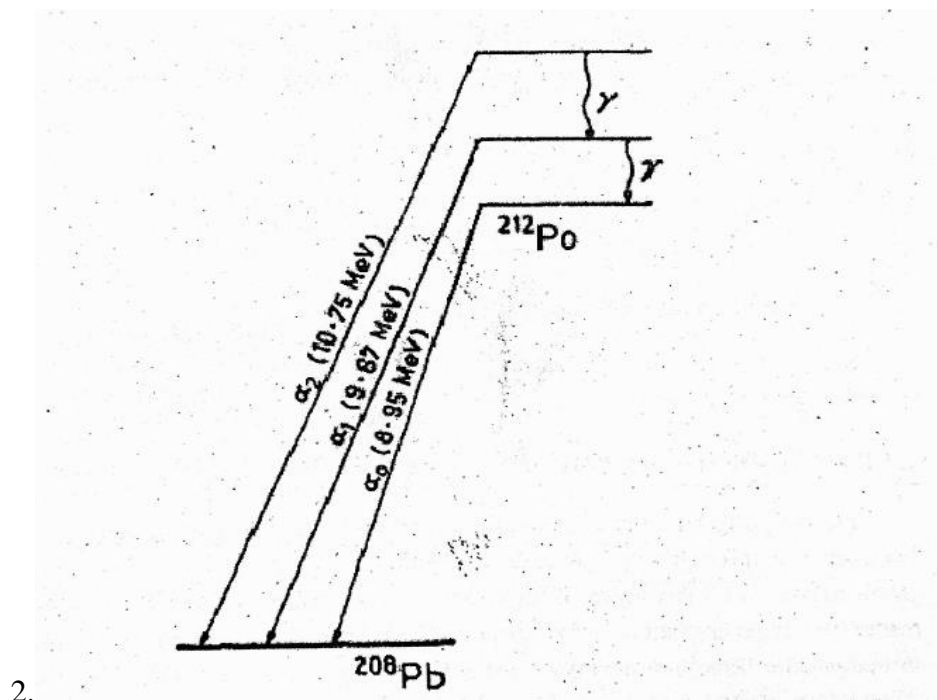


Fig. 2 Long Range α -particles emitted by ^{212}Po

This is because the energy of excitation becomes available to the α -particle, when the daughter nucleus reaches its ground state. The α -emitter is known to emit a few α -particles (α_1 , α_2) with much greater energies than those of main group (α_0) as shown in Fig.2

This is due to the fact that when the daughter nucleus reaches its ground state, the energy of excitation becomes available to the α -particle. According to Fig. 2, the α -emitter is known to emit a few α -particles (α_1 , α_2) with energies that are significantly higher than those of the main group (α_0).

Remember, in case of normal emission, the transition is from ground state of parent to ground state of daughter. In the case of long-range α -particle emission, the transitions are from excited states of parent nucleus to ground state of daughter nucleus directly. The emission of long-range α particles provides information about energy levels of the parent nucleus. The emission of short range α particles due to transition from ground state of parent nucleus to excited states of daughter nucleus (Fig.1) tells about the energy levels of daughter nucleus.

7.4 Q- VALUE IN THE α -DECAY

The Q value in the α -decay process is the total energy released in the decay process and is called "disintegration energy" which can be calculated by considering the conservation of momentum and the energy Q-value is given by

$$Q_{\alpha} = K_{\alpha} \left(\frac{m_{\alpha}}{m_d} + 1 \right) \quad \dots\dots(1)$$

with

K_{α} = Kinetic energy of α particle

m_{α} = mass of α particle

m_d = mass of the daughter nucleus

It is quite reasonable to replace the ratio of masses with the ratio of the mass numbers. The new expression for Q_{α} can be written as

$$Q_{\alpha} = K_{\alpha} \left(\frac{A}{A-4} \right) \quad \dots\dots(2)$$

with A = mass number of parent nucleus

Since usually A is large and so $Q_{\alpha} \approx K_{\alpha}$, i.e. the α particle carries away most of the disintegration energy.

7.5 RANGE OF α -PARTICLES

The α -particles released by naturally occurring radioactive elements are quickly absorbed by matter. They are able to pass through thin foils made of mica or aluminium as well as thin papers. They can only get through the top few layers of such material, though. As they go through stuff, they lose energy. After a given distance in matter, the mono energetic particles lose all of their energy by subsequent collisions (substance). The term "range" refers to the maximum distance a particle in a substance can go before losing all of its energy. It is expressed in unit of length(cm).The range is seen to be very small in solids and liquids (10^{-3} mm for energy of a few MeV) and large in gases (few centimetres). Increases in temperature and pressure have opposite effects on the range of the gas, respectively.

By measuring the ionisation that particles cause along their passage through the medium at various points along their range, they can be identified. Another approach is to take photos of their tracks in the Cloud Chamber.

7.6 RANGE-ENERGY (VELOCITY) RELATIONSHIP

Rutherford pointed out the fact that there appears to be a systematic relationship between the half-period and the range (R) of emitted α -particles. If the half-period is shorter, it seems like the particle velocity (energy) will be higher (large decay constant). The following mathematical relationship between the measured values of the ranges and energy of the α -particles has been established.

$$R = \alpha E^{\frac{3}{2}} \quad \dots(3)$$

This empirical relationship, valid in a limited energy range (3 to 7 MeV) is known as Geiger's law. For the energy (E) in MeV and the range (R) in meter, the value of the constant $a = 3.15 \times 10^{-3}$ m/MeV.

If v is the velocity of α particles then

we have

$$R = bv^3 \quad \text{.....(4)}$$

With $b = 9.416 \times 10^{-24}$ unit.

In the case of a solid, the range R_s (in meter) is related to the range R_s in air. As

$$R_s = \frac{0.312A^{\frac{1}{2}}}{\rho} \quad \text{.....(5)}$$

where ρ is the density of the solid of mass number A.

For higher and lower energies, more appropriate relationships are give by Eqs. 4 and 5 respectively.

$$R = v^4$$

$$\text{and } R \propto v^{3/2} \quad \text{.....(6)}$$

7.7 GEIGER-NUTTAL LAW

Geiger and Nuttal had discovered an empirical relationship between the decay constant (λ) of naturally occurring radioactive elements and the range (R) of α -particles. Geiger-Nuttal law, which may be written as, describes this relationship between R and λ as

$$\log \lambda = A + B \log R \quad \text{.....(7)}$$

with A and B as constants:

According to this law, the α -particles emitted by elements having larger disintegration constant (i.e. shorter half-period) have longer ranges and vice-versa. Geiger - Nuttal law tells that the graph of $\log \lambda$ versus $\log R$ is a straight line with slope B.

As illustrated in Fig.3, distinct straight parallel lines are obtained for various radioactive series, suggesting that the constant B has the same value across the spectrum while the constant A varies.

The Geiger-Nuttall law can also be expressed as, given that the range is related with energy as indicated by Eq.1

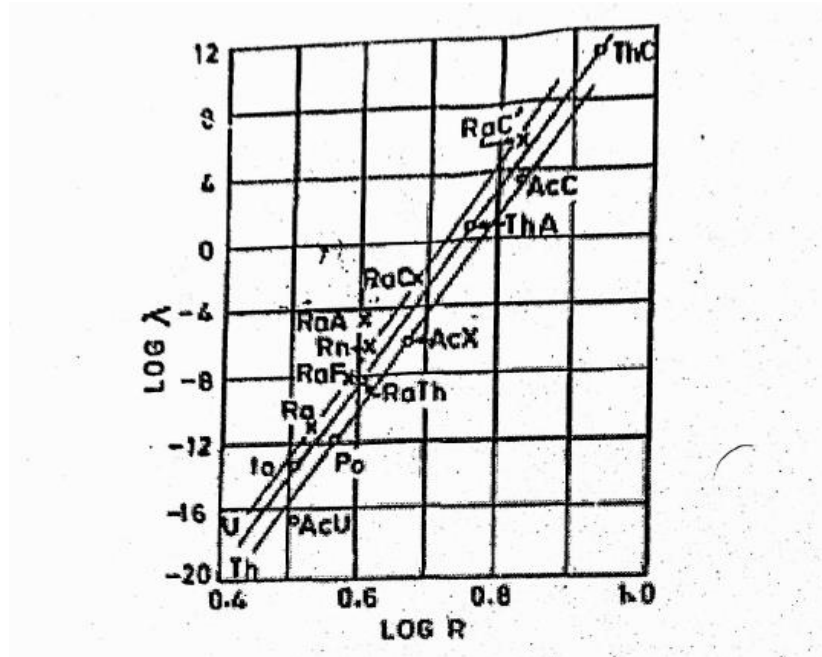


Fig.3 Variation of log λ with log R

$$\log \lambda = C + D \log E \quad \dots(8)$$

where C and D are other new constants.

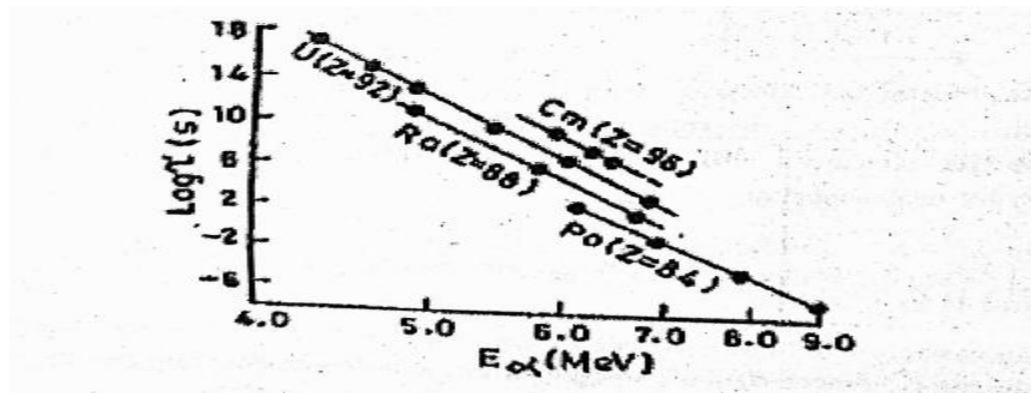


Fig.4. Variation of log τ with E

Since the half-life (T) is inversely proportional to decay (λ) constant, Geiger - Nuttall law gives relation between half-life and range or energy. The variation of half-life with energy is shown in Fig.4.

It shows that for a small increase of energy (by a factor of 2) the half-life decreases enormously. Remember (by a factor of 10^{24}). This enormous variation in half-life cannot be explained by the classical theory.

7.8 GAMOW'S THEORY OF α - DECAY

In 1928, G. Gamow, R. W. Gurney, and E.U. Candon independently and nearly simultaneously provided an acceptable explanation of decay using the principles of quantum mechanics. The α -particle is believed to be trapped inside the nucleus by the Coulomb barrier in a basic form of this theory, but it only has a small chance of exiting the nucleus.

The disintegration of heavy nuclei reveals that two protons and two neutrons can occasionally group together to form a particle within the nucleus. The strong, short-range, and attractive nuclear forces must act on a α -particle when it is inside the nucleus. However, as it leaves the nucleus, the Coulomb repulsion caused by the remaining positive nucleus $(Z-2)e$, where $+Ze$ is the charge of the parent nucleus, acts on it.

Outside the nucleus the Coulomb potential energy $V(r)$ at a distance r of α -particle from the centre of the nucleus is given by

$$V(r) = \frac{2(Z - 2)^2}{4\pi\epsilon_0 r} \quad \text{.....(9)}$$

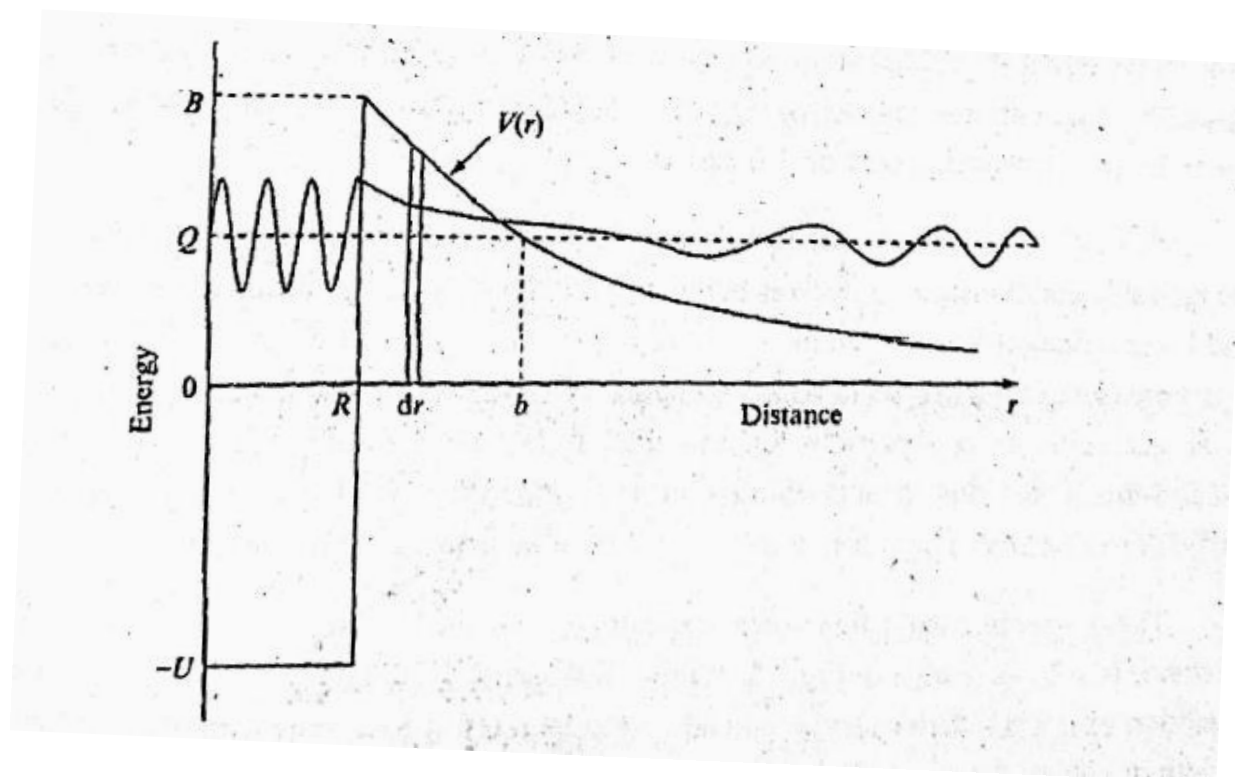


Fig.5. Potential well and Coulomb potential barrier of a nucleus. The α particle wave function is oscillatory nature inside the nucleus ($r < R$) and at large distances $s(r > b)$.

Fig.5 depicts a schematic representation of the Coulomb potential barrier for disintegration (the potential energy between the particle and daughter nucleus). The α -particle is bound and cannot escape at all if the decay Q -value is positive.

The Coulomb potential given by equation, ($r > R$) $V(r)$, describes what happens outside the nucleus eq. (9). If we neglect the recoil energy of the daughter nucleus, the kinetic energy of the α -particle asymptotically equals Q . The α -particle encounters the attractive nuclear forces close to the nuclear surface and develops a kinetic energy that is dependent on the depth of the nuclear potential well (U). In Fig. 5, the nuclear potential is shown as a square well with a sharp edge and a width R (nuclear radius).

The value of $V(r)$ turns out to be 34 MeV in the case of $^{88}\text{Ra}226$ emitting α -particles that produce $^{83}\text{Ra}223$ as a daughter nucleus. However, $Q = 4.88$ MeV, which is much lower than V , is the energy of α -disintegration (r) According to classical mechanics, a particle must have an energy at

least equal to $V(r)$ in order to leave or enter the nucleus, which is contradictory to the energy levels that have been measured.

However, such a classical forbidden occurrence is possible in quantum mechanics. According to the Schrödinger wave equation, the particle is represented as a wave. By substituting the relevant particle potentials, we can write the wave equation down for the various regions. An initial particle inside the nucleus ($r < R$) has a finite probability of leaving the nucleus when these equations are solved with the proper boundary conditions. We can imagine a particle escaping from the nucleus as if there were tunnels in the potential barrier. Thus, the phenomenon is known as the "quantum mechanical tunnel effect." The initial energy and the half-life of the dissolving nucleus can be calculated according to the quantum mechanical theory (parent nucleus). As a result, this quantum mechanical barrier penetration theory provides an explanation of Geiger -Nuttall law.

The probability that a particle exists as a recognizable entity inside the nucleus prior to emission is a significant aspect of the decay problem. The structure of the states of the parent and daughter nuclei strongly affects the formation probability (P), which is what it is called. It is a measure of how much the daughter nucleus in its final state plus a particle "looks like" the initial state. Although there are certain errors in determining the barrier penetration factor, it is theoretically possible to derive it from experimental measurement. P is difficult to calculate, so we assume it is equal to unity for the sake of simplicity.

The α -particle wave function is oscillatory inside the nucleus ($r < R$) and also at large distance ($r > b$) as shown in Fig.5. Within the barrier region ($R < r < b$), it is a decreasing function of r . The barrier can be considered to be a set of narrow rectangular barriers of width dr , which the α particle has to penetrate in order to escape from the nucleus. For $r > R$, $V(r)$ varies as $1/r$, therefore $R/b = Q/H$ where H being the barrier height.

Given that for existence of α -particle we can imagine it moving back and forth inside the nucleus with a speed ' v ' given by its kinetic energy and reaches the barrier with a frequency f given by

$$f \approx \frac{v}{2R} \quad \dots(10)$$

where R is the radius of the nucleus.

The value of f for α particle is

$$f = \frac{1.7 \times 10^7}{2 \times 10^{-14}} = 8.5 \times 10^{20} \text{ s}^{-1}$$

As mentioned earlier, quantum mechanically, the α -particle in a nucleus is represented by a wave function ψ . Inside the nucleus ψ is oscillatory with a wave number

$$K = \frac{\sqrt{2m(Q + U)}}{\hbar^2} \quad \dots(11)$$

with $(Q+U)$ as the total kinetic energy of a-particle inside nucleus.

and m is the particle's mass. If we assume that any reflected wave from the barrier's outer edge at $r=b$ is negligible, the wave function within the barrier region ($R > r > b$) is not zero and takes the shape of a falling exponential. The wave function is once more oscillatory outside of the nucleus, with a wave number at very long distances.

By treating the barrier as a series of narrow rectangular barriers in one dimension and assuming that the element's barrier height is given below, we can estimate the penetration probability.

$$V(r) = \frac{2Ze^2}{4\pi\epsilon_0 r} \quad \dots(12)$$

where Z is the atomic number of the daughter nucleus. The wave function at each barrier has the form $e^{-\beta r}$

where
$$\beta = \sqrt{\frac{2m[V(r)-Q]}{\hbar^2}} \quad \dots(13)$$

and after the barrier the wave function is reduced to have the form $e^{-\beta(r+dr)}$

The transmission probability through a single element of the barrier is $e^{-2\beta dr}$.

Now, the total of these infinitesimal probabilities, calculated for the set of barrier elements between $r = R$ and $r = b$, is the probability of breaking through the entire barrier. The entire "transmission probability" (T) can be expressed as because the product of such infinitesimal exponentials is equal to the exponential of the integral of the exponents, i.e., it equals the exponential of the sum of their exponents.

$$\begin{aligned} T &= \exp \left[-2 \int_R^b R(dr) \right] \\ &= \exp(-2G) \end{aligned} \quad \text{.....(14)}$$

where G is called the 'Gamow factor' which can be evaluated, with $V(r)$ given by equation (11) and obtained as

$$G = \sqrt{\frac{(2m)}{\hbar^2 Q}} \left(\frac{2Ze^2}{4\pi\epsilon_0} \right) \left[\cos^{-1} \sqrt{x} - \sqrt{x(1-x)} \right] \quad \text{.....(15)}$$

$$\text{with } x = r/b = Q/H \quad \text{.....(16)}$$

Combining the results of Eqns.10 and 14 with the preformation probability, we can write an expression for the α - decay rate or the decay constant (λ) as

$$\lambda = pf \exp(-2G) \quad \text{.....(17)}$$

In order to estimate the dependence of the half-life on the decay Q -value, as shown in Fig.4, we take ^{238}U as the parent nucleus and calculate the penetrability factor (transmission probability) given by Eqn 14 for $Q = 4.268$ MeV (which is the actual α decay Q -value of ^{238}U and $Q = 5.268$ MeV i.e. for two values that differ by 1 MeV.

Now slightly rewriting Eqn 13 as

$$G = 2\alpha z \sqrt{\frac{2mc^2}{Q}} \left[\cos^{-1} \sqrt{\frac{Q}{H}} - \sqrt{\left(\frac{Q}{H}\right) \left(1 - \frac{Q}{H}\right)} \right] \quad \text{.....(18)}$$

where the quantity $\alpha = e^2/4\epsilon_0\hbar c$ is the usual fine-structure constant and is equal to 1/137.

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In the case of ^{238}U we take the following data $Z = 90$, the rest-mass energy m_e of α particle equal to 3727 MeV, the Coulomb barrier height $H = 27.87$ MeV. (Obtained using Eqn 10) with $r = R = 1.2 (A_1^{1/3}, A_2^{1/3}) = 9.3$ fm, for the effective separation distance between the daughter nucleus with $A_1 = 234$ and the α -particle with $A_2 = 4$, at the peak of the barrier.

Substituting this data into Eqn 16, we obtain the $G = 44.391$ and 36.047 for $Q = 4.268$ and 5.268 MeV, respectively.

By using Eqn 15 and since half-life is related to decay constant as, $t_{1/2} \propto \frac{1}{\lambda}$

we obtain

$$t^{1/2} (5.268) / t^{1/2} (4.628) = \exp(-16.69) = 5.7 \times 10^{-8}$$

This suggests that an increase of 1 MeV in Q-value changes the half-life more by seven orders in magnitude.

We can utilize on another alternate method by using the following data for the α -decay.

Barrier height = $H = V(r) = 15\text{MeV}$, $Q = 5\text{MeV}$, $m = 4 \times 1.66 \times 10^{-27}$ kg. From equation 13 we have

$$\begin{aligned} \beta &= \sqrt{\frac{2m}{\hbar^2} [V(r) - Q]} \\ &= 1.388 \times 10^{+15} \end{aligned}$$

Then the value of $T = 2.55 \times 10^{-24}$ s

Accordingly, the probability of the α -particle colliding with the barrier wall and passing through it is around 2.5 in 10^{24} collisions.

.

$$\lambda = fT = 8.5 \times 10^{20} \text{ s}^{-1} \times 2.55 \times 10^{-24} = 2.17 \times 10^{-3} \text{ s}^{-1}$$

So, the mean life time against α decay is

$$\tau_m = \frac{1}{\lambda} = \frac{1}{1.17 \times 10^{-3}} = 461 \text{ sec} = 7 \text{ min } 41 \text{ sec}$$

7.9 SELECTION RULES IN ALPHA DECAY

. When the spins of the initial and final states differ, it can be shown that the α -particle loses some orbital angular momentum, which prevents the emission. However, it often has little effect on the decay probability. There are two key elements that control the intensity variations. The first is caused by the substantial energy dependence of barrier penetrability, which falls off as the final state's energy rises. The second component, which may affect the chance of preformation, is the structural variations between the initial and final states. Now, the effect caused by a particle losing a few units of angular momentum is minimal in comparison to these two effects.

The parity of the wave function is $(-1)^L$, where L is the orbital angular momentum quantum number, and there is a parity selection criterion for decay that depends on angular momentum. The parities of the initial and final nuclear states will differ if L is odd and remain the same if L is even due to the conservation of parity in α -decay. As a result, this rule forbids some decays. For an example, a 0^+ parent state cannot decay into odd parity state with even spin or an even parity state with odd spin.

7.10 SUMMARY

After studying the unit learners have learnt about

- The α -Spectrum and Fine Structure
- Long Range α - Particles
- Q- value in the α -decay
- Range of α particles
- Range-Energy (velocity) relationship
- Geiger-Nuttal law.
- Gamow's theory of α -decay

- Selection rules in Alpha Decay

7.11 REFERENCES

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4. Nuclear Physics An Introduction by S.B.Patel
5. Nuclear Physics by D. C Tayal.
6. Nuclear Physics by S.N.Ghoshal

7.12 SUGGESTED READINGS

1. "The Two Nucleon Problem" by M. Sugrwarra and Hulthen, Encyclopedia of Physics, Berlin: Springer Ver.
2. "Nuclear Two Body Problems and Elements of Nuclear Forces" Experimental Nuclear Physics by N. F. Ramsey, Wiley: New York.
3. Lectures on Nuclear Theory (translated from the Russian) by Landau, Plenum Press, New York.
4. Elementary Nuclear Theory, 2nd ed. by Bethe and Morrison, Wiley: New York.

7.13 TERMINAL QUESTIONS

1. Discuss the quantum mechanical tunnelling of a particle.
2. Write about 'fine structure of a spectrum and its importance.
3. Write a note on the selection rules for a particle emission.
4. Explain how the range and energy of the α -particle and the half-life period are related through Geiger-Nuttall Law.
5. Discuss the Gamow theory of α -decay and how it explains the main features of α particle emission process.

UNIT 8

BETA DECAY

Structure of the Unit

8.1 Introduction

8.2 Objectives

8.3 What is Beta Decay?

8.3.1 What happens in Beta Decay?

8.3.2 Types of Beta Decay

8.4 Q-Value in Beta Decay

8.5 Neutrino Hypothesis

8.6 Fermi's Theory of β -Decay

8.7 Fermi-Kurie Plots

8.8 Selection Rules for β -Decay

8.9 Parity Violation in Beta Decay

8.10 Summary

8.11 References

8.12 Suggested Readings

8.13 Terminal Questions

8.1 INTRODUCTION

The emission of electrons is associated with several nuclear processes. Some nuclei have been seen to emit positrons. In some nuclei, electron capture—in which the nucleus captures an orbital electron, usually from the K-shell—occurs. The term "beta decay" refers to all three closely related phenomena.

The process by which an unstable nucleus modifies itself to achieve the best possible neutron-to-proton ratio for a given mass number is known as β -decay. In contrast to α -decay, which results in a 4 unit decrease in mass number, β -decay has no change in the mass number ($A = 0$). The β -spectrum, one of the nuclear spectra, has the unusual quality of being "continuous," whereas the α -spectrum is a line spectrum. Every β - spectrum contains electrons (β - particles) with energies ranging from zero to a specific maximum. This radionuclide's unique property is its maximum energy value

Fermi proposed a successful theory of the shape of the spectrum and the lifetime of emitters in 1934. Many of the fundamental issues relating to β -particle emission were resolved by 1950.

This continuous nature of β -spectrum gave rise to serious difficulties, in understanding the β -decay process. The chief problem is that, just like the α -decay, β -decay also an energy transition between two definite energy states as shown in Fig.1

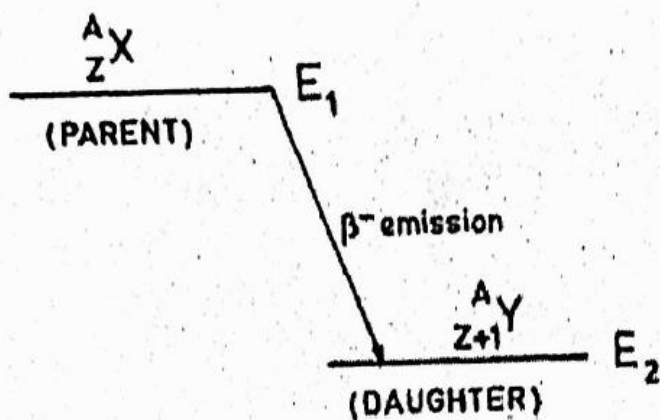


Fig.1 β -decay transition between two definite energy states.

8.2 OBJECTIVES

After studying this unit, learners will be able to

- Explain what is Beta Decay?
- Describe the types of Beta Decay
- Evaluate the Q-Value in Beta Decay
- Discuss neutrino hypothesis
- Describe Fermi's Theory of β -Decay
- Understand the various selection rules for β -Decay
- Explain parity violation in Beta Decay

8.3 WHAT IS BETA DECAY?

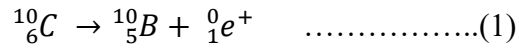
Beta Decay is a type of radioactive decay in which a proton is transformed into a neutron or vice versa inside the nucleus of the radioactive sample. Processes like beta decay and alpha decay allow the nucleus of the radioactive sample to get as close as possible to the optimum neutron/ proton ratio. While doing so, the nucleus emits a beta particle which can either be an electron or positron. Remember that there either a proton can turn into a neutron or a neutron into a proton. Electron and the positron are generated to obey the law of conservation of charge. Beta-decay occurs via weak interaction.

8.3.1 What happens in Beta Decay?

Beta decay is a radioactive decay in which a beta ray is emitted from an atomic nucleus. During beta decay, the proton in the nucleus is transformed into a neutron and vice versa. If a proton is converted to a neutron, it is known as β^+ decay. Similarly, if a neutron is converted to a proton, it is known as β^- decay. Due to the change in the nucleus, a beta particle is emitted. The beta particle is a high-speed electron when it is a β^- decay and a positron when it is a β^+ decay. Beta particles are used to treat health conditions such as eye and bone cancer and are also used as tracers.

Example:

One of the examples of beta decay is the β^- decay of the carbon atom. Here, a neutron of carbon is converted into a proton, and the emitted beta particle is an electron. Similarly, the β^+ decay of carbon-10 can be represented by an equation as follows:



Here, the proton of the carbon atom is converted into a neutron, and the emitted beta particle is a positron.

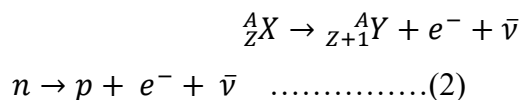
8.3.2 Types of Beta Decay

There are two types of beta decay types:

- (i) beta minus (β^-) and
- (ii) beta plus (β^+).

(i) Beta-Minus Decay

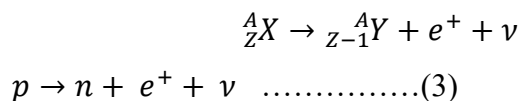
- In beta minus, a neutron is transformed to yield a proton, causing an increase in the atom's atomic number. The neutron is neutral, but the proton is positive.
- To maintain the conservation of charge, the nucleus in the process also produces an electron and an antineutrino.
- Antineutrino is the antimatter counterpart of neutrino. Both of these are neutral particles with negligible mass. They interact with matter very weakly and can even pass through the entire earth without being disturbed.
- In beta minus decay, the change in atomic configuration is:



Examples of beta minus decay include the decay of ^{14}C into ^{14}N and it usually occurs in neutron-rich nuclei.

(ii) Beta-Plus Decay

- In beta plus decay, the proton disintegrates to yield a neutron causing a decrease in the atomic number of the radioactive sample. The nucleus experiences a loss of proton but gains a neutron.
- Again, conservation of charge is important. The beta plus decay in order to obey the conservation law also yields a positron and a neutrino.
- A positron is the antimatter equivalent of an electron; the same in all aspects except that a positron has a positive charge.
- A neutrino's behaviour is the same as the antineutrinos. As expressed in the equation, it is:



Beta plus decay can happen only if the daughter nucleus is more stable than the mother nucleus. This difference goes into the conversion of a proton into a neutron, a positron and a neutrino. There is no increase in mass number because a proton and a neutron have the same mass.

The below image depicts the example of beta minus (β^-) decay and beta plus (β^+) decay.

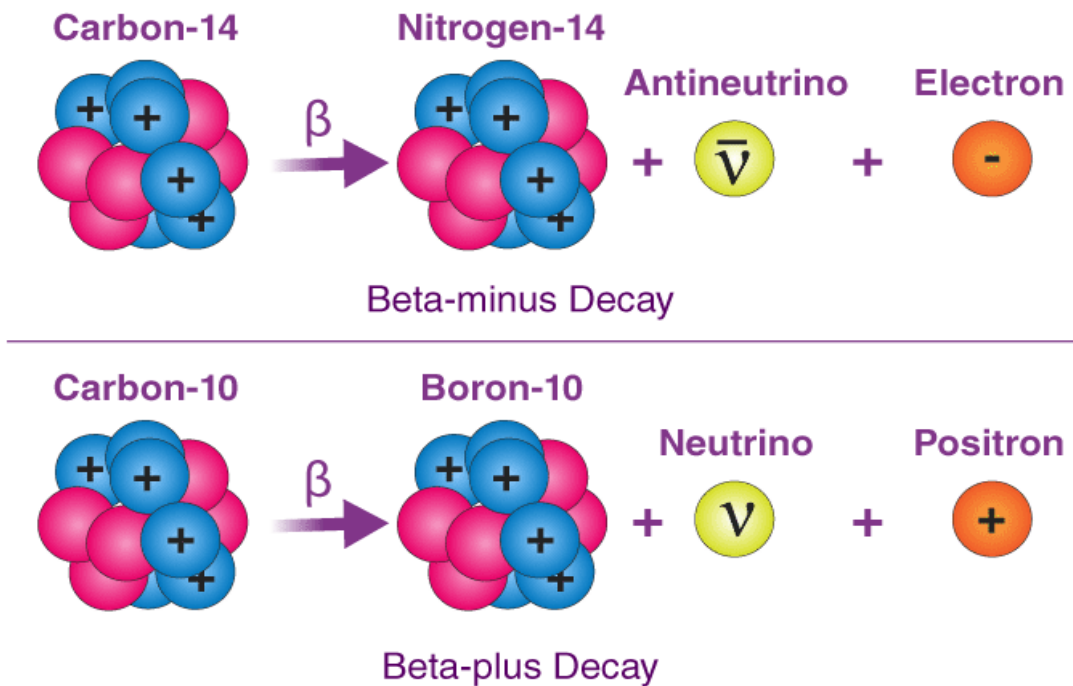


Fig 1: Beta decay types (<https://cdn1.byjus.com>)

8.4 Q-VALUE IN BETA DECAY

In nuclear and particle physics, the **energetics of nuclear reactions** is determined by the **Q-value** of that reaction. The **Q-value** of the reaction is defined as the **difference** between the sum of the **rest masses** of the **initial reactants** and the sum of the **masses** of the **final products** in energy units (usually in MeV).

Consider a typical reaction in which the projectile a and the target A give place to two products, B and b . This can also be expressed in the notation we have used so far, $a + A \rightarrow B + b$, or even in a more compact notation, $A(a,b)B$.

The **Q-value** of this reaction is given by:

$$Q = [ma + mA - (mb + mB)]c^2 \quad \dots\dots\dots(4)$$

When describing beta decay (reaction without projectile), the disintegrating nucleus is usually referred to as the parent nucleus and the nucleus remaining after the event as the daughter nucleus. The emission of a beta particle, either an electron, β^- , or a positron, β^+ , changes the atomic number of the nucleus without affecting its mass number. The total rest mass of the daughter nucleus and the nuclear radiation released in a beta disintegration, $m_{\text{Daughter}} + m_{\text{Radiation}}$, is always less than that of the parent nucleus, m_{parent} .

The mass-energy difference,

$$Q = [m_{\text{parent}} - (m_{\text{Daughter}} + m_{\text{Radiation}})]c^2 \dots\dots\dots(5)$$

appears as the disintegration energy, liberated in the process. For example, the Q-value of typical beta decay is:

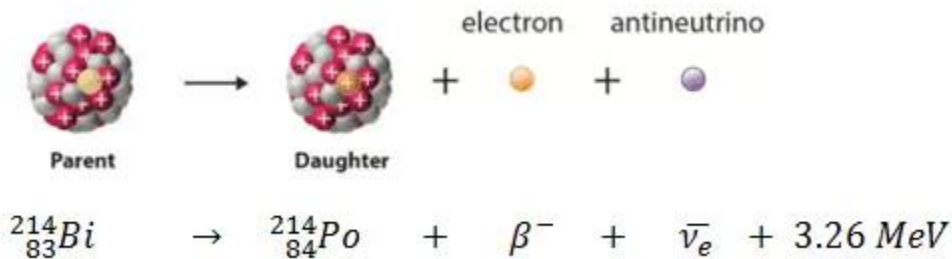


Fig.3: Q-value in beta decay (<http://nuclear-power.com>)

In the process of beta decay, either an electron or a positron is emitted. This emission is accompanied by the emission of **antineutrino** (β^- decay) or **neutrino** (β^+ decay), which share the energy and momentum of the decay. The beta emission has a characteristic spectrum. This characteristic spectrum is caused by the fact that either a neutrino or an antineutrino is emitted with the emission of a beta particle. The shape of this energy curve depends on what fraction of the reaction energy (**Q value**-the amount of energy released by the reaction) is carried by the massive particle. Beta particles can therefore be emitted with any kinetic energy ranging from **0 to**

Q. After an alpha or beta decay, the daughter nucleus is often left in an excited energy state. To stabilize itself, it subsequently emits high-energy photons, γ -rays

8.5 NEUTRINO HYPOTHESIS

Pauli, in 1930, postulated the existence of a new particle called neutrino denoted by ν and should be emitted in the β -decay process. This new particle carries away an amount of energy equal to the difference between the observed β -particle energy and the endpoint energy $r(E_{\max})$ of the continuous spectrum, in order to satisfy the conservation of energy. So, there can be 3-ways of energy sharing between the β -particle and neutrino as mentioned below:

(i) β -particle can carry the maximum energy ($E_{\beta} = E_{\max}$) and neutrino carrying zero energy ($E_{\nu} = 0$)

(ii) β -particle carrying some medium energy ($E_{\beta} < E_{\max}$) and neutrino carrying low energy ($E_{\nu} < E_{\beta}$). So that $E_{\beta} + E_{\nu} = E_{\max}$

(iii) β -particle with very low energy ($E_{\beta} \ll E$) and neutrino carrying maximum energy ($E_{\nu} > E_{\beta}$)
So that $E_{\beta} + E_{\nu} = E_{\max}$

Now, in order to satisfy the conservation of angular momentum, charge and nuclear statistics, the following properties are assigned to neutrino. Remember, the existence of neutrino is observed experimentally, after about 25 years of its proposal.

(i) Neutrino has zero charge and hence negligible magnetic moment.

(ii) The mass of neutrino is zero or almost zero

(iii) Neutrino has a spin of $\frac{1}{2}$

(iv) Neutrino must be a fermion in order to obey nuclear statistics.

Just like other elementary particles like electron, neutrino should have an antiparticle, called anti-neutrino denoted by $\bar{\nu}$ and has same properties as that of neutrino. The only difference between neutrino and antineutrino particles is the following. A neutrino has spin always antiparallel to its momentum, whereas anti-neutrino has its spin parallel to its momentum. In other words, neutrino is a left-handed particle while antineutrino is a right-handed particle, as shown in Fig.4.

To describe the handedness of the neutrino, the idea of helicity, (which is presently out of the scope of this course) is used. Remember that the helicity (H) is -1 for neutrino and +1 for anti-neutrino

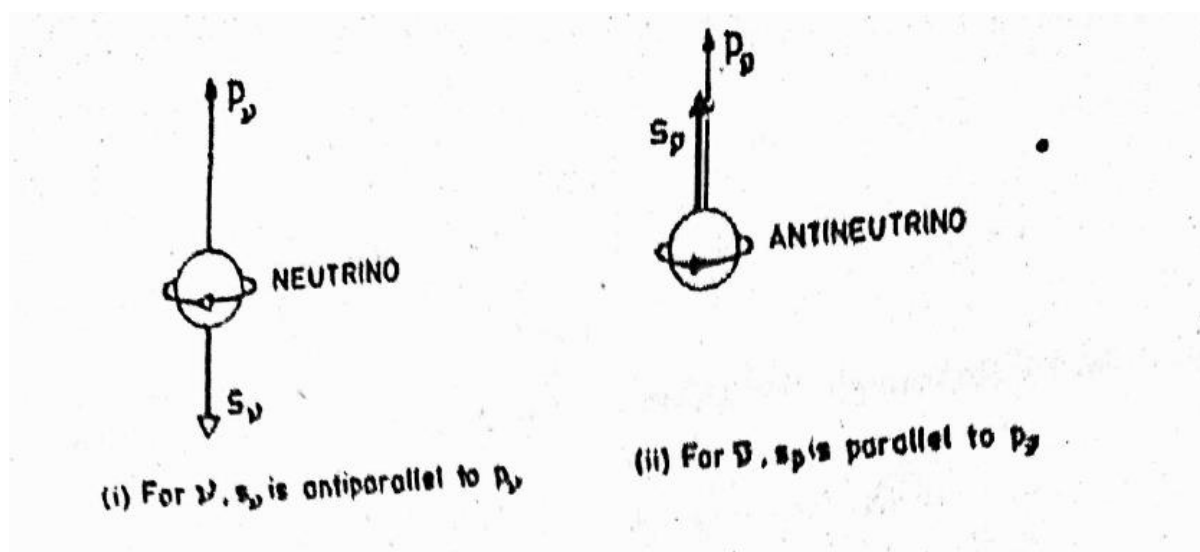


Fig.4: Neutrino and Antineutrino

8.6 FERMIS THEORY OF β -DECAY

Now, let us understand the features of Fermi's theory to explain the β -decay process.

According to Fermi's theory, the electron and neutrino are produced during the process of β -particle emission. Keep in mind that the electron is created, not present, in the nucleus. The neutrino is also referred to as the β -particle's "birth partner." The basic concept In order to use the perturbation theory, Fermi's hypothesis requires that the interaction responsible for the decay process be "weak."

The Hamiltonian that causes the decay provides the formula for the transition rate between the initial and final states in quantum mechanics. The likelihood of the decay is affected by a number of variables, including the energy released during the process, the angular momentum of the particles released, the nuclear charge, and the characteristics of the beginning and final nuclear states. The Fermi's Golden Rule, which is a foundation of quantum mechanics and is written as the transition probability as

$$dW_{fi} = \frac{2\pi}{\hbar} |\langle f | H_W | i \rangle|^2 \rho(E) \quad \text{.....(6)}$$

where H_W is the interaction Hamiltonian, $\rho(E)$ is called the 'density of states factor' or 'phase space factor', f and i indicate the final and initial states.

The expression for density of states $\rho(E)$ given by

$$\rho(E) = \frac{dN}{dE} \quad \text{.....(7)}$$

where N is the number of states of particles involved in the decay that can be put in a given volume and E is the total energy of the particles, dN is the number of final states with energy in the interval E to $(E + dE)$.

The phase-space factor is mainly responsible for how the energy produced during the transition is transformed into the kinetic energy of the particles. The distribution of energy is also influenced by the Coulomb field. For the sake of simplicity, it is assumed that the particles are released when the Coulomb field is absent.

If the kinetic energy of the remaining heavy nucleus is neglected, the neutrino and the β - particle each receive a portion of the total energy released during the transition, i.e. $E_{\max} = E_{\beta} + E_{\nu}$

The density of final states $\rho(E)$ completely determines the form of β -spectrum. In order to compute $\rho(E)$, we have to consider 'phase-space'. The graph of position of the particle(x) against its momentum (p_x) constitute a phase-space as shown in Fig.5

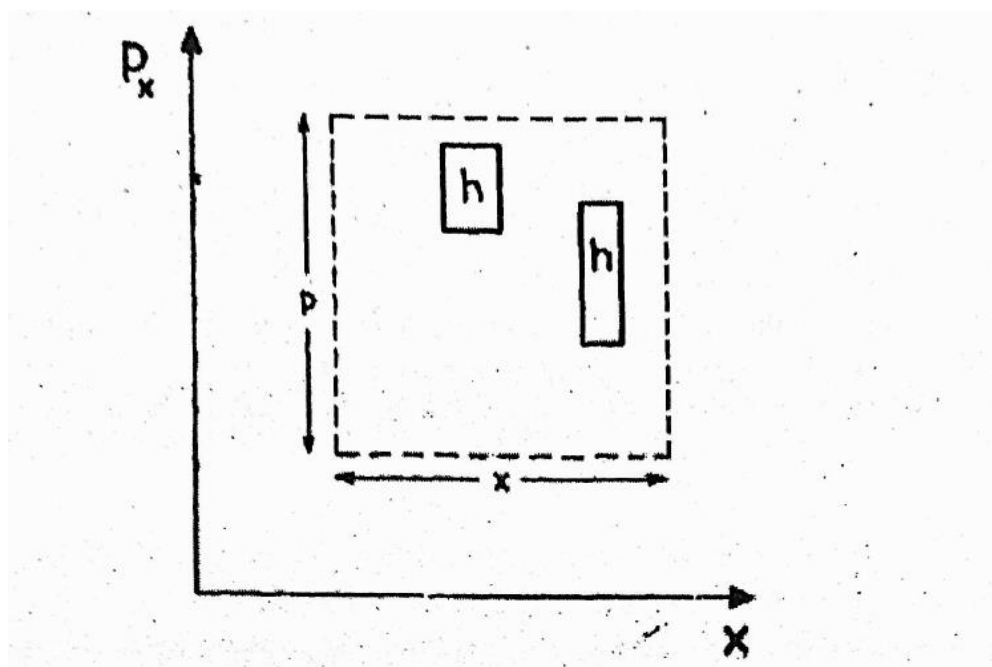


Fig.5: Phase space in quantum mechanics.

A particle in a phase-space is represented by a 'cell' and not by a point. The volume of the cell is always same and equal to h .

The transition rate (λ) between an initial state (i) and a final state (f) is given by

$$\lambda = \frac{2\pi}{h} |M|^2 \frac{dN}{dE} \quad \text{.....(8)}$$

With M is the matrix element of the transition (it gives an energy times volume) and is equal to $|\langle f|H_w|i\rangle|$ given in Eqn.2.3 and is independent of energy of the electrons (β -particles) emitted and can be treated as constant for most of the β -transitions.

The number of energy states available to a β -particle with momentum between $p\beta$ and $(p\beta + dp\beta)$ and constrained to a volume V is given by

$$[\rho(P)dP]_{\beta} = \frac{4\pi\rho_{\beta}^2 d\rho_{\beta} V}{h^3} \quad \text{.....(9)}$$

A similar formula applies to the particle neutrino, so we can write

$$[\rho(P)dP]_{\nu} = \frac{4\pi\rho_{\nu}^2 d\rho_{\nu} V}{h^3} \quad \dots(10)$$

Assuming that the phase-space available for either particle is independent of the other i.e. there is no angular correlation between their directions of emission, the total number of energy states available for the two particles is

$$dN = \frac{(4\pi)^2 V^2 \rho_{\beta}^2 d\rho_{\beta} \rho_{\nu} d\rho_{\nu}}{h^6} \quad \dots(11)$$

The energy of neutrino can be written as

$$E_{\nu} = \rho_{\nu} c = E_{max} - E_{\beta} \quad \dots(12)$$

For a fixed β particle energy (E_{β}) we can write

$$d\rho_{\nu} = \frac{dE}{c} \quad \dots(13)$$

Substituting Eqn.12 and 13 into Eqn.11 gives an expression for the phase-space factor in terms of the energy and momentum of the β -particle as

$$\frac{dN}{dE} \propto \rho_{\beta}^2 (E_{max} - E_{\beta})^2 d\rho_{\beta} \quad \dots(14)$$

Now, we have to consider a correction factor for the effect of Coulomb field on the emitted particles. This correction factor is called 'Fermi function', denoted by $F(Z, E_{\beta})$ where Z is the atomic number of the final (daughter) nucleus. This function has been evaluated for many different situations and tabulated values are available elsewhere.

Now, substituting the result of Eqn14, including the Fermi function, into Eqn.6, we obtain finally an expression, for the differential β decay probability per unit time for β -particle momenta in the range p_{β} to $(p_{\beta} + dp_{\beta})$ as

$$dW_{\beta} = d\lambda(\rho_{\beta}) = C|M|^2 F(Z, E_{\beta}) \rho_{\beta}^2 (E_{max} - E_{\beta})^2 d\rho_{\beta} \dots\dots\dots(15)$$

where C is a constant

By integrating the equation (15), the total decay rate can be obtained and we get

$$\lambda = C|M|^2 \int F(Z, E_\beta) \rho_\beta^2 (E_{max} - E_\beta)^2 d\rho_\beta \dots\dots\dots(16)$$

$$\text{with } f(Z, E_{max}) \int F(Z, E_\beta) \rho_\beta^2 (E_{max} - E_\beta)^2 d\rho_\beta \dots\dots\dots(17)$$

Accurate calculations can be made for the function (Z, Emax). Thus, it is convenient to describe the β -decay results in terms of the product of f(Z, Emax) and the transition half-life (t). The "comparative half-life" is the result of ft. According to the Eqn16, which separates the phase-space effects from the transition's important details, the product ft is proportional to $\frac{1}{|M|^2}$. It is more convenient to describe the value of $\log_{10}ft$ instead of ft with t in seconds because the value of ft changes over many orders of magnitude (10^3 to 10^{20} sec) for different transitions.

In general, the largest matrix elements and hence the smallest ft values result from the most fundamental β -decays (of low z elements). Since the matrix element produces energy multiplied by volume, as was already established, the interaction energy necessary for the β -decay process is on the order of 10^{-4} MeV. This suggests that the interaction of decay is weak.

8.7 FERMI-KURIE PLOTS

Eqn15 can be written as below gives a straight line graph plotted against electron (β -particle) energy E_β .

$$\left[\frac{dW_{fI}}{F(Z, E_\beta) \rho_\beta^2 d\rho_\beta} \right]^{1/2} = (\text{Constant})|M|(E_{\beta_{max}} - E_\beta) \dots\dots\dots(18)$$

The function is called 'Kurie function' (K) written as

$$K = \left[\frac{dW_{fI}}{F(Z, E_\beta) \rho_\beta^2 d\rho_\beta} \right]^{1/2} \dots\dots(19)$$

The Fermi-Kurie plot is the graph that K is plotted on and is found to be a straight line over a wide energy range. Experimentally, the transition dw_{fi} is the quantity of β -particles per energy interval, and the particles may be found and counted using a Geiger counter. A magnetic spectrograph may be able to determine the energy (or momentum) interval.

The Fermi-Kurie plot's linear nature supports the notion that leptonic particles do not carry orbital angular momentum. The linear dependence is not present in Fermi-Kurie graphs for decays in which the leptons take orbital angular momentum.

8.8 SELECTION RULES FOR β -DECAY

Because the matrix element determines the probability of a transition and, hence, the order of transition. So, a sequence of terms can be used to express the matrix element (M). When they don't disappear, these terms get smaller and smaller. It's important to focus on the first non-vanishing term. The transitions that result from the first term being unity and energy-independent are known as "permitted transitions."

The first term in the expansion of the matrix element becomes zero for some transitions because of the initial and final nuclear wave functions. These transitions are known as "first forbidden" transitions, which is what the expansion's second term refers.

We must use the third non-vanishing term, which is much smaller, if the first and second terms both disappear during the expansion of the matrix element. Second forbidden transitions are these transitions.

For β -decay, the criteria that link the transition's properties, such as changes in angular momentum and parity, with the transition order are referred to as "selection rules." It is possible to determine the type of interaction producing the β -decay by comparing the experimentally found order of - transition with the theoretical values order. There are two types of selection rules to be applied for β -decay process

CASE I : L_i and L_f are the nuclear spin vectors for the initial and final states in the transition, then the change in angular momentum (L) is given as

$$\Delta L = L_f - L_i \quad \dots(20)$$

where i and f refer to initial and final states respectively

The orbital angular momentum carried away by the electron-neutrino pair must be equal to the change L of the nucleus in β -transition. So, for allowed transition $\Delta L = 0$ The particles can be aligned either parallel or anti-parallel to each other, so that the total spin angular momentum carried away by them can be either $S = 0$ (parallel) or $S=1$ (anti-parallel) when $S = 0$, the spin change $\Delta S = \Delta L = 0$ and this rule is called 'Fermi Selection Rule'.

When $S = 1$, then $|\Delta S| = |\Delta I| = 1$ (in Unit of \hbar) and this rule is called 'Gamow-Teller selection Rule.'

CASE II: The vector difference between initial and final angular momentum of the nucleus will be equal to 1. So, the selection rule can be written as $\Delta L = \pm 1$. But there cannot be transition from zero spin angular momentum states to zero angular momentum states i.e. $L_i = L_f = 0$. This is excluded in 'Gamow-Teller section rule.'

Example: Check whether the transition $n \rightarrow p^+ e^-$ is allowed by both the type of selection rules.

Solution: Since parity associated with orbital angular momentum is $(-1)^L$, and in allowed transitions ($L = 0$) there will be no parity change.

The selection rules forbidden transitions are less simple than the rules for allowed transitions.

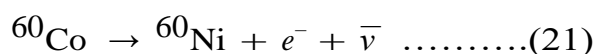
Here the leptons carry away an additional spin angular momentum of one unit.

Here, there will be a parity change if the orbital angular momentum quantum number L is odd and no change in parity if L is even.

For a first forbidden transition $L = (1)$ there will be a parity change.

8.9 PARITY VIOLATION IN BETA DECAY

β -decay exhibits a further peculiarity. This was discovered in 1957 by C.S. Wu who observed the decay of radioactive cobalt into nickel



The cobalt sample was kept at a low temperature and placed in a magnetic field so that the spin of the cobalt was pointing in the direction of the magnetic field.

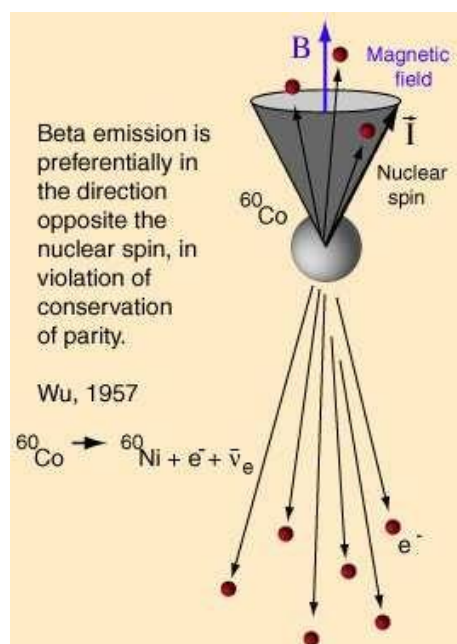


Fig.6: Beta emission from Cobalt

She discovered that most of the electrons emerged in the opposite direction from the applied magnetic field. If we write s for the spin of the parent nucleus and p_e for the momentum of an emitted electron, this means that the average value of the scalar product $s \cdot p_e$ was negative. In order to balance the momentum the antineutrinos are usually emitted in the direction of the magnetic field, so that the average value of $s \cdot p_{\bar{\nu}}$ was positive.

Under the parity operation

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$$r \rightarrow -r$$

and

$$p \rightarrow -p$$

but angular momentum which is defined as a vector product

$$L = r \times p,$$

is unchanged under parity

$$L \rightarrow L.$$

Spin is an internal angular momentum and so it also is unchanged under parity.

But this means that the scalar product $s \cdot p_e$ does change under parity

$$s \cdot p_e \rightarrow -s \cdot p_e$$

so that the fact that this quantity has a non-zero average value (or expectation value in quantum mechanics terms) means that the mechanism of β -decay violates parity conservation

If we view the above diagram in the corner of a mirrored room so that all the directions were reversed the spin would point in the same direction, but the electron direction would be reversed so that in that world the electrons would prefer to emerge in the direction of the magnetic field.

The spin of the daughter nucleus ${}^{60}_{28}\text{Ni}$ is 4 (it is produced in an excited state) whereas that of the parent ${}^{60}_{27}\text{Co}$ was $\frac{5}{2}$, so that in order to compensate for unit of angular momentum lost (in the direction of the magnetic field) the angular momentum the antineutrinos and electrons have their spins in the direction of the magnetic field. This means that the antineutrinos have a spin component $+\frac{1}{2}$ in their direction of motion (in units of \hbar) whereas the electrons have a spin component $-\frac{1}{2}$ in their direction of motion. The sign of the component of the spin of a particle in its direction of motion is called

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the “helicity” of the particle. Neutrinos always have negative helicity (antineutrinos always have positive helicity). An electron can have component of spin either $+\frac{1}{2}$ or $-\frac{1}{2}$ in its direction of motion (either positive or negative helicity). However, the electrons emitted in β -decay usually have negative helicity (positrons emitted in β -decay usually have positive helicity). This means that the mechanism responsible for β -decay (called the “weak interaction”) distinguish between positive and negative helicity and therefore violate parity.

8.10 SUMMARY

After studying this unit, learners have learnt about how to

- Explain what is Beta Decay?
- Describe the types of Beta Decay
- Evaluate the Q-Value in Beta Decay
- Discuss neutrino hypothesis
- Describe Fermi's Theory of β -Decay
- Understand the various selection rules for β -Decay
- Explain parity violation in Beta Decay

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8.12 SUGGESTED READINGS

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5. Nuclear Physics by D. C Tayal.

8.13 TERMINAL QUESTIONS

1. Discuss the postulates of Pauli's neutrino hypothesis.
2. What are Fermi-Kurie plots? Discuss their importance.
3. What are allowed and forbidden β - transitions?
4. Give the Fermi theory of β decay discuss how it explains the important features of β - spectrum.

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5. State and explain the selection rules of β -emission.
6. Discuss the parity violation in beta decay.
7. Discuss the types of beta decay with suitable examples.

UNIT 9**GAMMA DECAY**

Structure of the Unit

9.1 Introduction

9.2 Objectives

9.3 Gamma Decay

9.4 COMPTON SCATTERING OF GAMMA RAYS

9.5 PAIR PRODUCTION AND ANNIHILATION

9.6 K-CAPTURE or ELECTRON CAPTURE

9.7 INTERNAL CONVERSION

9.8 MULTIPOLARITY IN GAMMA TRANSITIONS

9.9 SELECTION RULES

9.10 Summary

9.11 References

9.12 Suggested Readings

9.13 Terminal Questions

9.1 INTRODUCTION

An excited or unstable nucleus transforms into a more stable structure and releases extra energy as radiation. An excited state is typically left over after a nucleus decays by alpha or beta emission. When emitting a particle from a nucleus is energetically impossible, the nucleus decays through electromagnetic interaction. Because γ -emission can be measured between more pairs of nuclear states, γ -spectroscopy can provide more about the structure of the nucleus. Gamma rays rarely absorb or scatter in air, which is crucial for determining whichever states are involved in the nuclear transition. The γ -rays can be easily detected with sensors having good energy resolution.

9.2 OBJECTIVES

After studying this unit, the learners will be able to how to

- Explain Gamma Decay
- Discuss Compton Scattering of Gamma Rays
- Describe Pair Production and Annihilation
- Explain K-capture or Electron Capture
- Describe Internal Conversion
- Discuss Multipolarity in Gamma Transitions
- Explain Selection Rules

9.3 GAMMADECAY

Gamma decay or γ decay represents the disintegration of a parent nucleus to a daughter through gamma rays (high energy photons) emission. This transition (γ decay) can be characterized as:

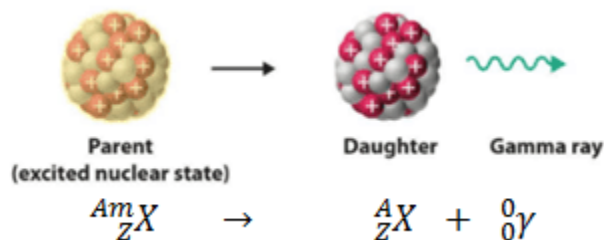


Fig.1: Gamma decay(<http://nuclear-power.com>)

As can be seen, if a nucleus emits a gamma-ray, atomic and mass numbers of the daughter nucleus remain the same, but the daughter nucleus will form a different energy state of the same element. Note that nuclides with equal proton number and mass number (thus making them by definition the same isotope) but in a different energy state are known as nuclear isomers. We usually indicate isomers with a superscript m, thus: ${}^{241m}\text{Am}$ or ${}^{110m}\text{Ag}$.

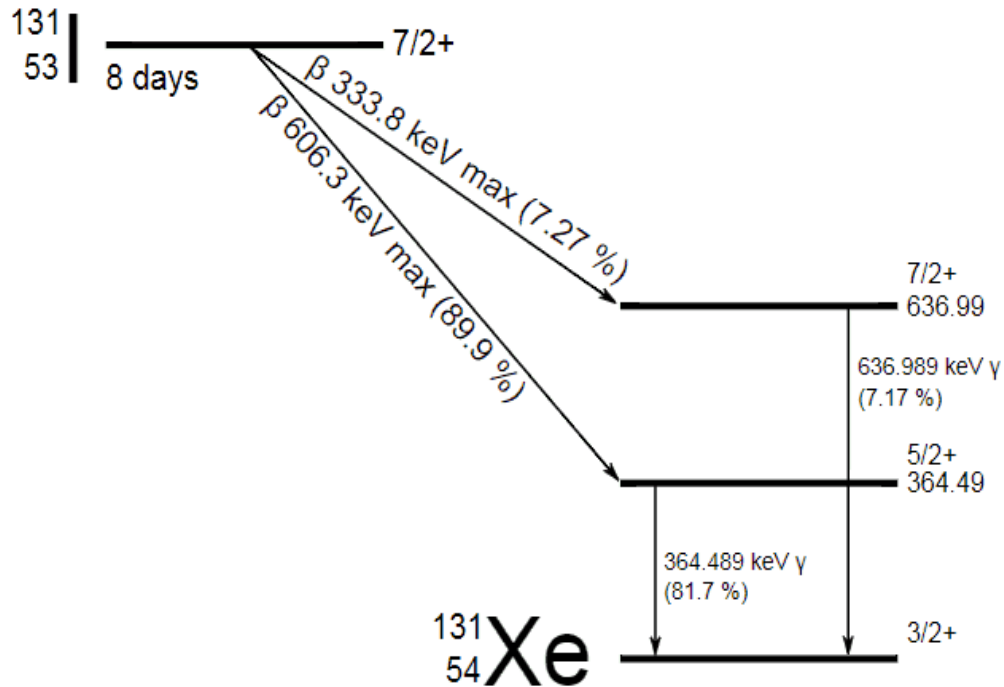


Fig. 02: Decay scheme of Iodine 131(<http://nuclear-power.com>)

In most practical laboratory sources, the excited nuclear states are created in the decay of a parent radionuclide. Therefore, gamma decay typically accompanies other forms of decay, such as alpha or beta decay. After a beta decay (isobaric transition), nuclei usually contain too much energy to be in their final stable or daughter state.

Gamma rays are high-energy photons with very short wavelengths and thus very high frequency. Gamma rays from radioactive decay are in the energy range from a few keV to ~8 MeV, corresponding to the typical energy levels in nuclei with reasonably long lifetimes. As was written, they are produced by the decay of nuclei as they transition from a high energy state to a lower state. Since the gamma rays are in substance only very high-energy photons, they are very penetrating matter and are thus biologically hazardous. Gamma rays can travel thousands of feet in the air and easily pass through the human body.

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In contrast to alpha and beta radioactivity, gamma radioactivity is governed by an electromagnetic interaction rather than a weak or strong interaction. As in atomic transitions, the photon carries away at least one unit of angular momentum (the photon, described by the vector electromagnetic field, has a spin angular momentum of \hbar), and the process conserves parity.

9.4 COMPTON SCATTERING OF GAMMA RAYS

Compton scattering involves the scattering of photons by charged particles where both energy and momentum are transferred to the charged particle while the photon moves off with a reduced energy and a change of momentum. Generally, the charged particle is an electron considered to be at rest and the photon is usually considered to be an energetic photon such as an X-ray photon or gamma ray photon. In this experiment gamma rays from a cesium-137 source are used for the source of photons that are scattered and each photon has an energy of 0.662MeV when incident on the target scatterer. The charged particle is assumed to be an electron at rest in the target. While the theory here is applied to gamma rays and electrons, the theory works just as well for less energetic photons such as found in visible light and other particles.

The theory of Compton scattering uses relativistic mechanics for two reasons. First, it involves the scattering of photons that are massless, and secondly, the energy transferred to the electron is comparable to its rest energy. As a result the energy and momentum of the photons and electrons must be expressed using their relativistic values. The laws of conservation of energy and conservation of momentum are then used with these relativistic values to develop the theory of Compton scattering.

From the special theory of relativity, an object whose rest mass is m_0 and is moving at a velocity v will have a relativistic mass m given by

$$m = \frac{m_0}{\sqrt{1-(v/c)^2}} \quad \dots\dots\dots(1)$$

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The relativistic momentum \vec{p} is defined as $m\vec{v}$ so that squaring both sides of Equation (1) and multiplying by c^4 leads to

$$\begin{aligned} m^2 - m^2(v/c)^2 &= m_0^2, \\ m^2 c^4 - m^2 v^2 c^2 &= m_0^2 c^4, \\ (mc^2)^2 - (pc)^2 &= (m_0 c^2)^2, \\ E^2 - (pc)^2 &= E_0^2, \end{aligned}$$

and

$$(pc)^2 = E^2 - E_0^2. \quad \dots\dots\dots(2)$$

Equation (2) then relates the magnitude of the relativistic momentum \vec{p} of an object to its relativistic energy E and its rest energy E_0 . From this equation it is readily seen that the magnitudes of momentum and energy of a massless particle such as a photon are related by

$$pc = E. \quad \dots\dots\dots(3)$$

or

$$p = \frac{E}{c} = \frac{hv}{c} = \frac{h}{\lambda}. \quad \dots\dots\dots(4)$$

Figure3 illustrates the scattering of an incident photon of energy $E = hv$ moving to the right in the positive x direction with a momentum $p = \frac{hv}{c} = \frac{h}{\lambda}$ and interacting with an electron at rest with momentum $p_e = 0$ and energy equal to its rest energy, $E_0 = m_0 c^2$. The symbols, h, v , and λ , are the standard symbols used for Planck's constant, the photon's frequency, and its wavelength. m_0 is the rest mass of the electron. In the interaction, the gamma ray is scattered in the positive x and y directions at an angle θ with momentum of magnitude $p' = \frac{hv'}{c} = \frac{h}{\lambda'}$ and energy $E' = hv'$. The electron is scattered in the positive x-direction and negative y-direction at an angle ϕ with respect to

the positive x-direction with momentum $p'_e = \frac{1}{c}\sqrt{E_e^2 - E_0^2}$ and energy $E_e = mc^2$ where m is the relativistic mass of the electron after the interaction.

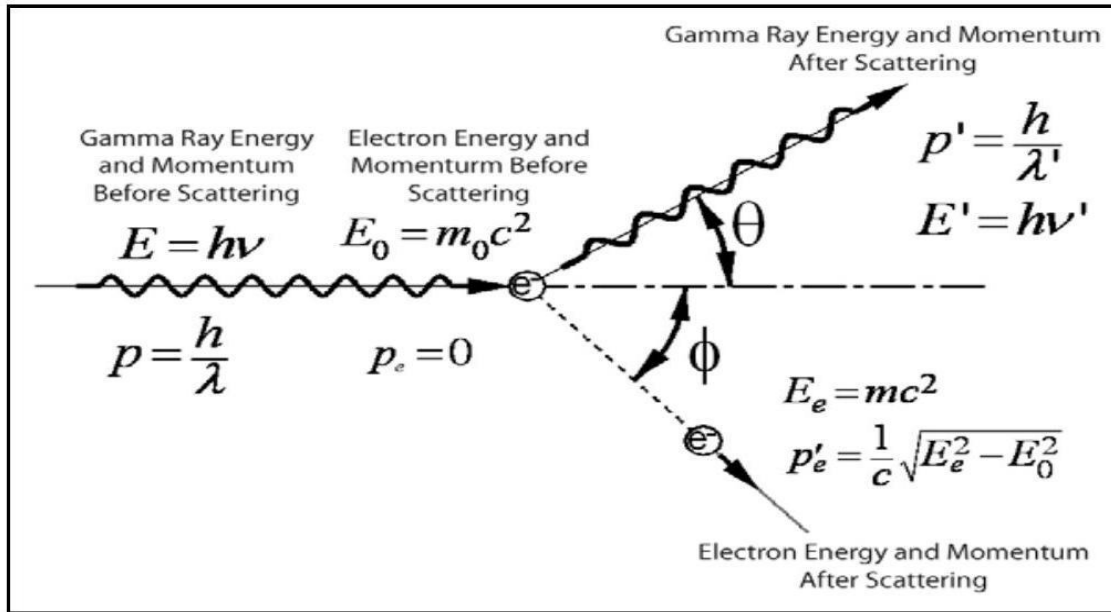


Figure 3:. Compton scattering diagram

From the law of conservation of energy, the energy of the incident gamma ray, $h\nu$, and the rest mass of the electron, E_0 , before scattering is equal to the energy of the scattered gamma ray, $h\nu'$, and the total energy of the electron, E_e , after scattering, or

$$h\nu + E_0 = h\nu' + E_e. \quad \dots\dots\dots(5)$$

From Equation (2), the relationship between the total energy, E_e , of the electron after scattering, its rest mass, E_0 , and its relativistic momentum, p_e , is given by

$$E_e^2 = (p'_e c)^2 + E_0^2 \quad \dots\dots\dots(6)$$

and

$$E_e = \sqrt{(p'_e c)^2 + E_0^2}. \quad \dots\dots\dots(7)$$

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Substituting Equation (7) into Equation (5) yields

$$hv + E_0 = hv' + \sqrt{(p'_e c)^2 + E_0^2}. \quad \text{.....(8)}$$

Using the relationship between the energy of a photon (massless particle) and its momentum from Equation (4) gives

$$pc + E_0 = p'c + \sqrt{(p'_e c)^2 + E_0^2}. \quad \text{.....(9)}$$

Rearranging gives

$$(p - p')c + E_0 = \sqrt{(p'_e c)^2 + E_0^2} \quad \text{.....(10)}$$

and

$$(p - p')^2 c^2 + E_0^2 + 2(p - p')cE_0 = (p'_e c)^2 + E_0^2 \quad \text{.....(11)}$$

that results in the following expression based on conservation of energy

$$p^2 + p'^2 - 2pp' + \frac{2(p-p')E_0}{c} = p_e'^2. \quad (12)$$

Equation (12) is then an expression relating the momentum p_e of the electron given to it by a scattered gamma ray whose initial momentum was p and whose final momentum is p' . The electron was assumed to be initially at rest and it was also assumed to be given enough energy for relativistic mechanics to apply. Equation (12) is solely based on the law of conservation of energy, but another independent expression for the momentum p_e can be found based on the law of conservation of momentum.

In the scattering process momentum must be conserved so that

$$\text{Total Momentum Before} = \text{Total Momentum After} . \quad \text{.....(13)}$$

Since momentum is a vector quantity,

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Total Momentum in X-Direction Before = Total Momentum in X-Direction After(14)

and

Total Momentum in Y-Direction Before = Total Momentum in Y-Direction After(15)

For an electron at rest, its initial momentum is zero and has no x and y components. For an incident gamma ray photon moving in the positive x direction and interacting with an electron at rest, the initial x-component of momentum is p and the y-component is zero so that

$$p = p' \cos \theta + p'_e \cos \phi \quad \text{.....(16)}$$

and

$$0 = p' \sin \theta + (-)p'_e \sin \phi \quad \text{.....(17)}$$

where p' and p'_e are the momenta of the scattered gamma ray and electron after interacting. Rearranging Equations (16) and (17) and squaring both sides of each produces

$$p'_e \cos \phi = p - p' \cos \theta, \dots \dots \dots (18)$$

$$p'_e \sin \phi = p' \sin \theta, \dots \dots \dots (19)$$

$$p_e'^2 \cos^2 \phi = p^2 + p'^2 \cos^2 \theta - 2pp' \cos \theta, \dots \dots \dots (20)$$

and

$$p_e'^2 \sin^2 \phi = p'^2 \sin^2 \theta. \dots \dots \dots (21)$$

Adding Equations (20) and (21) yields

$$p_e'^2 (\sin^2 \phi + \cos^2 \phi) = p^2 + p'^2 (\sin^2 \theta + \cos^2 \theta) - 2pp' \cos \theta \dots \dots \dots (22)$$

That can be simplified using the identity $\cos^2 x + \sin^2 x = 1$ to further yield

$$p_e'^2 = p^2 + p'^2 - 2pp' \cos \theta. \dots \dots \dots (23)$$

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Equation (23) is then an expression based on the law of conservation of momentum that relates the momentum given to the electron from its rest position by the incident gamma ray of momentum p interacting with the electron so that it is scattered off at angle θ with momentum p' .

Equating Equations (12) and (23), one based on conservation of energy and the other on conservation of momentum, gives

$$p^2 + p'^2 - 2pp' + \frac{2(p-p')E_0}{c} = p^2 + p'^2 - 2pp' \cos \theta \quad \dots\dots\dots(24)$$

that reduces to

$$\frac{2(p-p')E_0}{c} = 2pp' - 2pp' \cos \theta, \quad \dots\dots\dots(25)$$

and

$$\frac{1}{p'} - \frac{1}{p} = \frac{c}{E_0} [1 - \cos \theta]. \quad \dots\dots\dots(26)$$

Using the relationships for momentum, energy, wavelength, and frequency for photons,

$$p = \frac{h}{\lambda} = \frac{h\nu}{c} = \frac{E}{c},$$

Equation (26) can be transformed into

$$\frac{1}{E'} - \frac{1}{E} = \frac{1}{E_0} [1 - \cos \theta] \quad \dots\dots\dots(27)$$

that relates the energy of a scattered photon E' to the energy of the incident photon E and the scattering angle θ .

Equation (27) is a simple equation that can be used to verify the theory for the Compton Effect. The energy of incident gamma rays E can be easily measured with a scintillatorphotomultiplier detector and multichannel analyzer system. The energy of the scattered gamma rays E' as a function of θ can also be easily measured with the same

system. A plot of measurements of $\frac{1}{E'} - \frac{1}{E}$ versus measurements of $[1 - \cos \theta]$ should result in a linear graph whose slope is the inverse of the electron's rest energy $\frac{1}{E_0}$.

9.5 PAIR PRODUCTION AND ANIHILATION

Pair production is a process in which a gamma ray of sufficient energy is converted into an electron and a positron. A fundamental law of mechanics, given by Newton, is that in any process total linear (as well as angular) momentum remains unchanged. In the pair-production process a third body is required for momentum conservation. When that body is a heavy nucleus, it takes very little recoil energy, and therefore the threshold is just twice the rest energy of the electron; i.e., twice its mass, m , times the square of the velocity of light, c^2 , or $2mc^2$.

Pair production also can occur in the field of an atomic electron, to which considerable recoil energy is thereby imparted. The threshold for such a process is four times the rest energy of an electron, or $4mc^2$. The total pair-production cross section is the sum of the two components, nuclear and electronic. These cross sections depend on the energy of the gamma ray and are usually calculated in an electron theory proposed by the British physicist P.A.M. Dirac through a method of approximation that is a simplification of a method (a "first approximation") devised by the German physicist Max Born (i.e., a "first Born approximation"). The process is envisaged by Dirac as the transition of an electron from a negative to a positive energy state. Corrections are required for these cross sections at high energy, at high atomic number, and for atomic screening (the intrusion of the field of the electrons in an atom); these are normally made via numerical procedures. The fraction of residual energy, symbolized by the Greek letter alpha, unexpended in conversion of energy to mass, that appears in any one particle (e.g., the electron) is thus given by the kinetic energy of that electron E_e minus its rest energy mc^2 divided by the energy of the gamma ray $h\nu$ (i.e., the product of Planck's constant and the frequency of the gamma ray) minus twice the rest energy of the electron $2mc^2$, or $\alpha = (E_e - mc^2)/(h\nu - 2mc^2)$.

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$2mc^2$). Because the same equation applies to each of the two electrons that are formed, it must be symmetric about the condition that each of the particles has half the residual energy, symbolized by the Greek letter alpha, α (in excess of that conveyed to the “third body”); i.e., that $\alpha = 0.5$. Below an energy of about 10,000,000 eV for the gamma ray, the probability for pair production (i.e., the pair-production cross section) is almost independent of the atomic number of the material, and, up to about 100,000,000 eV of energy, it is also almost independent of the quantity α . Even at extremely high energies the probability that a certain fraction of the total available energy will appear in one particle is almost independent of the fraction as long as energy is comparably distributed between the two particles (excepting in cases in which almost all energy is dumped into one particle alone). Typical pair-production cross sections at 100 MeV (million electron volts) are approximately 10^{-24} to 10^{-22} square centimetre, increasing with atomic number. At high energies, approximately equal to or greater than 100 MeV, pair production is the dominant mechanism of radiation interaction with matter.

Clearly, as the photon energy increases, the dominant interaction mechanism shifts from photoelectric effect to Compton scattering to pair production. Rarely do photoelectric effect and pair production compete at a given energy. Compton scattering, however, at relatively low energy competes with the photoelectric effect and at high energy competes with pair production. Thus, in lead, interaction below 0.1 MeV is almost exclusively photoelectric; between 0.1 MeV and 2.5 MeV both photoelectric and Compton processes occur; and between 2.5 MeV and 100 MeV Compton scattering and pair production share the interaction. In the pair process the photon is annihilated, and an electron–positron pair is created. On the other hand, an electron or positron with energy approximately equal to or greater than 100 MeV loses its energy almost exclusively by production of high-energy bremsstrahlung (X rays produced by decelerating electric charges) as the result of interaction with the field of a nucleus. The cross section for bremsstrahlung production is nearly independent of energy at high energies, whereas at low energies the dominant energy-loss mechanism is by the creation of ionizations and excitations. A succession of

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bremsstrahlung and pair-production processes generates a cascade or shower in the absorber substance. This phenomenon can be triggered by an electron, a positron, or a photon, the triggering mechanism being unimportant as long as the starting energy is high. A photon generates a pair through pair production, and the charged particles generate photons through bremsstrahlung, and so on repeatedly as long as the energy is kept sufficiently high. With penetration into the substance, the shower increases in size at first, reaches a maximum, and then gradually decreases. Loss of particles by degradation to lower energies (in which the yield of bremsstrahlung is low), ionization loss, and production and absorption of low-energy photons eventually reduce the size of the cascade. The mathematical theory of cascades has been developed in great detail.

X rays and gamma rays

When light of sufficiently high frequency (or energy equal to $h\nu$), independent of its source, is absorbed in a molecular system, the excited molecular state so produced, or some excited state resultant from it, may either interact with other molecules or decompose to produce intermediate or ultimate products; i.e., chemical reactions ensue. Study of such processes is encompassed in the subject of photochemistry (see below Molecular activation).

Electromagnetic waves of energy greater than those usually described as ultraviolet light (see Figure 2) are included in the classes of X rays or gamma rays. X-ray and gamma-ray photons may be distinguished by definition on the basis of source. They are indistinguishable on the basis of effects when their energy is absorbed in matter.

9.6 K-CAPTURE or ELECTRON CAPTURE

The capture of electrons is a nuclear process. An atom's inner electron is absorbed by a nucleus rich in protons. The nucleus emits high-energy gamma rays as part of this process. The weak force causes electron capture, which is a relatively minor decay phase. If the number of protons in a nucleus exceeds the number of neutrons, electron capture occurs. The nucleus of an atom absorbs an electron from the atom's K-shell or L-shell. The absorbed electron and one proton then unite to generate a neutron, bringing the number of protons and neutrons in the nucleus back to a healthy balance.

The capture of a 1s electron by the nucleus of an unstable isotope is known as K-electron capture. When the nucleus of an unstable isotope captures an inner-orbital electron, it is known as electron capture. A proton combines with an electron to make a neutron, and an X-ray is produced as a result of the process.

The atomic number decreases by one unit, but the mass number stays the same. Because they are nearest to the nucleus, the captured electron usually comes from the 1s or 2s orbitals.

The process is known as K-electron capture if the electron arrives from the 1s level (the K-shell). The mass number of a nucleus does not change when it undergoes an electron capture reaction, but the atomic number lowers by one. The total number of protons and neutrons inside the nucleus is known as the mass number, while the number of protons or electrons in an atom is known as the atomic number. The electron is grabbed by the nucleus in an electron capture reaction.

9.7 INTERNAL CONVERSION

Internal conversion is another electromagnetic process which can occur in the nucleus and which competes with gamma emission. Sometimes the multipole electric fields of the nucleus interact with orbital electrons with enough energy to eject them from the atom. This process is not the same as emitting a gamma ray which knocks an electron out of the atom. It is also not the same as beta decay, since the emitted electron was previously one of the orbital electrons, whereas the electron in beta decay is produced by the decay of a neutron.

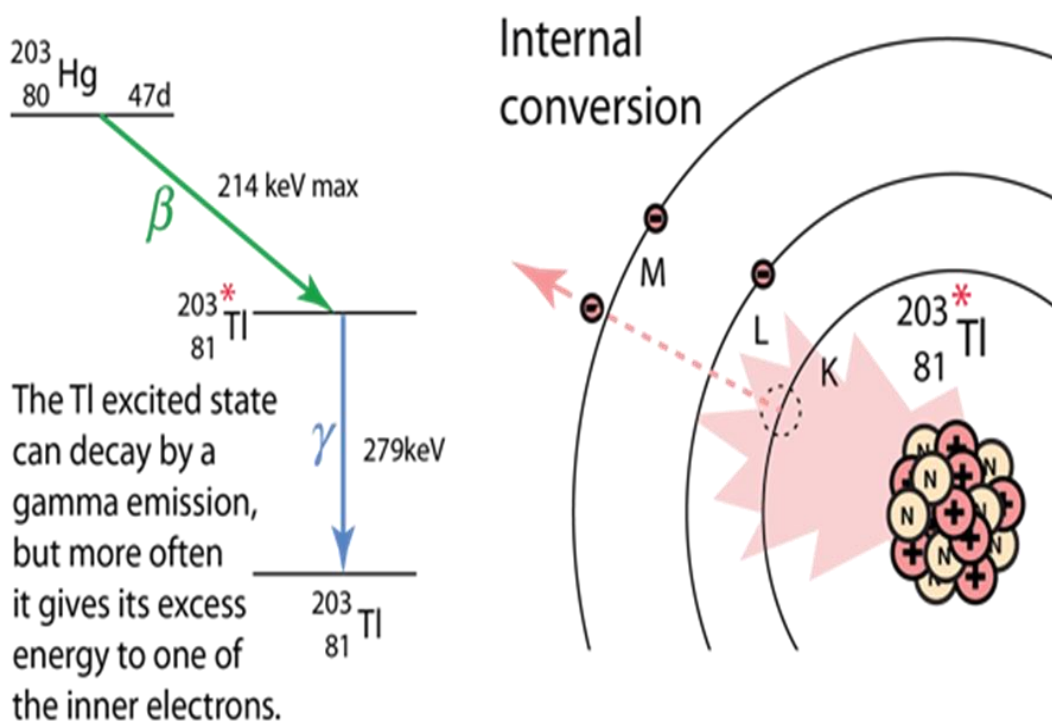


Fig.4: Internal Conversion

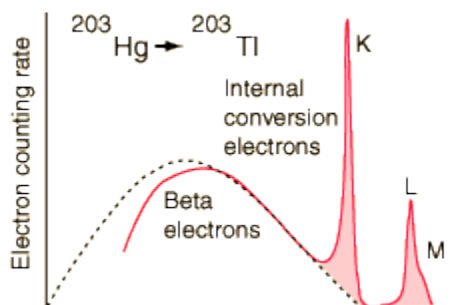
An example used by Krane is that of ^{203}Hg , which decays to ^{203}Tl by beta emission, leaving the ^{203}Tl in an electromagnetically excited state. It can proceed to the ground state

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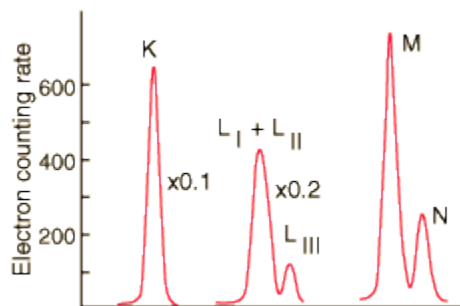
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by emitting a 279.190 keV gamma ray, or by internal conversion. In this case the internal conversion is more probable. Since the internal conversion process can interact with any of the orbital electrons, the result is a spectrum of internal conversion electrons which will be seen as superimposed upon the electron energy spectrum of the beta emission. The energy yield of this electromagnetic transition can be taken as 279.190 keV, so the ejected electrons will have that energy minus their binding energy in the ^{203}Tl daughter atom.

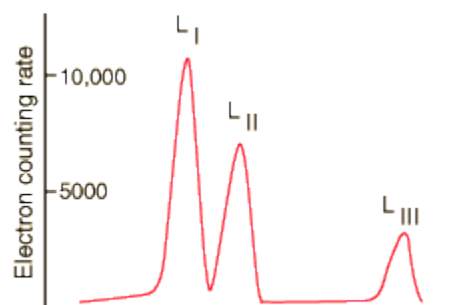
The diagram above is of course conceptual only and not to scale since the nuclear radius of thallium is modeled to be about 0.7×10^{-14} m and the radius of the atom is about 1.76×10^{-10} m, a factor of about 25,000 larger! And of course the planetary type orbits of the electrons are unrealistic since the wave properties of the electrons lead to charge distributions that give a finite probability that the K electron shown above will actually extend inside the nucleus so that the nucleus can interact with it and hand off its excess energy. An examination of the electron distribution for the simplest atom, hydrogen, can give the perspective that the electron has a small but finite probability of extending into the nucleus. From the table of binding energies below, you can see that the binding energy of the K-shell electron is over 85,000 electron volts compared to 13.6 eV for the hydrogen electron, or over 6,000 times larger.



Electron emissions from the Hg-203 to Tl-203 decay, measured by A. H. Wapstra, et al., *Physica* 20, 169 (1954).



At higher resolution, the internal conversion electrons from the L, M and N shells can be resolved. Z. Sujkowski, Ark. Fys. 20, 243 (1961).



At even higher resolution, the three L shells can be resolved. From C. J. Herrlander and R. L. Graham, Nucl. Phys. 58, 544 (1964).

The resolution of the electron detection is good enough that such internal conversion electron spectra can be used to study the binding energies of the electrons in heavy atoms. In this case, the measured electron energies can be subtracted from the transition energy as indicated by the gamma emission, 279.190 keV.

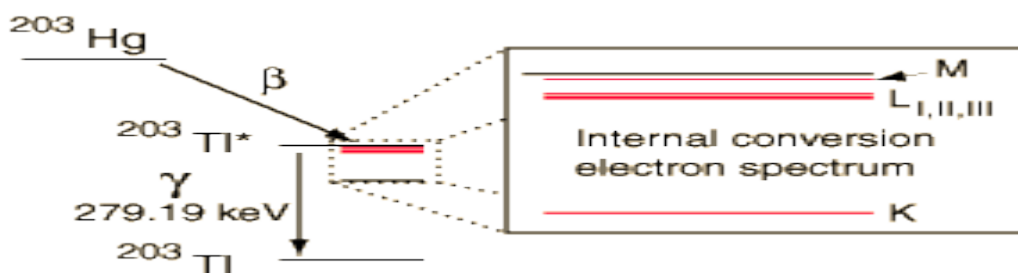


Fig.05: Internal conversion electron spectrum

In addition to information from the internal conversion electrons about the binding energies of the electrons in the daughter atom, the relative intensities of these internal

conversion electron peaks can give information about the electric multipole character of the nucleus.

9.8 MULTIPOLARITY IN GAMMA TRANSITIONS

Transitions between excited states (or excited states and the ground state) of a nuclide lead to the emission of gamma quanta. These can be classified by their multipolarity. There are two kinds: electric and magnetic multipole radiation. Each of these, being electromagnetic radiation, consists of an electric and a magnetic field.

The conservation of angular momentum and parity between the emitting system and the radiation field forms the basis for categorising the various photon emission processes. The potential (V_L) resulting from an electric multiple of an order (L) at the origin is often described by

$$V_L = \frac{1}{4\pi\epsilon_0 R_0^{L+1}} \sum q_i r_i^2 P_L(\cos \theta_i) \quad \dots(28)$$

Where R_0 is the position vector of the field-point, $P_L(\cos)$ is the Legendre polynomial of order L , q_i is the i th charge, r_i is the position of i th charge from origin, and there is an angle between r_i and R_0 . In Eqn.28, the various terms represent the electric vectors of the radiation fields resulting from various orders of multipoles. The radiation caused by an electric monopole, which does not radiate energy, is represented by the first term. The second term is the electric dipole's (E1) time-varying moment's radiation field. The following word denotes the radiation field of the magnetic dipole (M1) of the time-varying moment as well as the radiation field of the electric quadruple (E2) of the time-varying moment.

In general, V_L is equivalent to the radiation field of a magnetic $2L-1$ pole or an electric $2L$ pole (EL) with time-varying moments. L is a measurement of the radiation's

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multipolarity and the amount of angular momentum that was transmitted during the radiative transition. The orbital angular momentum transferred in the case of particle emission is equivalent to this L.

The multiple order of a transition is referred to as 2^L , as was already mentioned earlier in this section. L stands for the multipolarity of photons, and multipole radiation and transition are referred to

- as
- (i) dipole (L=1) i.e., $|\Delta\bar{I}| = |\bar{L}| = 1$ or $2^1 = 2$
 - (ii) quadrupole (L=2) i.e., $|\Delta\bar{I}| = |\bar{L}| = 2$ or $2^2 = 4$
 - (iii) octupole (L=3) i.e., $|\Delta\bar{I}| = |\bar{L}| = 3$ or $2^3 = 8$ and higher order transitions

For any order, the transitions can be either Electric or Magnetic, in nature.

9.9 SELECTION RULES

In simple definition we have

$$|I_i - I_f| \leq L \leq |I_i + I_f| \quad \text{.....(29)}$$

So, we have the following selection rules for the γ -ray emission process.

- i) the γ -photon must carry away at least one unit of angular momentum, and
- ii) in γ -decay process, the polarity must always be conserved.

The change in the magnitude of the angular momentum is given by

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$$L = \Delta I = |I_i \pm I_f| \quad \dots(30)$$

Suppose, the nuclear spin changes from 4 to 2 during a transition. Then ΔI is simply the scalar difference 2. However, $|\Delta I|$ which is the vector change in angular momentum ($\neq \Delta I$) can have values i.e. $|I_i - I_f|$ to $|I_i + I_f|$ 2,3,4,5 and 6 in this case, since any integral value from 4-2 to 4+2 is possible. Remember $L = 0$ γ -emission is not allowed.

The parity change for the nucleus determines the nature of the multipole transition.

The γ -transitions are classified as electric 2^L pole and magnetic 2^L pole according to

$$\pi = (-1)^L \text{ for electric } 2^L \text{ pole radiation}$$

$$\pi = (-1)^{L+1} \text{ for magnetic } 2^L \text{ pole radiation}$$

The radiation field can have even or odd parity for a given L , depending on whether the nature is electric or magnetic.

The electric multipole radiation, denoted by $E1, E2, \dots, EL$ has parity $(-1)^L$ and magnetic multipole radiation $M1, M2, \dots, ML$ has parity $(-1)^{L+1}$. The electric and magnetic/multipoles of the same order L have opposite parity. The electric and magnetic dipole moment are shown in Fig.

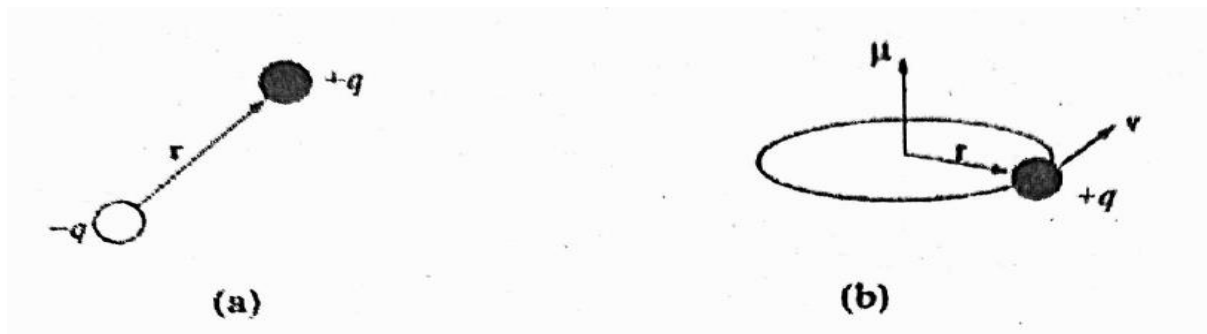


Fig.6 (a) Electric dipole moment (b)Magnetic dipole moment

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An electric dipole, as shown in Fig6 (a) is a displacement of charge q and is of the form which under the parity operation of inversion transforms into $-$ and therefore, has 'odd' (negative) parity. A magnetic dipole like a circulatory charge q which forms a current loop (Fig.6(b)), has a magnetic moment proportional to $(q \mathbf{r} \times \mathbf{v})$, which under inversion, transforms into $[q (-\mathbf{r}) \times (-\mathbf{v})]$ i.e. it does not change sign and so has even (positive) parity. The emission rate strongly depends on photon energy as well as on multipolarity. This means that normally, radiation will be emitted with lowest multipolarity, allowed by the selection rules.

9.10 SUMMARY

After studying this unit, the learners have learnt about, how to

- Explain Gamma Decay
- Discuss Compton Scattering of Gamma Rays
- Describe Pair Production and Annihilation
- Explain K-capture or Electron Capture
- Describe Internal Conversion
- Discuss Multipolarity in Gamma Transitions
- Explain Selection Rules

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9.12 SUGGESTED READINGS

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2. R. M. Singru, " Introduction to Experimental Nuclear Physics", Wiley Eastern Pvt. Ltd., 1972, Chs. 1, 4, and 7.
3. G. F. Knoll, "Radiation Detection and Measurement" John Wiley & Sons, 1979, Chs. 2, 10, 16, 17, and 18.
4. S. S. Kapoor and V. S. Ramamurthy, "Nuclear Radiation Detector", Wiley Eastern Ltd., 1986, Ch V, page 176, 185.
5. GDM 10, Handbook and other related manuals.

9.13 TERMINAL QUESTIONS

1. Explain Gamma decay.
2. Discuss the multipolarity of a gamma transition and explain the selection rules.
3. Describe Compton theory of scattering in detail.
4. What is pair production and annihilation in gamma rays?
5. What do you understand by electron capture in gamma decay?
6. Write a note on Internal Conversion.

UNIT 10**NUCLEAR REACTIONS**

Structure of the Unit

10.1 Introduction

10.2 Objectives

10.3 Types of Nuclear Reaction

10.4. Conservation Laws for Nuclear Reactions

10.5 Charged Particle Induced Nuclear Reactions

10.5.1 Proton Induced Nuclear Reactions.

10.5.2 Deuteron Induced Nuclear Reactions:

10.5.3 α -particle induced reactions

10.6 Neutron Induced Reactions

10.7 Photo-disintegration (Nuclear Reactions Induced by Photons or γ -rays)

10.8 Reaction Energetics - the Q-value Equation

10.9 Nuclear Reaction Cross-Sections

10.10 Partial Wave Analysis of Nuclear Reaction Cross-sections

10.11 Summary

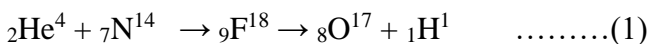
10.12 References

10.13 Suggested Readings

10.14 Terminal Question

10.1 INTRODUCTION

We have preliminary idea that our present-day knowledge about the nuclear structure involves many experiments involving bombardment of heavy nuclei by means of lighter energetic particles. Rutherford used particles from a radioactive source to create the first nuclear reaction in his lab in 1919. The artificial transmutation of elements was discovered as a result of the experiment that generated artificial nuclei disintegration. A composite system was created in Rutherford's experiment as a result of a very high velocity particle colliding head-on collision with nitrogen nuclei N^{14} . This composite system disintegrated in matter of 10^{-15} seconds, generating a high-velocity proton and a small amount of remnant nuclear material. An equation analogous to the equation for a chemical reaction that is given below can be used to represent the nuclear transmutation process mentioned below



In Eqn.1 the composite system ${}_9\text{F}^{18}$ is known as compound nucleus.

.Any such collision, in which the characteristic of the struck particle is changed from original identity, is known as a nuclear reaction.

In general, a nuclear reaction can be represented by



or in compact notation it can be written as $X(x, y) Y$

A compound nucleus is created when the incident particle (or projectile) x collides with the target nucleus X . This compound nucleus quickly disintegrates to form a residual nucleus Y and a product particle y . Most nuclear reactions involve light particles like neutrons, protons, deuterons, and particles as the incident and product particles. Nuclear

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reactions offer a testing ground for theories regarding nuclear forces as well as a crucial function in elucidating the structure of nuclei. Nuclear spectroscopy puts a high value on the information that nuclear reactions convey regarding the energy levels and decay patterns of the excited product nucleus.

10.2 OBJECTIVES

After studying this unit learners will be able to how to

- Describe the Types of Nuclear Reaction
- Explain the Conservation Laws for Nuclear Reactions
- Discriminate between Charged Particle Induced Nuclear Reactions
- Understand the Photo-disintegration process
- Evaluate the Reaction Energetics - the Q-value Equation
- Explain Nuclear Reaction Cross-Sections
- Describe Partial Wave Analysis of Nuclear Reaction Cross-sections

10.3 TYPES OF NUCLEAR REACTION

Various types of nuclear reactions are possible when a target nucleus is bombarded by means of energetic projectile particles, the bombardment process may give rise to the following events singly or jointly.

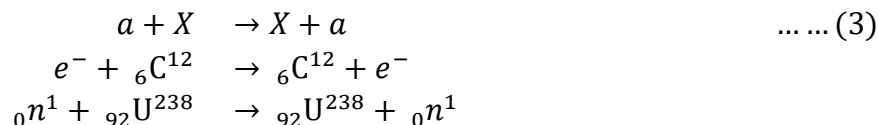
(i) Elastic Scattering :It is a process in which the incident and the outgoing particles are the same and the kinetic energy is conserved. Thus, if the incident particles and the target nuclei are simply scattered by each other without any change in their relative energy, the process is said to be elastic scattering.

If the incident particle is denoted by ' a ' and the target nucleus by X , then the elastic scattering process may be symbolically written as

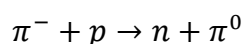
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e.g.,



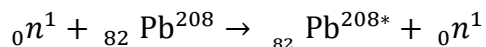
Sometimes the charge exchange reactions like the one given below, are also referred to as elastic scattering.



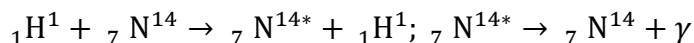
(ii) Inelastic Scattering: The bombardment process may result into a change in the relative energy without change in the internal structure of the target nucleus. This type of process is known as inelastic scattering and in it although the incident and the outgoing particles are the same but kinetic energy is not conserved during the process. In this process the target nucleus, after the interaction, is not left in its initial state but is raised to an excited state and so the emergent particle, although the same as the incident particle, emerges with an energy lower than the energy of incident particle by an amount equal to the excitation energy given to the target nucleus. Symbolically inelastic scattering is represented as



where * indicates that the target nucleus is in an excited state. To quote specific examples



Subsequently ${}_{82}\text{Pb}^{208*}$ emits γ -rays.



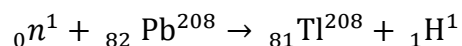
(iii) Nuclear Reactions: Although in the broadest sense, nuclear reactions may be taken to include all types of processes which result from the bombardment of target

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nuclei by means of an energetic particle but the term nuclear reaction is generally used to indicate the processes in which the incident and the emergent particles are not the same but are different i.e. an initial state involving nucleons is converted into a different final state involving nucleons. Therefore, a nuclear reaction is a process in which a change in the composition (internal structure) and/ or energy of the target nucleus is brought about through bombardment with a nuclear projectile. The projectiles may be nucleons, complex nuclei, photons, electrons or any particle with a life time $\gg 10^{-8}$ sec. The target also may be a single nucleon or a complex nucleus. Symbolically, a nuclear reaction is written as



To quote a particular example;



It is a standard practice to designate a nuclear reaction symbolically by enclosing the incoming and outgoing particles in parentheses separated by a comma and writing the target nucleus on the left of the parentheses and product nucleus to the right of it. Thus, the reactions (11.3) may be written as



or in particular ${}_{82}\text{Pb}^{208}(n, p){}_{81}\text{Tl}^{208}$

Ever since the discovery of the first artificial nuclear reaction by Rutherford [${}_2\text{He}^4 + {}_7\text{N}^{14} \rightarrow {}_8\text{O}^{17} + {}_1\text{H}^1$], experimental nuclear physicists have been concerned with the investigations of various kinds of nuclear reactions. As we have seen above, a nuclear reaction is usually initiated by exposing a suitable target material to a collimated beam of mono-energetic lighter nuclear particles (protons, neutrons, α -particles etc.). While using charged particles as projectiles, it is be noted that the projectile must have sufficient

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energy to overcome the Coulomb potential barrier projectile must have sufficient energy to overcome the Coulomb potential barrier surrounding the target nucleus. This necessitated the development of charged particle accelerators to accelerate the projectiles to high enough energies. Therefore, with the development of particle accelerators in 1932 onwards and now nuclear reactions may be produced with any of the element of the periodic table. Studies on the cross-sections of nuclear reactions and the angular distributions of the disintegration fragments coupled with the various theoretical led to a better understanding of the nature of nuclear forces.

The richness of phenomena encountered in the study of nuclear reactions is revealed by the colorful terminology in current use such as, the cloudy crystal ball model of the nucleus, boil off processes, stripping (${}_2\text{He}^4 + {}_8\text{O}^{16} \rightarrow {}_9\text{F}^{18} + {}_1\text{H}^2$) i.e. projectile losing nucleons to the target and pick up reactions, (${}_1\text{H}^2 + {}_8\text{O}^{16} \rightarrow {}_8\text{O}^{15} + {}_1\text{H}^3$) i.e. projectile gaining nucleons from the target, knock out, spallations (nuclear reaction in which several particles are emitted) etc. The study of nuclear physics covers one of the largest areas of nuclear and subnuclear physics and in a brief survey (as in this chapter) with any pretensions to coherences it is impossible therefore to make detailed mention of all the phenomena : Ideally, the task of a comprehensive theory of nuclear reactions is to predict the cross-sections for individual energetically allowed processes that can be initiated by the bombardment of a given target nucleus with a given projectile. In the search for such a theory, we shall be limiting ourselves to a modest goal of predicting the absorption and scattering cross-sections of the nuclear reactions in this unit.

The study of nuclear reactions involves the measurement of the following quantities:

- (i) Intensity, energy and identity of the incident beam of particles.
- (ii) The number of particles which are emitted from the target per unit time.
- (iii) The energy and identity of the emitted particles and of the residual (product) nucleus.

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(v) Induced activity of the product nucleus if any. type:

Nuclear reactions at low excitation energies (below 10MeV) are mostly of the



where Q is the energy released or absorbed during the reaction. It is equal to the total rest-mass of the left-hand side minus the total rest mass of the right-hand side of the equation, multiplied by c^2 . In these reactions the major share of available kinetic energy is carried away by the emitted particle. If the product nucleus Y happens to be in an excited state after the emission of the light particle hitherto designated by b , it subsequently decays through one or more γ -emissions. The product nucleus may also be beta unstable. In such a case, it decays by electron or positron emission at some later time, possibly also followed by γ -emission.

Nuclear reactions at low excitation energies include following types of reactions:

(i) (p, γ) , (ii) (p, n) , (iii) (p, α) , (iv) (n, γ) , (v) (n, p) , (vi) (n, α) ,

(vii) (d, p) , (viii) (d, n) , (ix) (α, n) ...

An approximate classification of nuclear reactions may be done on the basis of energy and wavelength of the incident particle. If the wavelength of incident particle is large in comparison with the size of the nucleus, the scattering is found to be insensitive to the detailed structure of the nucleus and the nucleus as a whole act coherently. As the energy of the incident particle increases, its wavelength decreases, the interaction takes place with a local cluster of nucleons only and not with the nucleus as a whole. When the energy of the incident particle becomes so high that the wavelength associated with it becomes about the size of a nucleon, the primary interaction takes place with a nucleon only although secondary interactions may raise the residual nucleus to an excited state

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which may evaporate nucleons and decay to ground state electro-magnetically. At an energy of several hundred MeV and greater, meson production predominates.

The wavelength associated with an incident nucleon of kinetic energy K is,

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mK}} \text{ and hence } K = \frac{h^2}{2m\lambda^2}$$

If $\lambda = 10^{-14}$ meters Nuclear Reactions

$$K = \frac{(6.625 \times 10^{-34} \text{Joule} - \text{Sec})^2}{2 \times (1.67 \times 10^{-27} \text{Kg})(10^{-14} \text{ m})^2(1.6 \times 10^{-19} \text{ J/eV})}$$

$$\approx 8 \times 10^6 \text{ eV} = 8 \text{ MeV}$$

So, we find that up to an energy $\approx 10 \text{ MeV}$, the nucleus as a whole act as one entity. If the wavelength becomes 1/10 of this value *i. e.* $\lambda = 10^{-15}$ meter, the kinetic energy has to increase by a factor of 100 as kinetic energy is inversely proportional.

As the energy of the projectile particle is increased, the type and variety of the possible type of nuclear reactions also increases. For example, if the bombarding energy of the nucleons lies between 20 and 30 MeV, two light particles instead of just one for 10 MeV projectile energies, may be emitted. Examples of such reactions are (p, pn) , (n, pn) reactions etc.

As the projectile energy is further increased and reaches beyond about 200 MeV which is the threshold energy for the creation of pions (π -mesons), we enter a new field - the field of sub-nuclear Physics. Thus, the distinctive boundary between nuclear and sub-nuclear physics is set by the threshold energy, for the creation of pions. Below this threshold energy we deal only with nuclear reactions in the strict sense of the word and above this threshold, mesic and particle phenomena become predominantly important until projectile energy reaches multi BeV region. In this region nuclear features become of secondary importance and the interactions between sub-nuclear particles like pions,

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nucleons, K -mesons etc. become most prominent features, while nuclear composition of targets becomes of secondary importance.

We can sum up these consideration as follows:

- 1 When the energy of the incident particle lies below about 10MeV, the nucleus as a whole act as one entity.
- 2 When the energy of the incident particle is of the order of few hundreds of MeV, interactions with local clusters of nucleons predominate.
- 3 At bombarding energy of several hundred MeV, meson production predominates and nuclear structure becomes of little importance.

10.4 CONSERVATION LAWS FOR NUCLEAR REACTIONS

Nuclear reactions like several other physical phenomena and like chemical reactions are governed by certain conservation laws which provide very valuable information about the nuclear reactions. These conservation laws represent certain features of the nuclear reactions which are common to all and are of great importance in analyzing the experimental data. We list below these conservation properties.

(i)Conservation of Electric Charge: The total electric charge is conserved in all nuclear reactions. This means for example that a proton can change itself into a neutron but a positron or a positive meson must appear or all electron must disappear in the process. A most straight forward confirmation of electric charge conservation comes from the creation of electron-positron pair.

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(ii) Conservation of Nucleons : So far no nuclear reaction has been observed in which the total number of nucleons i.e. the value A is not conserved. On the contrary in all nuclear reactions, nucleon number is always conserved.

(iii) Conservation of the Constants of motion of Classical and Quantum Mechanics : In all nuclear reactions, at least to an order of accuracy beyond experimental interest, the total energy (sum of the rest mass energy and the kinetic energy), the linear and angular momenta, spin angular momentum, isotopic spin to be defined later in chapter 13 and parity (provided weak interactions like β and meson decay are excluded but not electro-magnetic processes), are conserved. The conservation of energy helps us to calculate Q -value of the reaction. The conservation of linear momentum guarantees that the incoming particle and the product nucleus move in a plane if only two product nuclei result.

(iv) Conservation of statistics: The conservation of statistics is also followed by all nuclear reactions. This law asserts that the spin character of a closed system cannot change.

10.5 CHARGED PARTICLE INDUCED NUCLEAR REACTIONS

Charged particle induced nuclear reactions are those which are induced by charged particles like protons, deuterons, α -particles etc. With the invention of charged particle accelerators capable of accelerating particles, to the energy range of several hundred BeV, the scope and variety of such reactions has become so large that even a brief mention of these all, will cover a very large volume. For a systematic study, these nuclear reactions are classified therefore in accordance with the energy of the incident particle as follows:

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- (i) Low energy reactions ($0 < E^i < 100\text{eV}$)
- (ii) Moderate energy reactions ($1\text{keV} < E^i < 500\text{keV}$)
- (iii) High energy reactions ($0.5\text{MeV} < E^i < 10\text{MeV}$)
- (iv) Very high energy reactions ($10\text{MeV} < E^i < 50\text{MeV}$)
- (v) Ultra high energy reactions ($E^i > 50\text{MeV}$)

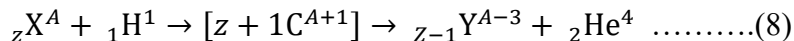
Target nuclei with $A < 25$, are called light nuclei, those with $25 < A < 80$, intermediate nuclei and those with $80 < A < 250$. are called heavy nuclei.

Under the charged particle induced reactions, we first put forth nuclear reactions induced by protons.

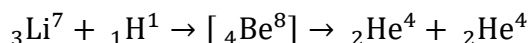
10.5.1 Proton Induced Nuclear Reactions.

Proton induced reactions include (p, α) , (p, n) , (p, γ) and (p, d) reactions.

(i) **(p, α) reactions.** (p, α) nuclear reactions can in general be represented as

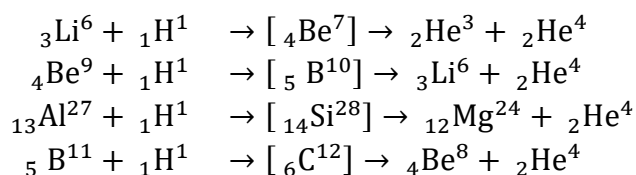


If we recollect, this type of reaction has the status of being the first artificially induced reaction, studied by Walton and Cock-croft in 1932. In the reaction studied by Walton and Cockcroft ${}_3 Li^7$ target was bombarded by means of 0.7MeV protons to produce two α -particles as follows.



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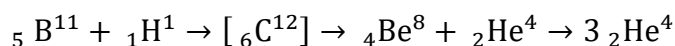
The reaction is important on account of another historical background in as much as it was this reaction which provided the first quantitative verification of Einstein's mass energy relation ($E = mc^2$). Further examples of reactions of this type are :



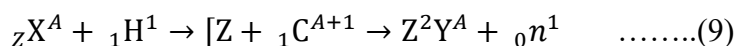
In the last reaction ${}_4\text{Be}^8$ nucleus is unstable and therefore breaks up giving rise to two α -particles.



Thus the final reaction reads

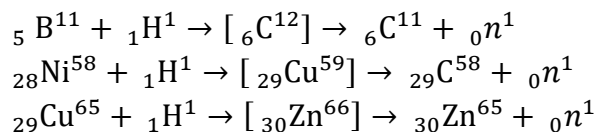


(ii) (p, n) reactions : In general scheme such reactions can be written as



It is clear from the above equation that these type of reactions produce a nucleus which is one atomic number higher than the original (target) nucleus and as such the nucleus rises higher up (to the right) in the periodic table.

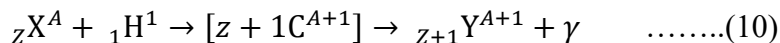
Examples of these reactions are :



All the (p, n) reaction are usually endoergic reactions.

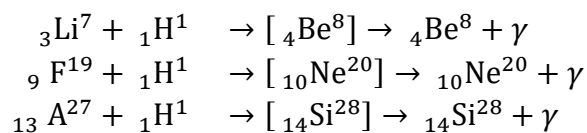
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(iii) (p, γ) reactions: The general scheme of such reactions is



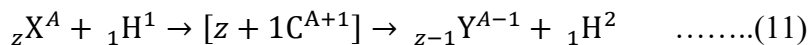
In this case also the product nucleus has an atomic number greater by 1 than the target nucleus and mass number also increases by unity.

Examples of this type are

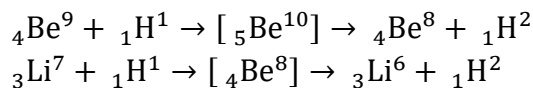


The γ -rays which are produced in these reactions are very energetic and can be used to induce nuclear reactions.

(iv) (p, d) reactions: The general scheme of these reaction is



Note that in this case, the atomic number of the product nucleus is one less than that of the target nucleus and therefore the product nucleus goes one place down (to the left) in the periodic table.

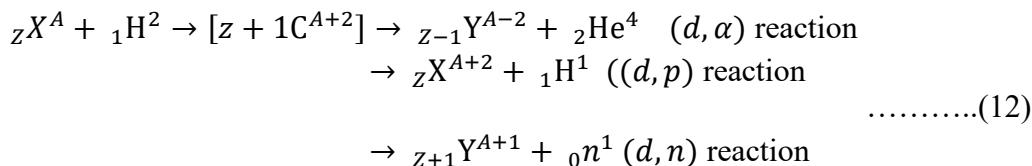
**10.5.2 Deuteron Induced Nuclear Reactions:**

These reactions include (d, α), (d, p) and (d, n) types of nuclear reactions.

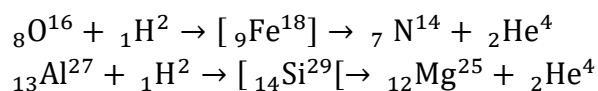
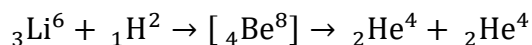
On general scheme, all these reactions may be written as;

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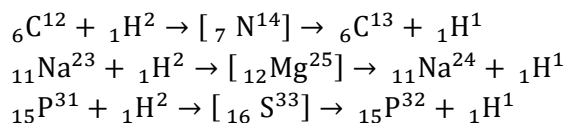
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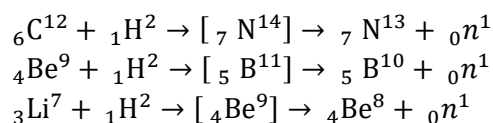
Examples: (i) (d, α) reactions



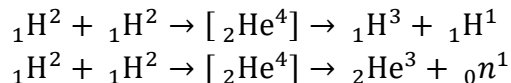
(ii) (d, p) reaction



(iii) (d, n) reaction



However most interesting which the target itself contains deuterons, e.g.

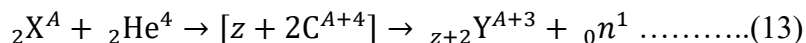


The Q values for (d, p) reactions are generally positive and so they are exoergic reactions.

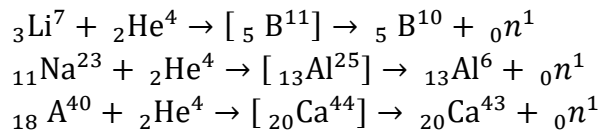
10.5.3 α -particle induced reactions

(i) (α , n) reactions: General scheme of such reactions is

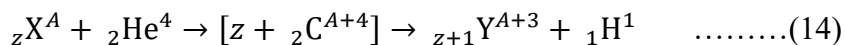
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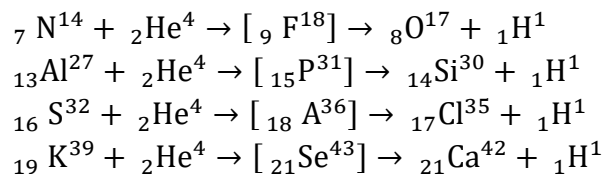
Particular examples are



(ii) (α, p) reactions. General scheme of such reactions is

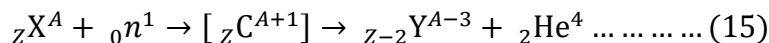


Particular Examples

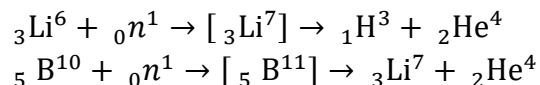
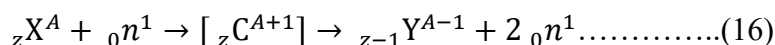


10.6 NEUTRON INDUCED REACTIONS

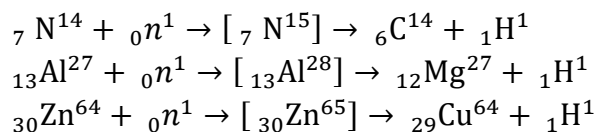
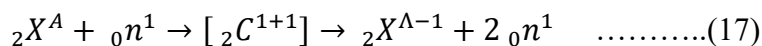
Neutron induced nuclear reactions are the most interesting ones. Since neutrons are uncharged particles, they do not have to overcome the Coulomb potential barrier surrounding a target nucleus and therefore even slow neutrons are very much effective in inducing nuclear reactions. Because of this property more very much effective in inducing nuclear reactions. Because of this property more induced reactions. Neutron induced reactions because of their use in nuclear reactors, have a special status. Types of the neutron induced reactions are given as (i)(n, α), (ii)(n, p), (iii)(n, γ) and ($n, 2n$).

NUCLEAR PHYSICS**MSCPH511****(i) (n, α) reactions:** (n, α) reaction

Particular examples are

**(ii) (n, p) reactions.**

Particular examples:

**(iii) (n, 2n) reactions.**

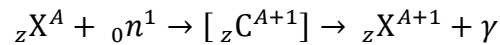
Particular examples:



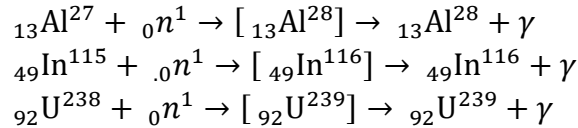
The Q -value of (n, 2n) reactions is always negative and therefore can be produced only by the fast neutrons.

Another type of reaction induced by neutrons is the so-called radiative capture of neutrons, in which photons (γ -rays) are emitted out and the product nucleus is an isotope of the target nucleus with a mass number one unit greater.

General scheme of such reactions is



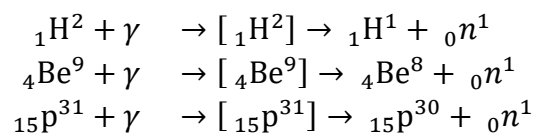
Typical examples are



10.7 PHOTO-DISINTEGRATION (NUCLEAR REACTIONS INDUCED BY PHOTONS OR γ -RAYS)

Photo-disintegration is a process in which disintegration is produced by bombardment through high energy photons. Photons possess zero rest-mass energy and therefore are capable of delivering only their kinetic energy and for the ejection of a particle from the target nucleus, the kinetic energy of the photon must exceed or be at least equal to the binding energy of the target nucleus. This means that photo-disintegration reactions are endoergic with threshold energies of the order of 10MeV.

Examples of photo-disintegration reactions are



These are the examples of $[\gamma, n]$ reactions. Another type of photo-disintegration reactions may be (γ, p) reactions. The (γ, p) reactions need photons of energies higher than those used in (γ, n) reactions.

10.8 REACTION ENERGETICS - THE Q-VALUE EQUATION

The fundamental principle of conservation of energy and momentum has a decisive influence upon the characteristics of all physical processes and so also of nuclear reactions whose energetics, we are going to study in this unit.

Most nuclear reactions involve two-body interactions of the type depicted in figure 01 [Symbolically written as $M^T(M^i, M^e)M^R$, in which a projectile or incident particle of rest mass M^i impinges with a velocity v^i upon a stationary target ($v^T = 0$) of rest mass M^T and as a result of this interaction a lighter particle of rest mass M^e is emitted out with a velocity v^e at an angle θ with the incident direction and the residual nucleus of rest mass M^R moves off with a velocity v^R at an angle ϕ . From energy conservation principle, the total energies of the initial and final systems must be equal.

$$\text{i.e. } E_{\text{tot}}^i + E_{\text{tot}}^T = E_{\text{tot}}^e + E_{\text{tot}}^R \quad \dots\dots(18)$$

$$\text{or } [K^i + E_0^i] + [K^T + E_0^T] = [K^e + E_0^e] + [K^R + E_0^R] \quad \dots\dots(19)$$

where K^i, K^T, K^e, K^R refer to the kinetic energies and $E_0^i, E_0^T, E_0^e, E_0^R$, refer to the rest mass energies, of the incident particle, the target, the emergent particle and the residual nucleus respectively. It is to be noted that K^T , the kinetic energy of the target, is zero for a target at rest in the L -system.

An important parameter of all the nuclear reaction experiments is the measurement of the energy released in the reaction which appears as kinetic energy of the residual nucleus and of emitted particles or as the energy of the photons etc., set free at the expense of the internal energy of the colliding systems. This energy is called the Q -value of the nuclear reaction. Equation (19) can be rearranged to read.

$$[E_0^i + E_0^T] - [E_0^e + E_0^R] = [K^e + K^R] - [K^i + K^T] \quad \dots\dots(20)$$

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The Q -value is then defined as Nuclear Reactions

or also

$$\begin{aligned} Q &= [K^e + K^R] - [K^i + K^T] \\ Q &= [E_0^i + E_0^T] - [E_0^e + E_0^R] \end{aligned} \quad \dots (21)$$

Now, since the target is stationary in the laboratory system so $K^T = 0$ and from (21), we get another useful relation viz.

$$Q = K^e + K^R - K^i \quad \dots\dots(22)$$

The Q -value as defined in equation (21), equals the difference between the rest mass energies of the pre-reaction quantities (incident particle and target) and the post-reaction quantities (emergent particle and the residual nucleus). Using Einstein's mass energy relations ($E = mc^2$), equation (21) can also be written in terms of the rest masses of the participants and the products of the nuclear reactions.

Thus

$$Q = c^2[M^i + M^T] - (M^e + M^R) \quad \dots\dots(23)$$

Every nuclear reaction possesses a characteristic Q -value and is termed as Exoergic or Exothermic if Q is +ve i.e. ($Q > 0$) meaning thereby that energy is released during the nuclear reaction and Endo-ergic or Endo-thermic, if Q is -ve i.e. ($Q < 0$) i.e. if energy is absorbed during the reaction. An exoergic reaction can take place (at least in principle) spontaneously but an endoergic reaction cannot take place unless the incident particle possesses a kinetic energy that exceeds the threshold energy of the nuclear reaction. This threshold energy is equal so the absolute Q -value in the centre of mass system. Exactly at threshold, the reaction can just commence.

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The Q -value of the reaction can be calculated by means of either of the equations (22) or (23) : If we use equation (21), then in the centre of mass system, measurements of K^i and K^e would suffice as $Q = K^i - K^e$. Then K^R and K^T could be calculated from the momentum conservation law. But measurements are made in L-system and the experimental observables which are simplest to record, are the values of K^i and K^c . In the L -system K^T is zero as the target is stationary in this system. The kinetic energy of the recoil nucleus K^R is usually very small and hard to measure and therefore is estimated by expressing it in terms of other measured quantities as follows:

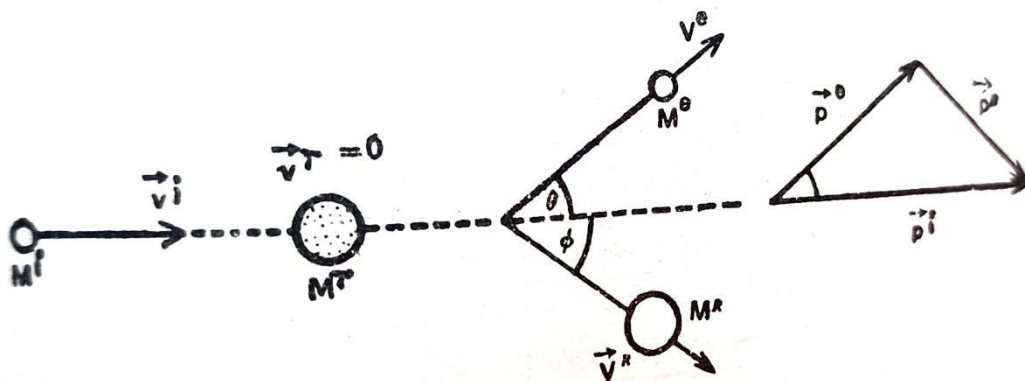


Fig.01: Schematic diagram depicting kinematics of a nuclear reaction.

The momentum conservation law in vector form may be written as

$$\mathbf{p}^i = \mathbf{p}^e + \mathbf{p}^R \quad \dots\dots(24)$$

The relativistic relationship between momentum and kinetic energy is*

$$p^2 = 2m_0K + K^2/c^2 \quad \dots\dots(25)$$

Then from the momentum triangle (fig.1) we have

$$(p^R)^2 = (p^i)^2 + (p^e)^2 - 2p^i p^e \cos \theta \quad \dots\dots\dots(26)$$

Substituting from equation (25) with appropriate suffixes we get

$$2M^R K^R + \frac{(K^R)^2}{c^2} = 2M^i K^i + \frac{(K^i)^2}{c^2} + 2M^e K^e + \frac{(K^e)^2}{c^2} - 2 \cos \theta \sqrt{\left[\left\{ 2M^i K^i + \frac{(K^i)^2}{c^2} \right\} \times \left\{ 2M^e K^e + \frac{(K^e)^2}{c^2} \right\} \right]} \quad \dots\dots(27)$$

(i) Non-Relativistic Q-value Equation

For low energy reactions, the term $(K^R)^2/c^2$ shall be very small and then we can find an approximate solution of equation (27). It is to be noted that neglecting K^2/c^2 term in the expression (25) for relativistic momentum gives,

$$p^2 = 2m_0 K \quad \dots\dots(28)$$

which is the classical expression for the momentum i.e. the expression for the momentum under non-relativistic limits. Under this case, equation (27) reduces to ;

$$\begin{aligned} 2M^R K^R &= 2M^i K^i + 2M^e K^e - 2 \cos \theta \sqrt{[(2M^i K^i)(2M^e K^e)]} \\ &= 2M^i K^i + 2M^e K^e - 4 \cos \theta \sqrt{(M^i M^e K^i K^e)} \\ \text{or} \quad K^R &= \frac{M^i}{M^R} K^i + \frac{M^e}{M^R} K^e - 2 \cos \theta \frac{\sqrt{(M^i M^e K^i K^e)}}{M^R} \quad \dots\dots(29) \end{aligned}$$

Now for a stationary target in L -system, $K^T = 0$ and from equation (22), we have

$$Q = K^e + K^R - K^i$$

Substituting the value of K^R in this equation from equation (29), we get

$$Q = K^e + \frac{M^i}{M^R} K^i + \frac{M^e}{M^R} K^e - \frac{2 \cos \theta \sqrt{(M^i M^e K^i K^e)}}{M^R} - K^i$$

$$\text{or } Q = \left(\frac{M^i}{M^R} - 1\right) K^i + \left(\frac{M^e}{M^R} + 1\right) K^e - \frac{2 \cos \theta \sqrt{(M^i M^e K^i K^e)}}{M^R} \dots \dots \dots (30)$$

This is what is called the 'non-relativistic Q -value equation'. All the masses appearing in this equation are the nuclear rest masses. With K^i, K^e and θ measured in a Laboratory System, Q of the reaction can be calculated from this equation. When $\theta = \pi/2$, i.e. when the emergent particles are observed at right angles to the incident beam of particles, the last term in equation (30) disappears and the equation reduces to,

$$Q = \left(\frac{M^i}{M^R} - 1\right) K^i + \left(\frac{M^e}{M^R} + 1\right) K^e$$

The formula is independent of the mechanism by which the nuclear reaction proceeds as also of any particular nuclear model and so can be applied to all types of two body non-relativistic nuclear reactions.

For example, we consider some particular processes:

(i) Elastic Scattering. In this process:

$$M^i = M^e \text{ and } M^T = M^R,$$

hence from equation (23) viz. $Q = [(M^i + M^T) - (M^e + M^R)]c^2$ we have,

$$Q = 0. \quad \dots \dots (31)$$

(ii) Inelastic Scattering. In this process,

$$\begin{aligned} M^i &= M^e \text{ and } M^T = M^R \\ \text{but } Q &= -E^* \quad \dots \dots (32) \end{aligned}$$

where E^* is the excitation energy imparted to the target nucleus.

(ii) **Relativistic Q-value Equation:** For finding out the relativistic Q -equation, the equation (27) is to be solved as such i.e. without the approximation ($K^2/c^2 = 0$). This adds a correction term δ_{rel} to the non-relativistic Q -equation (30). The relativistic Q -value equation may then be written as:

$$Q = \left(1 + \frac{M^e}{M^R}\right) K^e + \left(\frac{M^i}{M^R} - 1\right) K^i - 2 \cos \theta \frac{\sqrt{M^i M^e \cdot K^i K^e}}{M^R} + \delta_{rel} \dots (33)$$

$$\text{where } \delta_{rel} = \frac{1}{2M^R c^2} \left[(K^i)^2 + (K^e)^2 - (K^R)^2 \right.$$

$$\left. - \cos \theta \sqrt{(M^i M^e \cdot K^i K^e)} \cdot \left(\frac{K^i}{M^i} + \frac{K^e}{M^e}\right) \right] \dots (34)$$

Mostly K^R is small as compared with K^i and K^e and therefore while evaluating δ_{rel} we can set $K^R = 0$. It is worthwhile to remark here that in all the relativistic formulae above, the case in which some particle denoted by the suffix i is a photon, can be obtained by replacing the total energy $E_{tot}^i = \frac{M^i c^2}{(1-v^2/c^2)}$ by the quantum energy $h\nu$ and neglecting M^i wherever it occurs, in comparison with factor $1/(1 - v^2/c^2)$

Solution Of Non-Relativistic Q-value Equation.

The variation of emergent particle kinetic energy K'' for a fixed Q -value can be studied by regarding the non-relativistic Q -value equation (30) as a quadratic equation in $\sqrt{K^e}$. We can re-write this equation as follows,

$$Q = \left(1 + \frac{M^e}{M^R}\right) K^e + \left(\frac{M^i}{M^R} - 1\right) K^i - \frac{2\sqrt{M^i M^e K^i K^e}}{M^R} \cos \theta$$

$$\text{or } Q M^R = (M^R + M^e) K^e + (M^i - M^R) K^i - \left[2\sqrt{(M^i M^e K^i) \cos \theta} \right] \sqrt{K^e}$$

$$\text{or } (M^R + M^e) K^e - \left[2\sqrt{(M^i M^e K^i) \cos \theta} \right] \sqrt{K^e}$$

$$-[QM^R + (M^R - M^i)K^i] = 0 \dots\dots(35)$$

which is a quadratic equation in $\sqrt{K^e}$. The solution of this equation can be written as,

$$\begin{aligned} \sqrt{K^e} &= \frac{2\sqrt{(M^i M^e K^i)} \cos \theta \pm \sqrt{4M^i M^e K^i [\cos^2 \theta + 4(M^e + M^R)\{QM^R + (M^R - M^i)\}K^i]}}{2(M^R + M^e)} \\ &= \frac{\sqrt{(M^i M^e K^i)} \cos \theta}{(M^R + M^e)} \pm \sqrt{\frac{M^i M^e K^i \cos^2 \theta}{(M^R + M^e)^2} + \frac{\{QM^R + (M^R - M^i)K^i\}}{M^R + M^e}} \end{aligned}$$

$$\text{or } \sqrt{K^e} = p \pm \sqrt{(p^2 + q)} \text{ (say)}$$

$$\text{where } p = \frac{\sqrt{(M^i M^e K^i)}}{(M^R + M^e)} \cos \theta, q = \frac{[QM^R + (M^R - M^i)K^i]}{(M^R + M^e)} \dots\dots(36)$$

Energetics Of Nuclear Reactions. We can now discuss energetics of nuclear reactions. Like chemical reactions, nuclear-reactions can also be of two types (i) Exo-ergic Reactions and (ii) Endo-ergic Reactions, depending upon whether Q -value for the reaction is respectively positive or negative.

We discuss their energetics below :

(i) **Exo-ergic Reactions** : For reactions of this type, Q -value for the reaction is necessarily positive. These reactions can take place spontaneously (at least in principle) i.e. even when the kinetic energy of the incident particle is zero ($K^i = 0$). We first study the variation of the kinetic energy of the emergent particles and their angular distribution as the kinetic emergent particles gradually increased from zero.

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(ii) **Zero Projectile Energy** : When kinetic energy of the incident particle $K^i \rightarrow 0$, p as defined in equation also tends to zero

i.e. $p \rightarrow 0$ and $q \rightarrow \frac{QM^R}{M^R + M^e}$. Hence

or

$$\sqrt{K^e} = [p \pm \sqrt{(p^2 + q)}] \rightarrow \sqrt{q} \text{ or } K^e \rightarrow q$$

$$K^e \rightarrow \frac{QM^R}{M^R + M^e} \dots \dots \dots (37)$$

It is evident from this expression for K^e that kinetic energy of the emergent particle viz. K^e is independent of θ , i.e. it is the same in all directions if the projectile energy is zero.

Finite Projectile Energy. We have defined q as follows.

$$q = \frac{QM^R + (M^R - M^i)K^i}{M^R + M^e}$$

Since in all nuclear reactions, rest mass of the recoil nucleus is always greater than the rest mass of the incident particle (projectile) i.e. $M^R > M^i$, hence q is positive for all values of K^i (kinetic energy of the incident particle). Therefore, only one of the two roots of the Q -value equation. viz. $p + \sqrt{(p^2 + q)}$, shall be acceptable. Thus in this case K^e is single valued for all possible values of incident particle energy K^i is single valued for all possible values of incident particle energy K^i and is given by,

$$K^e = p + \sqrt{(p^2 + q)} \dots \dots \dots (38)$$

(ii) **Endo-ergic Reactions** : For every nuclear reaction with a positive Q -value, the inverse reaction has always a negative Q -value of exactly the same absolute magnitude. Reactions having negative Q -values are termed as "endo-ergic reactions". When

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projectile (incident) particle energy is zero, viz. $K^i = 0$, the term $(p^2 + q)$ becomes negative since Q is necessarily negative. Therefore $\sqrt{K^e}$ becomes "imaginary" i.e. the reaction can not take place even in principle. As the kinetic energy of the incident particle K^i is increased from zero, one finds a minimum value of the kinetic energy of the incident particle at which the reaction is just initiated. This minimum value of the kinetic energy of the incident particle K^i , required to initiate a necessarily endo-ergic reaction is called the "threshold energy" which is different for different endo-ergic reactions.

From our above analysis, it is at once clear that the reaction becomes first possible when the kinetic energy of the incident particle K^i is large enough to make the term $(p^2 + q)$ equal to zero i. e.

$$p^2 + q = 0$$

or

$$\frac{M^i M^i K^i \cos^2 \theta}{(M^R + M^i)^2} + \frac{Q \cdot M^R + (M^k - M^i) K^i}{(M^k + M^i)} = 0$$

or

$$[M^i M^e \cos^2 \theta + (M^R + M^e)(M^R - M^e) K^i = -Q M^R M^R + M^e]$$

or

$$\begin{aligned} (K^i)_\theta &= \frac{-Q M^R (M^R + M^e)}{M^i M^e \cos^2 \theta + (M^R + M^e)(M^R - M^i)} \\ (K^i)_\theta &= \frac{-Q M^R (M^R + M^e)}{M^i M^e \cos^2 \theta + (M^R)^2 - M^R M^i + M^R M^e - M^i M^e} \\ (K^i)_\theta &= \frac{-Q (M^R + M^e)}{M^R + M^e - M^i - \frac{M^i M^e}{M^R} \sin^2 \theta} \dots \dots \dots (39) \end{aligned}$$

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If the emergent particle is observed at $\theta = 0$, then the term $\frac{M^i M^e}{M^R} \sin^2 \theta$ becomes zero and in this case $(K^i)_{\theta=0}$ assumes its minimum possible value $(K^i)_{\theta=0}$, called "threshold energy" which is given by,

$$(E_{th})_{\theta=0} = (K^i)_{\theta=0} = \frac{-Q(M^R + M^e)}{M^R + M^e - M^i} \dots\dots(40)$$

Now, we know that the general relationship between Q -value and rest masses of reaction participants and products is given by equation (23) which is reproduced below,

or

$$Q = [(M^i + M^T) - (M^R + M^e)]c^2$$

$$M^i + M^T = M^R + M^e + \frac{Q}{c^2}$$

but as is usually the case, $M^T \gg \frac{Q}{c^2}$, then as a very good approximation, we can write from the above relation,

$$\begin{aligned} M^i + M^T &= M^R + M^e \\ M^R + M^e - M^i &= M^T \dots\dots\dots(41) \end{aligned}$$

Substituting these values in equation (40), we obtain

$$(E_{th})_{\theta=0} = \frac{-Q(M^i + M^T)}{M^T} \dots\dots\dots(42)$$

This is the threshold energy for $\theta = 0^\circ$

At the threshold of the reaction, product particles first emerge in the direction $\theta = 0$ and we can calculate the kinetic energy of these emergent particles as under,

$$\text{We know } \sqrt{K^c} = p^2 + \sqrt{(p^2 + q)}$$

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but at threshold, $p^2 + q = 0$, hence in the direction $\theta = 0$,

$$K^e = p^2 = \frac{M^i M^e (K^i)_{\theta=0}}{(M^R + M^e)} = 0$$

where we have substituted for p from equation (36).

$$K^e = (E_{th})_{\theta=0} \frac{M^i M^e}{(M^R + M^e)^2} \dots\dots\dots(43)$$

As K^i increases from the threshold value, product particles start emerging in the directions $\theta > 0$ also. At $\theta = 90^\circ$, $p = 0$ and then

$$\begin{aligned} \sqrt{K^e} &= \sqrt{q} \text{ or } K^e = q \\ K^e &= \frac{QM^R + (M^R - M^i)(K^i)_{\theta=90^\circ}}{(M^R + M^e)} \dots\dots\dots(44) \end{aligned}$$

Now at Threshold, $p^2 + q = 0$ and if $\theta = 90^\circ$, from equation (36), it can be seen that p is also zero. Hence, we conclude that $q = 0$

$$\begin{aligned} \frac{QM^R + (M^R - M^i)(K^i)_{\theta=90^\circ}}{(M^R + M^e)} &= 0 \\ QM^R + (M^R - M^i)(K^i)_{\theta=90^\circ} &= 0 \dots\dots\dots(45) \\ (K^i)_{\theta=90^\circ} &= (E_{th})_{\theta=90^\circ} \frac{-QM^R}{M^R - M^i} \end{aligned}$$

which is the threshold energy for 90° .

Threshold Energy: (CASE I): The minimum kinetic energy that an incident particle must possess in order to bring about a necessarily endo-ergic reaction, is called the Threshold Energy E_{th} for the nuclear reaction. Threshold energy just suffices to initiate the nuclear reaction process in which the reaction products are formed with zero mutual velocity in the CM system. It therefore follows that at threshold, the final net amount of

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kinetic energy in the CM -system is zero. However, in the L -system K^e and K^R are not zero because the velocity of the CM itself is not zero in L -system.

In the L -system the target is stationary and therefore the total kinetic energy of the reaction participants is given by K^i only.

In the CM -system both the incident particle and the target nucleus are moving towards each other such that total linear momentum in CM -system is zero.

i.e.

$$\begin{aligned} M^i v^i &= M^T v^T \\ v &= \frac{M^i v^i}{M^T} \dots\dots\dots(46) \end{aligned}$$

Since the target is stationary in the L -system and hence the total kinetic energy in the L -system is equal to the kinetic energy of the incident particle only viz. K^i and total kinetic energy in the CM -system is

$$[K]_{CM} = [K^i]_{CM} + [K^T]_{CM} \dots\dots\dots(47)$$

The relationship between the kinetic energy of a particle in the CM -system and in L -system is

$$(K)_{CM} = K^i \left(\frac{M^T}{M^i + M^T} \right) \dots\dots\dots(48)$$

The net final kinetic energy is obtained by adding Q to this, which gives us what is called threshold condition

$$\left[K^i \left(\frac{M^T}{M^i + M^T} \right) + Q \right]_{ih} = 0$$

Therefore, the positive threshold energy E_{th} in the L -system is

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with $Q < 0$ i.e. negative.

$$E_{th} = [K^i]_{th} = (-Q) \left(\frac{M^i + M^T}{M^T} \right) \dots\dots\dots(49)$$

The threshold energy can be determined experimentally and the result can be used to calculate the Q -value from the above equation. The equation (49) holds good in quite a general case. However, if the incident particles are γ -rays, then $M^i = 0$ and in that case, threshold energy will be just equal to $-Q$.

The masses entering the Q -value equation (31) and the threshold energy equation are the nuclear rest masses. However, in actual calculations, they may be replaced by the masses of the neutral atoms. The electrons that are added to the nuclei to form the neutral atom, cancel on the two sides of the nuclear reaction equation as the number of electrons on the two sides of the equation is the same.

CASE II : If the incident particle of mass M^i strikes the target with a velocity v and forms a compound nucleus of mass M^c which moves off with a velocity v^c , then from momentum conservation principle, we have

$$M^i v^i = M^c v^c = (M^i + M^T) v^c \text{ since } M^c = M^i + M^T$$

$$\therefore v^c = \frac{M^i v^i}{M^i + M^T} \dots\dots\dots(50)$$

Now, the energy balance equation can be written as:

Since

$$Q = K^c - K^i$$

$$\text{or } Q = \frac{1}{2} M^c (v^c)^2 - \frac{1}{2} M^i (v^i)^2 \dots\dots\dots(51)$$

$$Q = \frac{1}{2} \frac{(M^i + M^T)(M^i v^i)^2}{(M^i + M^T)^2} - \frac{1}{2} M^i (v^i)^2 = \frac{1}{2} M^i (v^i)^2 \left[\frac{M^i}{M^i + M^T} - 1 \right]$$

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where we have substituted the value of v^c from equation (50)

or

$$Q = \frac{1}{2} M^i (v^i)^2 \left(-\frac{M^T}{M^i + M^T} \right)$$

or

$$\frac{1}{2} M^i (v^i)^2 = (-Q) \left(\frac{M^i + M^T}{M^T} \right)$$

\therefore Threshold energy

$$E_{Th} = \frac{1}{2} M^i (v^i)^2 = (-Q) \left(\frac{M^i + M^T}{M^T} \right) \dots\dots\dots(52)$$

Binding Energy And Q-value: The Q -value of a nuclear reaction can be expressed in terms of the binding energy of the interacting nuclei. If W represents the mass of the constituents of an atom in free state, then

$$W = Zm_p + (A - Z)m_n + Zm_e$$

where Z = Atomic number, A = mass number and m_p, m_n and m_e refer to the proton, neutron and electron mass respectively.

The binding energy is then defined to be the energy equivalent of the difference of masses of the constituent particles of the atom in the free state and the mass of bound atom (M).

$$\therefore \text{B.E. (binding energy)} = (W - M)c^2$$

$$\therefore M^i = Z^i m_p + (A^i - Z^i) m_n + Z^i m_e - (\text{B.E.})^i / c^2$$

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$$M^T = Z^T m_p + (A^T - Z^T) m_n + Z^T m_e - (\text{B.E.})^T / c^2$$

$$M^e = Z^e m_p + (A^e - Z^e) m_n + Z^e m_e - (\text{B.E.})^e / c^2$$

$$\text{and } M^R = Z^R m_p + (A^R - Z^R) m_n + Z^R m_e - (\text{B.E.})^R / c^2$$

where the raised suffixes i, T, e, R refer to incident particle, target, emitted particle and the residual nucleus respectively.

$$\begin{aligned} \therefore Q &= [(M^i + M^T) - (M^e + M^R)] c^2 = -[(\text{B.E.})^i + (\text{B.E.})^T] \\ &\quad + [(\text{B.E.})^e + (\text{B.E.})^R] \\ &= [\Sigma(\text{B.E.})_{\text{final}} - \Sigma(\text{B.E.})_{\text{initial}}] \end{aligned}$$

10.9 NUCLEAR REACTION CROSS-SECTIONS

We know that for all types of nuclear reactions, certain conservation laws are satisfied. It is a very difficult task to ascertain whether a particular projectile will initiate a particular nuclear process in a given target or not? But somehow we can predict with calculable accuracy the fraction of the incident particles that induces a particular nuclear process i.e. we can determine the probability of occurrence of a particular nuclear process. This then introduces the concept of nuclear reaction cross-section.

Cross-section, for a particular nuclear reaction,

$$\sigma = \frac{\text{Number of given type of events per unit time per nucleus}}{\text{Number of projectile particle per unit area per unit time}}$$

The total cross-section σ_{tot} may be written as

$$\sigma_{tot} = \sigma_{sc} + \sigma_a \quad \dots\dots\dots(53)$$

with σ_{sc} = scattering cross-section

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and $\sigma_a =$ absorption cross-section

Cross-section has the dimensions of area, and is consequently measured in a unit called 'barn' (1 barn = 10^{-24} cm² = 10^{-28} m²). The results of all the experimental studies may be expressed quite conveniently in terms of the concept of cross-section.

Level Width (Γ): The compound nucleus, on the average, remains in a given excited state for a certain time before decaying either through particle or γ -ray emission. The reciprocal of this mean life time τ , is called the decay constant which represents the probability per unit time of the emission of a particle or γ -ray. In the study of energy states excited by nuclear reactions, it is customary to use, instead of disintegration constant, a quantity proportional to it which is called the level width and defined as

$$\Gamma = \frac{\hbar}{\tau} = \hbar\lambda \quad \dots\dots\dots(54)$$

where λ is the decay constant.

Level width is expressed in energy units and the basis for its use is the uncertainty principle. In case of energy-time measurements, the uncertainty principle may be written as :

$$\Delta E \cdot \Delta t \approx \hbar \quad \dots\dots\dots(55)$$

The mean life time of an excited state of the compound nucleus may be identified with the time uncertainty Δt corresponding to an energy uncertainty ΔE . The level width Γ is then identified with this energy uncertainty ΔE . It is at once clear from equation (11.40) that a state having a short mean life time, has a large level width and is thus very poorly defined in energy whereas a long-lived excited state is quite sharply defined in energy. Different decay modes possess different level widths. The total width may be written as the sum of the partial level widths

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$$\Gamma = \Gamma_d + \Gamma_\alpha + \Gamma_n + \Gamma_\gamma + \dots \quad \dots\dots\dots(56)$$

where $\Gamma_d, \Gamma_\alpha, \Gamma_n, \Gamma_\gamma$ etc. are the partial level widths for the emission of deuteron, α -particle, neutron, γ -rays etc.

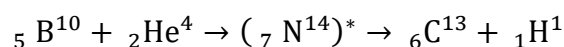
The concept of level width is useful in that it can be obtained from the measurement of resonances. By knowing the total level width, the mean life time can be calculated from the relation:

$$\begin{aligned} \tau(\text{sec}) &= \frac{\hbar(\text{Joule-sec})}{\Gamma(\text{Joule})} = \frac{1 \cdot 106 \times 10^{-34}}{\Gamma(\text{eV}) \times 1 \cdot 6 \times 10^{-19}} \text{sec.} \\ &= \frac{6 \cdot 9 \times 10^{-16}}{\Gamma(\text{eV})} \text{sec.} \quad \dots\dots\dots(57) \end{aligned}$$

Experimental values of the average life time of the compound nucleus are found to be of the order of 10^{-14} sec. Comparing this life time with the natural nuclear time which is the time required for a nuclear projectile to traverse a target nucleus, it is found that the mean life time is considerably longer than the natural nuclear time. To get an estimate, if we take a nuclear diameter of $\sim 10^{-15}$ meter and a velocity 10^7 m/sec. for absorbed projectile, then the natural time is $\approx 10^{-15}/10^7 \text{sec} \cong 10^{-22} \text{sec}$. This means that the mean life of the compound nucleus is about 10^8 times as long as the natural nuclear time.

Example: The ${}_5\text{B}^{10}(\alpha, p){}_6\text{C}^{13}$ reaction shows among others a resonance for an excitation energy of the compound nucleus of 13.23MeV. The width of this level as found experimentally is 130KeV. Calculate the mean life of the nucleus for this excitation.

Solution: The reaction is



The mean life time is given by :

$$\begin{aligned}\tau &= \frac{h}{2\pi\Gamma} = \frac{6.625 \times 10^{-34} \text{ Joule-sec.}}{2(3 \cdot 14) \times 130 \times 10^3 \text{ eV} \times 1 \cdot 6 \times 10^{-19} \text{ Joule/eV}} \\ &= 5 \times 10^{-21} \text{ sec.}\end{aligned}$$

Example: Neutron capture of slow neutrons by U^{235} shows a resonance for an excitation energy of 0.29eV. The compound nucleus can become de-excited either through γ -emission or by fission into larger nuclear fragments. The mean life time of the compound nucleus was found to be 4.7×10^{-15} sec. and the partial width for γ -emission $\Gamma_\gamma = 3.4 \times 10^{-2}$ eV. Calculate the partial fission width T_f .

Solution: Total width

$$\begin{aligned}\Gamma &= \frac{h}{2\pi\tau} = \frac{(6.625 \times 10^{-34} \text{ Joule/sec})}{2 \times 3.14 \times (4.7 \times 10^{-19} \text{ sec})(1.6 \times 10^{-19} \text{ Joule/eV})} \\ &= 0.14 \text{ eV}\end{aligned}$$

Now since $\Gamma = \Gamma_\gamma + \Gamma_f$

$$\begin{aligned}\therefore \Gamma_f &= \Gamma - \Gamma_\gamma \\ &= 0.14 \text{ eV} - 0.034 \text{ eV} \\ &= 0.106 \text{ eV}\end{aligned}$$

10.10 PARTIAL WAVE ANALYSIS OF NUCLEAR REACTION CROSS-SECTIONS

When a beam of particles strikes against a target, the individual nuclei in the target behave as independent interaction centres. The incident particle beam is partly scattered and partly absorbed and therefore the total cross-section of the process, σ_{tot} , is composed of the scattering cross-section σ_{sc} and the reaction (absorption) cross-section σ_a . i.e.

$$\sigma_{tot} = \sigma_{sc} + \sigma_a \quad \dots\dots\dots(58)$$

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The free particles wave equation describing the process may be written as

$$\nabla^2\psi + k^2\psi = 0 \quad \dots\dots\dots(59)$$

The beam of incident particles may be thought of or described by a plane wave moving in the Z-direction and for particles of constant momentum, the wave function describing the beam of incident particles as

$$\psi_{inc} = e^{ikz} = e^{ikr\cos\theta} \quad \dots\dots\dots(60)$$

The wave number k is given by

$$k = \frac{Mv}{\hbar} = \frac{1}{\hbar} [2ME]^{\frac{1}{2}} \quad \dots\dots\dots(61)$$

where M is the reduced mass of the incident particle and target nucleus and v is the velocity of the particle. The incident plane wave can be expressed as superposition of waves with different orbital angular momenta as follows

$$\psi_{inc} = e^{ikr\cos\theta} = \sum_{l=0}^{l=\infty} B_l(r)Y_{l,0}(\theta) \quad \dots\dots\dots(62)$$

The expansion is obtainable in this form because the expression for the wave function is independent of the azimuthal angle ϕ and therefore the only spherical harmonics entering the expansion are

$$Y_{l,0}(\theta) \text{ i.e. with } m = 0$$

The coefficients $B_l(r)$ can be expressed in terms of Spherical Bessel functions $j_l(kr)$ as follows :

$$B_l(r) = i^l \sqrt{[4\pi(2l+1)]} j_l(kr) \quad \dots\dots\dots(63)$$

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The spherical Bessel function $j_l(kr)$ is related to the ordinary Bessel function $J_{l+1/2}(kr)$ through the formula

$$j_l(kr) = \left(\frac{\pi}{2kr}\right)^{1/2} J_{l+1/2}(kr) \dots\dots\dots(64)$$

and can be represented as

$$j_{\text{doqn}} 10l(kr) = (-kr)^l \left[\frac{1}{kr} \cdot \frac{d}{d(kr)}\right]^l \left[\frac{\sin kr}{kr}\right] \dots\dots\dots(65)$$

whence asymptotically.

$$j_l(kr) = \begin{cases} \lambda|kr| \ll l & \frac{(kr)^l}{(2l+1)!!} \\ kr \gg l & \frac{1}{kr} \sin\left(kr - \frac{l\pi}{2}\right) \end{cases} \dots\dots\dots(66)$$

where $(2l+1)!! = (2l+1)(2l-1) \dots 7.5.3.1$.

$$\psi_{inc} = e^{ikr \cos \theta} = \sum_{l=0}^{l=\infty} i^l \sqrt{[4\pi(2l+1)]} j_l(kr) \cdot Y_{l,0}(\theta) \dots\dots\dots(67)$$

At large distances from the interaction centre i.e. $|kr| \gg l$, the expansion (62) of the incident wave function with the aid of equation (66), may be written as

$$\begin{aligned} \psi_{inc} &= e^{jkr \cos \theta} = \sum_{l=0}^{l=\infty} \sqrt{[4\pi(2l+1)]} \cdot \frac{1}{kr} \sin\left(kr - \frac{l\pi}{2}\right) Y_{l,0}(\theta) \\ &= \frac{\pi^{1/2}}{kr} \sum_{l=0}^{l=\infty} \sqrt{(2l+1)} i^{l+1} [e^{-i(kr-l\pi/2)} - e^{i(kr-l\pi/2)}] Y_{l,0}(\theta) \dots (68) \end{aligned}$$

where l is the orbital momentum quantum number which designates each incoming partial wave. Equation (68) contains both the incoming and the outgoing spherical waves. The first of the exponential terms *viz.* $e^{-i[(kr-l\pi/2)]}$ represents a series of incoming spherical waves impinging at the interaction centre i.e. $z = 0$ and second of the

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exponential terms viz. $e^{i[(kr-(l\pi/2)]}$ represents outgoing spherical waves. The above equation (68) represents undistorted waves in the absence of any absorbing or scattering centre. If however a nucleus is located at the origin ($z = 0$), the amplitudes of the outgoing spherical waves from the origin, shall undergo changes.

The stream of particles in the above picture shall have all sorts of impact parameters with the target nucleus. The impact parameter is the separation at which the collision partners (i.e their centers) pass one another in the *CM* system if there were no interaction between them is given by

$$m v b = l \hbar \quad \dots\dots\dots(69)$$

l , is orbital quantum number and can take up values 0,1,2,3,

From the above equation, the impact parameter is then given by:

$$b = \frac{l \hbar}{m v} = l \lambda \quad \dots\dots\dots(70)$$

where λ (read as lambda cross) is the rationalised de Broglie wavelength for the non-relativistic particle. The impact parameter b could also be conceived of as quantized Then only certain discrete could be assumed by the incident particle with respect to the interacting nucleus e.g.

particle waves with $l = 0$ (*S*-waves) have impact parameter $b = 0$.

particle waves with $l \neq 1$ (*P*-waves) have impact parameter $b = \lambda$

particle waves with $l = 2$ (*F*-waves) have impact parameter $b = 2\lambda$ and so on..

To understand the significance of the angular momenta in a plane wave, the incident beam can be conceived of as arranged in concentric cylindrical zones, each having discrete value of the orbital angular momentum. The inner-most zone then contains particles with impact parameter less than λ i.e. (*S*-waves). The next zone contains all

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particles with impact parameters between λ and 2λ . The l th zone contains all particles with impact parameters between $l\lambda$ and $(l + 1)\lambda$. The particles in the l th zone, then have angular momenta between $l\hbar$ and $(l + 1)\hbar$.

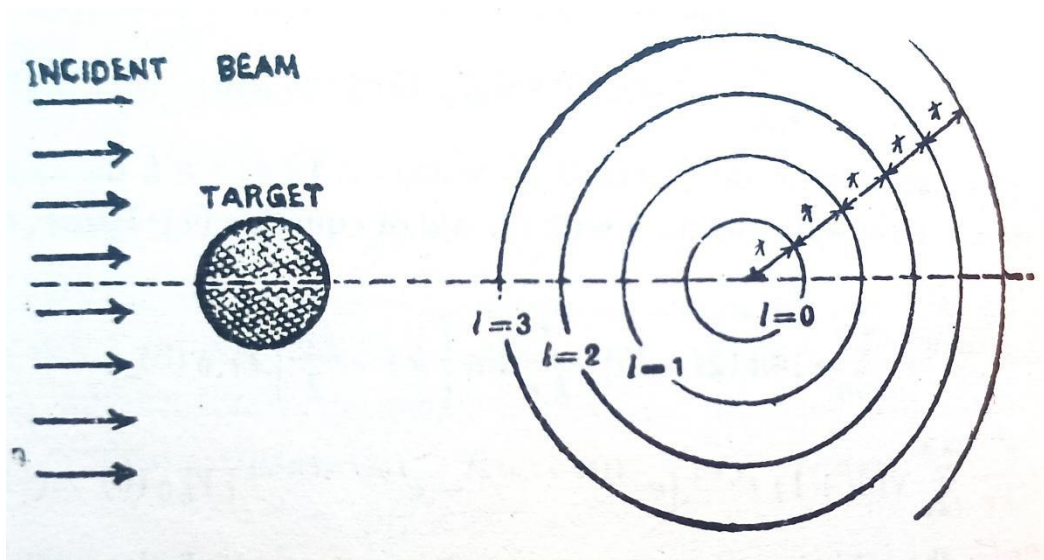


Fig. 1. Illustration of the quantization of the incident particle orbital angular momentum.

The cross-sectional area of the l th zone

$$= \pi(l + 1)^2(\lambda)^2 - \pi l^2(\lambda)^2 = (2l + 1)\pi(\lambda)^2$$

i.e., the reaction cross-section = $(2l + 1)\pi(\lambda)^2$ (71)

Since no more particles can be removed from the incident beam by the interaction centre than are originally present in it and therefore the upper limit to the reaction cross-section is,

$$\sigma_{a,l} \leq (2l + 1)\pi(\lambda)^2 \quad \dots\dots\dots(72)$$

where $\sigma_{a,l}$ stands for the reaction cross-section for the l th partial wave.

. The interaction can change the phase and amplitude of the outgoing wave only and not of the incoming wave. To make provision for these changes in the outgoing wave, we modify the wave by introducing a complex multiplicative scattering coefficient η_l , where l refers to the orbital angular momentum. Naturally then $|\eta_l| \leq 1$, since the number of outgoing particles cannot exceed the number of incoming particles. Incorporating this factor in equation (68), the new wave function is

$$\psi(r) = \frac{\pi^{1/2}}{kr} \sum_{l=0}^{l=\infty} \sqrt{(2l+1)} i^{l+1} \left[e^{-i|kr - (\frac{l\pi}{2})|} + \eta_l e^{i[kr - (l\pi/2)]} \right] Y_{l,0}(\theta) \quad \dots \dots \dots (73)$$

$$= \psi_{inc} + \psi_{out} = \psi_{inc} + \psi_{sc} \quad \dots \dots \dots (74)$$

This equation (73) holds good for neutrons only. If the incident particles are charged, the equation must be replaced by an appropriate Coulomb wave function.

The elastically scattered wavefunction ψ_{sc} is the difference of the new total wave function and the original total wave function i.e. $\psi_{sc} = \psi(r) - \psi_{inc}$

Substituting for $\psi(r)$ and ψ_{sc} from equation (73) and (68), we get,

$$\psi_{sc} = \frac{\pi^{1/2}}{kr} \sum_{l=0}^{l=\infty} \sqrt{(2l+1)} i^{l+1} (1 - \eta_l) e^{i[kr - (l\pi/2)]} Y_{l,0}(\theta) \quad \dots \dots \dots (74)$$

The differential elastic scattering cross-section in the direction θ , which is defined as

$$\sigma_{sc}(\theta) = \frac{\text{outgoing scattered flux of particles through the solid angle } d\Omega \text{ in a direction } \theta}{\text{incident flux of particles} \times d\Omega}$$

To obtain $N_{sc}(\theta)d\Omega$, we enclose the interaction centre by a large sphere of radius r_0 . The flux corresponding to ψ_{sc} , through a sphere of radius r_0 , according to the rules of quantum mechanics, is calculated as follows

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The number of particles scattered per second into the solid angle $d\Omega$ at an angle θ , is equal to the number of particles scattered through an area $dA = r_0^2 d\Omega$, which is given by

$$N_{sc}(\theta)d\Omega = j_{sc}dA \quad \dots\dots\dots(75)$$

where j_{sc} = scattered probability current density and $dA = r_0^2 d\Omega$

$$\begin{aligned} \therefore N_{sc}(\theta)d\Omega &= \frac{\hbar}{2Mi} \left(\psi_{sc}^* \frac{\partial \psi_{sc}}{\partial r} - \psi_{sc} \frac{\partial \psi_{sc}^*}{\partial r} \right)_{r=r_0} \cdot r_0^2 d\Omega \\ &= \frac{\hbar}{Mi} \left(\psi_{sc}^* \frac{\partial \psi_{sc}}{\partial r} \right)_{r=r_0} \cdot r_0^2 d\Omega \\ &= \frac{\hbar k}{M} |\psi_{sc}(r_0, \theta)|^2 r_0^2 d\Omega \\ &= v |\psi_{sc}(r_0, \theta)|^2 r_0^2 \cdot d\Omega, \text{ Since } \frac{\hbar k}{M} = v \dots\dots\dots (76) \end{aligned}$$

With the help of equation (74), this equation may be written as,

$$\begin{aligned} N_{sc}(\theta)d\Omega &= v \cdot \frac{\pi}{k^2 r_0^2} \left| \sum_{l=0}^{\infty} \sqrt{(2l+1)} i^{l+1} (1 - \eta_l) e^{i(kr - l\pi/2)} Y_{l,0}(\theta) \right|^2 \times r_0^2 d\Omega \\ &= v \frac{\pi}{k^2} \left| \sum_{l=0}^{\infty} \sqrt{(2l+1)} i^{l+1} (1 - \eta_l) e^{i(kr - l\pi/2)} Y_{l,0}(\theta) \right|^2 d\Omega \dots\dots\dots (77) \end{aligned}$$

And, the incident flux is given by

$$N = \frac{\hbar}{Mi} \left(\psi_{inc}^* \frac{\partial \psi_{inc}}{\partial r} \right)$$

$$\Psi_{inc} = e^{ikz}$$

$$\therefore N = \frac{\hbar}{Mi} \left[e^{-ikz} \frac{\partial}{\partial z} (e^{ikz}) \right]$$

$$= \frac{\hbar k}{M} = v \quad \dots\dots\dots(78)$$

Hence

$$Nd\Omega = v = d\Omega$$

$$\begin{aligned} \therefore \sigma_{sc}(\theta) &= \frac{N_{sc}(\theta)d\Omega}{vd\Omega} \\ &= \frac{\pi}{k^2} \left| \sum_{l=0}^{\infty} \sqrt{(2l+1)} i^{l+1} (1-\eta_l) e^{i(kr-l\pi/2)} Y_{l,0}(\theta) \right|^2 \\ &= \frac{\pi}{k^2} \left| \sum_{l=0}^{\infty} \sqrt{(2l+1)} (1-\eta_l) Y_{l,0}(\theta) \right|^2 \dots\dots\dots (79) \end{aligned}$$

The total elastic scattering cross-section is accordingly given by

$$\sigma_{sc} = \int \sigma_{sc}(\theta) d\Omega = \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) |1-\eta_l|^2 \dots\dots\dots (80)$$

$$\text{Now since } \sigma_{sc} = \sum_{l=1}^{\infty} \sigma_{sc,l} \dots\dots\dots (81)$$

where σ_{sc} = total scattering cross-section and $\sigma_{sc,l}$ is the scattering cross-section for the l^{th} partial wave of angular momentum corresponding to l .

From equations (80) and (81), we have

$$\begin{aligned} \sum_{l=0}^{\infty} \sigma_{sc,l} &= \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) |1-\eta_l|^2 \\ \therefore \sigma_{sc,l} &= \frac{\pi}{k^2} (2l+1) |1-\eta_l|^2 \dots\dots\dots (82) \end{aligned}$$

Reaction Cross-Section: We can also calculate the reaction cross-section by taking account of particle loss. In the case of absorption (reaction), the number of particles coming out of the interaction centre is less than the number coming in and so $|\eta_l| < 1$.

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Therefore, total absorption cross-section σ_a can be obtained by calculating the number N_a of the particles removed from the beam per sec. and dividing it by the number N of the incident particles per unit area per second.

Then N_a is the number of particles that enter this sphere without leaving it again through the entrance channel. It is therefore equal to the total net flux into this sphere $\psi(r)$ as contained in equation (73).

$$\text{Thus } N_a = - \iint j_a(r_0, \theta) r_0^2 d\Omega$$

where j_a refers to the absorbed probability current density.

$$\begin{aligned} \therefore N_a &= -\frac{\hbar}{2Mi} \iint \left(\psi^*(r) \frac{\partial \psi(r)}{\partial r} - \psi(r) \frac{\partial \psi^*(r)}{\partial r} \right)_{r=r_0} \times r_0^2 d\Omega \\ &= -\frac{\hbar}{Mi} \iint \left(\psi^*(r) \frac{\partial \psi(r)}{\partial r} \right)_{r=r_0} \times r_0^2 d\Omega \end{aligned}$$

Making use of equation (11 · 109), and carrying out integration.

$$\begin{aligned} N_a &= -\frac{\hbar}{Mi} \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) [\exp \{i(kr - l\pi/2)\} \\ &\quad - \eta_l^* \exp \{-i(kr - l\pi/2)\}] [(-ik) \exp \{-i(kr - l\pi/2)\} \\ &\quad - (ik) \eta_l \exp \{i(kr - l\pi/2)\}] \end{aligned}$$

On account of the orthonormality of $Y_{l,0}(\theta)$

$$N_a = \frac{\hbar \pi}{M k} \sum_{l=0}^{\infty} (2l+1)(1-|\eta_l|^2)$$

$$N_a = \frac{\hbar k}{M} \cdot \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1)(1-|\eta_l|^2)$$

$$N_a = v \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1)(1-|\eta_l|^2)$$

Hence absorption cross-section,

$$\sigma_a = \frac{N_a}{N} = v \frac{\pi}{k^2} \sum_{l=0}^{\infty} \frac{(2l+1)(1-|\eta_l|^2)}{v}$$

$$\sigma_a = \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1)(1-|\eta_l|^2) \quad \dots\dots\dots(83)$$

$$\sigma_a = \sum_{l=0}^{\infty} \sigma_{a,l} \quad \dots\dots\dots(84)$$

where σ_a = total absorption (reaction) cross-section and $\sigma_{a,l}$ is the absorption cross-section for the l^{th} partial wave. From equations (83) and (84)

$$\therefore \sigma_{a,l} = \frac{\pi}{k^2} (2l+1)(1-|\eta_l|^2) \quad \dots\dots\dots(85)$$

The cross-section (scattering and absorption) given by equations (82) and (85), are valid for neutron scattering and absorption only as we have not included Columb wave function in our analysis.

Total Cross-Section: The total crosssection is the sum of the elastic scattering cross-section σ_{sc} and the reaction cross-section σ_a and is given by

$$\sigma_{tot} = \sigma_{sc} + \sigma_a = \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1)(|1-\eta_l|^2 + 1-|\eta_l|^2) \quad \dots\dots\dots(86)$$

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To evaluate the above expression we know that if Z_1 and Z_2 are two complex quantities, then

$$\begin{aligned} |Z_1 - Z_2|^2 &= |Z_1|^2 - 2\operatorname{Re}(Z_1 Z_2^*) + |Z_2|^2 \\ \therefore |1 - \eta_l|^2 &= |1|^2 - 2\operatorname{Re}(\eta_l^*) + |\eta_l|^2 \quad \dots \dots \dots (86) \end{aligned}$$

Substituting it in the above equation (86), we get

$$\begin{aligned} \sigma_{tot} &= \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1)[1 - 2\operatorname{Re}(\eta_l^*) + |\eta_l|^2 + 1 - |\eta_l|^2] \\ &= \frac{2\pi}{k^2} \sum_{l=0}^{\infty} (2l+1)[1 - \operatorname{Re}(\eta_l^*)] \\ &= 2\pi(\lambda)^2 \sum_{l=0}^{\infty} (2l+1)[1 - \operatorname{Re}(\eta_l^*)] \end{aligned}$$

or also $\sigma_{tot} = \pi(\hbar)^2 \sum_{l=0}^{\infty} (2l+1)[2 - (\eta_l + \eta_l^*)]$ The symbol $\operatorname{Re}(\eta_l^*)$ stands for the real part of η_l^* . The total cross-section for the l^{th} partial wave may then be written as

$$\begin{aligned} \sigma_{tot,l} &= \pi(\lambda)^2 (2l+1)[\eta_l + \eta_l^*] \\ \sigma_{tot} &= \sum_{l=0}^{\infty} \sigma_{tot,l} \quad \dots \dots \dots (87) \end{aligned}$$

Conclusions. We can infer that

(i) If $\eta_l = -1$, $\sigma_{a,l} = 0$ but $\sigma_{sc,l}$ is at its maximum and is given by

$$[\sigma_{sc,l}]_{\max} = 4\pi(\hbar)^2 (2l+1)$$

This shows that scattering can take place without absorption.

(ii) If $\eta_l = 0$, $\sigma_{a,l}$ is at its maximum and is given by,

$$\sigma_{sc,l} = [\sigma_{a,l}]_{\max} = \pi(\hbar)^2(2l + 1)$$

meaning thereby that absorption is always accompanied by scattering, although scattering may and may not be accompanied by absorption. Thus the scattering and absorption cross-sections for the l^{th} partial wave, lie between the limits :

$$0 \leq \sigma_{sc,l} \leq 4\pi(\hbar)^2(2l + 1)$$

$$0 \leq \sigma_{a,l} \leq \pi(\hbar)^2(2l + 1)$$

10.11 SUMMARY

After studying this unit learners have learnt about how to

- Describe the Types of Nuclear Reaction
- Explain the Conservation Laws for Nuclear Reactions
- Discriminate between Charged Particle Induced Nuclear Reactions
- Understand the Photo-disintegration process
- Evaluate the Reaction Energetics - the Q-value Equation
- Explain Nuclear Reaction Cross-Sections
- Describe Partial Wave Analysis of Nuclear Reaction Cross-sections

10.12 REFERENCES

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3. Elements of Nuclear Physics by M.L.Pandya,R.P.S.Yadav
4. Nuclear Physics An Introduction by S.B.Patel

5. Nuclear Physics by S.N.Ghoshal

10.13 SUGGESTED READINGS

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- 2.“Nuclear Two Body Problems and Elements of Nuclear Forces” Experimental Nuclear Physics by N. F. Ramsey, Wiley: New York.
- 3.Lectures on Nuclear Theory (translated from the Russian) by Landau, Plenum Press, New York.
- 4.Elementary Nuclear Theory, 2nd ed. by Bethe and Morrison, Wiley: New York.
- 5.Nuclear Physics by D. C. Tayal.

10.14 TERMINAL QUESTIONS

1. Describe the Types of Nuclear Reaction with suitable examples.
2. Explain the Conservation Laws for Nuclear Reactions with examples.
3. Discriminate between different Charged Particle Induced Nuclear Reactions along with suitable examples.
4. What do you understand by the Photo-disintegration process.
5. Derive an expression for the Q-value of the nuclear reaction.
6. Explain Nuclear Reaction Cross-Sections
7. Describe in detail the Partial Wave Analysis of Nuclear Reaction Cross-sections.

UNIT 11**FISSION AND FUSION**

Structure of the Unit

11.1 Introduction

11.2 Objectives

11.3 Compound Nucleus

11.3.1 Excitation energy of the Compound Nucleus

11.4 Direct Reactions

11.5 Theory of Stripping and Pick-up Reactions

11.6 Resonance Scattering and Reaction Cross-sections

11.6.1 Reaction Cross-Section

11.7 Continuum Theory of Nuclear Reactions

11.8 Optical Model Theory of Nuclear Reactions

11.9 Nuclear Fission

11.9.1 Discovery of Nuclear Fission

11.9.2 Energy Release in Fission

11.10 Nuclear Fusion

11.10.1 Source of Energy in Stars

11.11 Summary

11.12 References

11.13 Suggested Readings

11.14 Terminal Questions

11.1 INTRODUCTION

. In this unit we shall study that there are many types of nuclear reactions but they all can be broadly classified into two categories :

- (i) Particle induced nuclear reactions.
- (ii) Electromagnetic radiation induced nuclear reactions.

The particle induced reactions may further be divided into two more or less distinct classes known as compound nucleus reactions and direct reactions.

. A unified theory of nuclear reactions simplified enough to be of practical utility is not available and therefore it is necessary to use different models and approximations according to the nature of the projectile particle and the target. We first consider particle induced reactions. The target nucleus as seen from the projectile may be taken to be a region with a potential and absorption coefficient. This potential is due to the single nucleons composing the target nucleus. The projectile particles after hitting the target, may be diffracted by the potential without any energy loss (elastic scattering). If the energy of the projectile is enough, it may enter the nucleus, hit one target nucleon and lift it to higher energy state or even to an unbounded state and still preserve enough energy to leave the target nucleus. This type of interaction is termed as 'direct interaction' or a direct reaction.

At lower energies, reactions may not involve a direct interaction of the type considered above but may proceed via an unstable intermediate state called the compound nucleus state. In this case the incident particle loses so much energy that it cannot escape the struck nucleus. The compound nucleus thus formed has its mass number equal to the sum

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of the mass numbers of the target nucleus and that of the projectile. When nucleons are the bombarding particles, the mass number of the compound nuclei shall be $A + 1$.

11.2 OBJECTIVES

After studying the unit learners will be able to

- Explain Compound Nucleus
- Describe Direct Reactions
- Understand the Theory of Stripping and Pick-up Reactions
- Explain Resonance Scattering and Reaction Cross-sections
- Discuss the Continuum Theory of Nuclear Reactions
- Describe Optical Model Theory of Nuclear Reactions
- Explain Nuclear Fission
- Discuss the Nuclear Fusion

11.3 COMPOUND NUCLEUS

The compound nucleus being in a highly excited state, is unstable and gives rise to a typical energy spectrum for the emitted particles having nearly a Maxwellian distribution of velocities and a practically isotropic angular distribution. In contrast, the direct processes show strong angular dependences and characteristic maxima of the cross-section as a function of energy. Perhaps the simplest quantitative difference between compound nucleus mechanism and the direct reaction mechanism is the time of interaction. In direct processes the interaction time is of the order of the transit time of the

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incident particle over a nuclear diameter which is of the order of 10^{-22} sec. whereas in compound nucleus formation mechanism, the interaction time is of the order of 10^{-14} sec. which is about 10^8 times greater than the direct reaction mechanism interaction time. We first discuss the compound nucleus theory of nuclear reaction mechanisms.

The compound nucleus theory of nuclear reactions was put forth by N. Bohr in 1936. Prior to the publication of this classic paper, attempts were made to explain the variation of neutron cross sections on the assumption that the incident projectile particle interacts with a simple square well type potential. This problem is easily handled within scattering theory in quantum mechanics and calculations predict that an interaction with a square potential well can neither change the energy of the particles nor can remove it from the beam; it can only deflect it. In other words in an interaction with a simple square potential-well, elastic scattering will be the dominant process. Calculations also predict that scattering resonances should occur at wide energy intervals of 10 to 20 MeV. But Fermi and others discovered that slow neutron resonances in medium and heavy nuclei occur as close as 1 eV. Thus the simple square potential-well proves to be inadequate to explain them.

According to the Bohr's compound nucleus theory, the nuclear reaction is a two stage process as outlined below :

(i) A low energy bombarding particle is absorbed by the target nucleus and both together form a compound nucleus which is in a highly excited state, i.e. incident particle + target nucleus \rightarrow compound nucleus. The kinetic energy of the incident particle together with its binding energy with the compound nucleus, represents the excitation energy of the compound nucleus. After the incident particle has merged completely with the target nucleus, its energy no longer remains concentrated on one particle but is rapidly shared

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by the collective motion of all the particles of the new system - the compound nucleus. Thus each of the nucleons in the compound nucleus will have some additional energy but none of them will have the likelihood of getting all the energy of the incoming particle or even any large fraction of it. The sharing of the incoming energy is exactly like the sharing of heat energy in a drop of water between all its constituent molecules in the form of kinetic energy. Because of this thorough sharing of energy among all the nucleons, the compound nucleus has no chances of immediate break up.

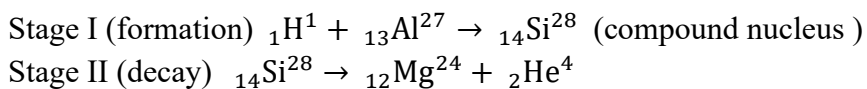
(ii) The second stage in the compound nucleus theory involves the break-up of the compound nucleus into a product nucleus and one or more emitted particles. Compound nucleus \rightarrow product nucleus + emitted particles.

The decay of the compound nucleus will occur only when sufficient energy gets concentrated on some single nucleon or a group of nucleons such as the α -particles, for it to break through the coulomb potential barrier and escape from the nucleus. As the life time of the nucleus is large on a nuclear scale (about 10^8 times larger than the time taken by the incident particle to traverse nuclear diameter), the intrinsically slow process of electromagnetic radiation (γ -emission) competes strongly with charged particle emission. Since the excitation energy is thoroughly shared between all the nucleons, it seems highly probable that then the mode of decay of the compound nucleus is completely independent of the mode of its formation i.e. the break up does not take place until the compound nucleus has completely forgotten its history of formation. In general, the emitted particle is different from the one which entered the target nucleus to form the compound nucleus and it will neither have the energy nor the direction of the incoming particle. However, even if the emitted particle is the same as the incident projectile particle ; e.g. a neutron escapes when a neutron enters, the process need not be an elastic collision.

The assumption of the formation of the compound nucleus and its subsequent break-up is in accordance with many facts of the nuclear reactions. The decay mode is independent

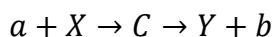
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of the formation mode is demonstrated by the fact that in general when a target nucleus is bombarded by means of a given type of projectile, several different nuclides are formed. For example when ${}_{13}\text{Al}^{27}$ is bombarded by means of protons : nuclides like ${}_{12}\text{Mg}^{24}$, ${}_{14}\text{Si}^{17}$, ${}_{14}\text{Si}^{28}$, ${}_{11}\text{Na}^{24}$ etc. are formed. The reaction involving ${}_{13}\text{Al}^{27}$ target and proton as projectiles, on the basis of compound nucleus formation theory, may be written as



We shall now derive and discuss the expression for the cross-section of a nuclear reaction that proceeds by way of compound nucleus formation hypothesis. Since according to the compound nucleus hypothesis as mentioned above, the formation and decay modes of the compound nucleus are independent of each other and therefore to calculate the cross-section for a nuclear reaction, it is necessary to determine the cross-sections for the two processes *i.e.* the formation and decay of the compound nucleus.

When the incident particle has a low energy, the energy level separation is greater than the level width of the excited levels, and therefore the decay takes place from well defined states of the compound nucleus. However at high projectile energies the excited levels may overlap and the life time of the compound nucleus may be comparable with the transit time of the projectile through the nucleus. In this case a departure from compound nucleus concept is expected. Here we shall assume the validity of the compound nucleus hypothesis *i.e.* consider two stage reactions. On the basis of this hypothesis, the cross-section for the nuclear reaction $X(a, b)Y$ through the formation of the compound nucleus C *i.e.*



may be written as

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$$\sigma(a, b) = \sigma_C(a)G_C(b) \quad \dots\dots\dots(1)$$

where $\sigma_C(a)$ is the cross-section for the formation of the compound nucleus C by the incident particle ' a ' with the target nucleus X and $G_C(b)$ is the relative probability that the compound nucleus C , once formed decays with the emission of the particle ' b ' leaving a residual nucleus Y . Probability $G_C(b)$ is also referred to as the branching ratio of the reaction into the emission of ' b ' and it is a pure number. It is implied that the particle ' a ' and the target nucleus are in a given total angular momentum state specified by the symbol ' a ' and that the emission of the particle ' b ' leaves the residual nucleus in a given state. During the reaction, total angular momentum and parity were conserved. The specified entrance and exit states were called channels by Weisskopf. In this terminology, the reaction is initiated through the channel ' a ' and the compound nucleus decays through the channel ' b '.

Consider now another case in which the compound nucleus C decays through the emission of another particle z leaving a residual nucleus Z so that

and

$$\begin{aligned} a + X &\rightarrow C \rightarrow Z + z \\ \sigma(a, z) &= \sigma_C(a)G_C(z) \quad \dots\dots\dots(2) \end{aligned}$$

According to Bohr's assumption, the disintegration of the compound nucleus into different channels depends only upon the excitation energy E_C , the total angular momentum J_C and the parity π_C , of the compound nucleus. Therefore for the same excitation energy, angular momentum and parity, we assume that the cross-section for the formation of the compound nucleus with particle a , viz. $\sigma_C(a)$ is the same. Consider now the reaction

and then

$$\begin{aligned} b + Y &\rightarrow C \rightarrow Z + z \\ \sigma(b, z) &= \sigma_C(b)G_C(z) \quad \dots\dots\dots(3) \end{aligned}$$

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where for the same compound nucleus C . we have assumed the same branching ratio $G_C(z)$.

Now, we have $\tau = \hbar/\Gamma$, where Γ is the total level width. If the partial level widths for the decay modes a, b, z are $\Gamma_a, \Gamma_b, \Gamma_z$, then $G_C(a)G_C(b)$ may be written, as

$$G_C(a) = \frac{\Gamma_a}{\Gamma} \text{ and } G_C(b) = \frac{\Gamma_b}{\Gamma} \quad \dots\dots\dots(4)$$

condition

Now since $G_C(a)$ or $G_C(b)$ is a probability, it must satisfy the normalisation

$$\Sigma G_C(a) = 1 \text{ and also } \Sigma \Gamma_a = \Gamma \quad \dots\dots\dots(5)$$

The mean life times for the modes a, b, \dots are given by [see equation

$$\tau_a = \frac{\hbar}{\Gamma_a}, \tau_b = \frac{\hbar}{\Gamma_b}, \tau_z = \frac{\hbar}{\Gamma_z} \quad \dots\dots\dots(6)$$

$$\text{Hence } \frac{1}{\tau} = \frac{\Gamma}{\hbar} = \frac{\Gamma_a + \Gamma_b + \dots}{\hbar} = \frac{1}{\tau_a} + \frac{1}{\tau_b} + \dots \quad \dots\dots\dots(7)$$

Now, we know that formation and decay processes of the compound nucleus are independent of each other, we find out a relationship between T_a and $\sigma_C(a)$ making use of the reciprocity theorem* which state that

$$k_a^2 \sigma(a, b) = k_b^2 \sigma(b, a) \quad \dots\dots\dots(8)$$

where $k = 1/\lambda$, is the wave number

(λ is to be read as 'lambda cross')

Making use of equations (1) and (4). this may be written as

$$k_a^2 \sigma_C(a) \frac{\Gamma_b}{\Gamma} = k_b^2 \sigma_C(b) \frac{\Gamma_a}{\Gamma} \quad \dots\dots\dots(9)$$

$$\frac{k_a^2 \sigma_C(a)}{\Gamma_a} = k_b^2 \frac{\sigma_C(b)}{\Gamma_b} = k_z^2 \frac{\sigma_C(z)}{\Gamma_z} = \dots = U \quad \dots\dots\dots(10)$$

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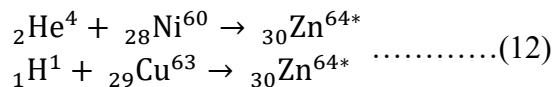
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where U is a function only of the state *i.e.* excitation energy, angular momentum and parity, of the compound nucleus and does not depend upon the decay channel. From equations (4) and (10), the relative probability of decay of the compound nucleus through the channel, ' a ' is given by,

$$G_C(a) = \frac{\Gamma_a}{\Gamma} = \frac{k_a^2 \sigma_C(a)}{\Gamma U} = \frac{k_a^2 \sigma_C(a)}{\Gamma_a U + \Gamma_b U + \Gamma_z U \dots} \dots \dots \dots (11)$$

$$= \frac{k_a^2 \sigma_C(a)}{(k_a^2 \sigma_C(a) + k_b^2 \sigma_C(b) + k_z^2 \sigma_C(z) + \dots)}$$

Of course a number of experimental evidences are available which support Bohr's hypothesis of compound nucleus formation and break up but the most interesting series of experiments to test it was made by Ghoshal. His experiments were designed to study the decay of the compound nucleus ${}_{30}\text{Zn}^{64*}$ which could either be produced by the bombardment of ${}_{28}\text{Ni}^{60}$ by means of α -particles or by bombarding ${}_{29}\text{Cu}^{63}$ by means of protons,



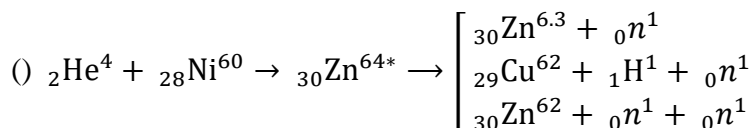
In order to produce the same state of excitation of the compound nucleus ${}_{30}\text{Zn}^{64*}$, the kinetic energy of the α -particles must exceed that of the protons by $\approx 7\text{MeV}$ *i.e.*

Thus

$$E_{kin}^\alpha = E_{kin}^p + 7\text{MeV}$$

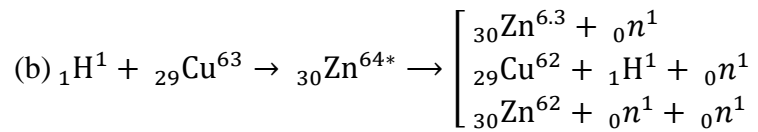
$$(M_\alpha + M_{Ni}^{60})c^2 + 7\text{MeV} = (M_p + M_{Cu}^{63})c^2 \dots \dots \dots (13)$$

The reactions studied were the following since, after the formation of the compound nucleus ${}_{30}\text{Zn}^{64*}$, it may decay in any of the following ways :



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The cross-section for these reactions with the aid of equations (1) and (4) may be written as

$$\sigma(a, b) = \sigma_c(a) \frac{\Gamma_b}{\Gamma} \quad \dots\dots\dots(14)$$

where $\sigma_c(a)$ is the cross-section for the formation of the-compound nucleus with particle a and Γ_a is the partial level width for the decay of compound nucleus through the emission of the particle b . If the decay of the compound nucleus ${}^{64*}_{30}\text{Zn}$ which is in a given excited state, is independent of its mode of formation, the ratio of the cross-section for the first three reactions given by (a) must be equal to the ratio of the cross-section for the last three reactions given by (b), provided the excitation energy of the compound nucleus ${}^{64*}_{30}\text{Zn}$, when it is formed through the bombardment of ${}^{60}_{28}\text{Ni}$ by means of α -particles and when it is formed by the bombardment of ${}^{63}_{29}\text{Cu}$ by protons is the same i.e.

$$\sigma(\alpha, n) : \sigma(\alpha, pn) : \sigma(\alpha, 2n) = \sigma(p, n) : \sigma(p, pn) : \sigma(p, 2n)$$

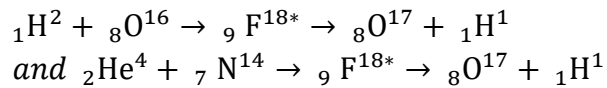
The excitation energy was made the same by making the kinetic energy of the α -particles to exceed that of protons by about 7MeV.

The above relation was actually verified by Ghoshal. The experimental results of Ghoshal with proton cross section curves shifted by 7MeV to the right are shown in fig. (1). The results were found to be accurate within 10%. A notable feature of these curves is that the probability for the decay of the compound nucleus by the emission of protons and neutrons (pn) is much greater than the probability for decay by the emission of two neutrons ($2n$).

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Another confirmation of the compound nuclear hypothesis. was done by R.R. Roy through the study of the decay of the compound nucleus ${}_9\text{F}^{18*}$ which could be formed in either of the following two ways :



The compound nucleus ${}_9\text{F}^{18*}$ formed by the bombardment of ${}_8\text{O}^{16}$ by deuterons was known to decay by the emission of two group of protons for a specified excitation energy. R.R. Roy was able to observe the same two groups of protons excitation energy. R.R. Roy was able to observe the same two groups of protons

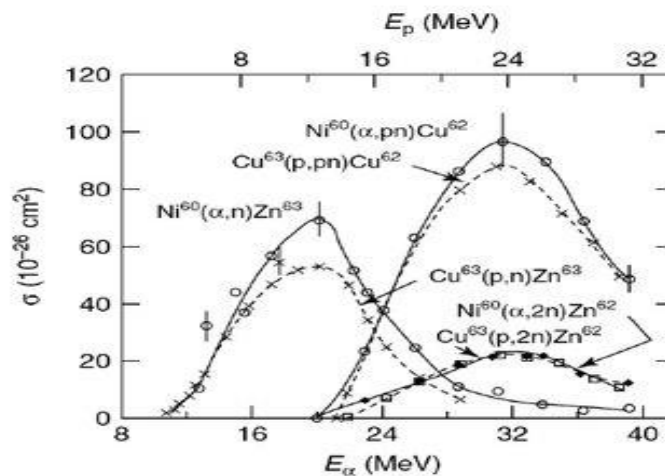


Fig. 1: . Cross-section curves (<https://www.researchgate.net>)

Similar confirmation was also made by John in heavy elements by comparing the excitation function of (α, xn) reaction in ${}_{82}\text{Pb}^{206}$ and (p, xn) reaction in ${}_{83}\text{Bi}^{209}$. The compound nucleus formed was ${}_{84}\text{Po}^{210*}$. According to Bohr's hypothesis we should have

$$\sigma(p, 2n) : \sigma(p, 3n) : \sigma(p, 4n) \dots = \sigma(\alpha, 2n) : \sigma(\alpha, 3n) : \sigma(\alpha, 4n) \dots$$

These results were confirmed experimentally by John.

11.3.1 Excitation Energy Of The Compound Nucleus

The relativistic relationship between the total energy E , momentum p , and rest mass m_0 is as follows :

$$\begin{aligned} E^2 &= p^2 c^2 + m_0^2 c^4 \\ \therefore m_0 c^2 &= [E^2 - p^2 c^2]^{1/2} \dots\dots\dots(15) \end{aligned}$$

Now let,

and M_{NC}^* = the rest mass of the compound nucleus in excited state

M_{CN} = the rest mass of the compound nucleus in ground state

Energy of the excited state = Kinetic energy of the incident particle

$$+ [\text{mass of the target nucleus} + \text{mass of the incident particle}] c^2$$

And excitation energy, $E_{exc} = (E_{\text{excited state}} - E_{\text{ground state}})$

$$= (M_{CN}^* - M_{CN}) c^2 \dots\dots\dots(16)$$

Rest mass energy equivalent of the compound nucleus in the excited state in accordance with equation (15) is

$$\begin{aligned} M_{CN}^* c^2 &= \left[\{K^i + (M^i + M^T) c^2\}^2 - (p^i)^2 c^2 \right]^{1/2} \\ \therefore E_{exc} &= \left[\{K^i + M^i + M^T\} c^2 \right]^2 - (p^i)^2 c^2 \Big]^{1/2} - M_{CN} \cdot c^2 \end{aligned}$$

here M^i and M^T refer to the masses of the incident particle and the target nucleus respectively

$$\begin{aligned} \therefore E_{exc} &= \left[(K^i)^2 + (M^i + M^T)^2 c^4 + 2K^i(M^i + M^T) c^2 \right] \\ &\quad - (p^i)^2 c^2 \Big]^{1/2} - M_{CN} \cdot c^2 \\ &= \left[(K^i)^2 + 2K^i M^i c^2 + (M^i + M^T)^2 c^4 + 2K^i M^T \cdot c^2 \right. \\ &\quad \left. - (p^i)^2 c^2 \right]^{1/2} - M_{CN} \cdot c^2 \\ &= \left[(M^i + M^T)^2 c^4 + 2K^i M^T \cdot c^2 \right]^{1/2} - M_{CN} \cdot c^2 \end{aligned}$$

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Since $(K^i)^2 + 2K^i \cdot M^i c^2 = (p^i)^2 \cdot c^2$

$$\begin{aligned} \therefore E_{exc} &= (M^i + M^T)c^2 \left[1 + \frac{2M^i K^T}{(M^i + M^T)^2 c^2} \right] - M_{CN} \cdot c^2 \\ &\approx (M^i + M^T)c^2 \left[1 + \frac{2K^i M^T}{2(M^i + M^T)^2 \cdot c^2} \right] - M_{CN} \cdot c^2 \\ &\approx (M^i + M^T)c^2 + \frac{K^i M^T}{(M^i + M^T)} - M_{CN} \cdot c^2 \\ &\approx (M^i + M^T - M_{CN})c^2 + \frac{K^i}{\left(1 + \frac{M^i}{M^T}\right)} \dots \dots \dots (17) \end{aligned}$$

This equation shows that the excitation energy of the compound nucleus is made up of two parts

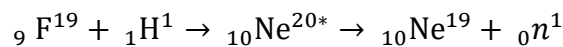
- (i) Part I is independent of the kinetic energy of the incident particle.
- (ii) Part II is proportional to the kinetic energy of the incident particle.

The first part as given above is just the Q -value of the reaction,

$$\text{viz, } Q = (M^i + M^T)c^2 - (M_{CN})c^2$$

where M_{CN} = (mass of the emitted particle + mass of the residual nucleus) and the second part is the kinetic energy available in the centre of mass system.

Example:. Protons of energy 4.99MeV, are made to bombard ${}_9\text{F}^{19}$ as a result of which a neutron is emitted according to the following reaction :



Calculate the excitation energy of the compound nucleus ${}_{10}\text{Ne}^{20*}$. By

Solution: Kinetic energy of the proton available in the C.M. system is given

$$\begin{aligned} &= \frac{K^i}{1 + \frac{M^i}{M^T}} = \frac{4.99\text{MeV}}{1 + \frac{1}{19}} = \frac{4.99 \times 19\text{MeV}}{20} \\ &= 4.74\text{MeV} \end{aligned}$$

To calculate the first part

$$\begin{aligned}
 M^i &= {}_1\text{H}^1 = 1 \cdot 008146 \text{ a.m.u.} & M_{CN} &= {}_{10}\text{Ne}^{20} = 19 \cdot 998772 \\
 M^T &= {}_9\text{F}^{19} = 18 \cdot 004444 \text{ a.m.u.} \\
 M^i + M^T &= 20 \cdot 012590 \text{ a.m.u.} \\
 \therefore (M^i + M^T) &= (20 \cdot 012590 - 19 \cdot 998772) \\
 &= 0.013818 \text{ a.m.u.}
 \end{aligned}$$

which is equivalent to $(0.013818 \times 931)\text{MeV} = 12.88\text{MeV}$

\therefore Excitation energy of the compound nucleus is given by

$$E_{exc} = 4 \cdot 74 + 12 \cdot 88 = 17 \cdot 62\text{MeV}$$

11.4 DIRECT REACTIONS

The compound nucleus process for the nuclear reactions is dominant only in the low energy region i.e. for comparatively slow projectiles and as such is very important in practical applications. However the information provided by it about the nucleus itself is minimal. In fact it was responsible for a 'bum steer' which held back the development of nuclear structure theory for more than a decade. However, back the development of nuclear structure theory for more than a decade. However, as the projectile energy is increased, direct reactions become increasingly and these reactions play a vital role in revealing the structure of nuclei.

The term direct reaction is used for a wide variety of nuclear interactions such as :

- (i) An incident nucleon enters a target nucleus, collides with it. During the collision it loses some of its energy and then comes off (Inelastic collision).
- (ii) An incident nucleon enters a target nucleus and in collision with it loses most of its energy and then comes off (Inelastic collision).

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(iii) An incident nucleon comes close enough to the target nucleus to set it into vibration or if the target nucleus is non-spherical, into rotation. The incident particle continues its way though in a different direction with different energy (Inelastic collision).

(iv) A proton entering a target nucleus transfers its charge to a neutron in the nucleus through meson exchange and then comes off as a neutron (Knock out process).

(v) An incident nucleon comes close enough to a target nucleus, picks up a nucleon from the nuclear surface and the two come off together as deuteron. (Pick up reaction).

(vi) An incident deuteron comes close enough to a target nucleus for one of its two constituent nucleons to settle in the nucleus

Thus direct reactions include inelastic nuclear collisions, stripping reactions and their inverse i.e. the pick up reactions and knock out processes etc. All these processes have one thing in common viz. the life time of the corresponding compound nucleus. Thus a direct reaction is the one which proceeds without the intermediate step of formation of the compound nucleus. Direct reactions are considered to be instantaneous processes and take place only at the surface of the target nucleus and therefore direct reaction products exhibit certain characteristics which are markedly different from those if the reaction proceeds by way of compound nucleus formation. In these reactions the impinging particle hits a specific nucleon in the target and the remainder of the nucleus simply acts as a "spectator".

11.5 THEORY OF STRIPPING AND PICK-UP REACTIONS

The term 'stripping' is used for a type of direct reaction in which an incident compound particle (an incident projectile having more than one nucleons) is split into two fragments, one of which is absorbed within the target nucleus while the other emerges at some angle θ with the incident direction. An example of such a reaction is provided by deuteron. The most common reactions involving deuteron are the (d,p) and the (d,n) reactions. Deuteron has a low binding energy of only 2.225MeV per nucleon and a comparatively large neutron-proton separation of about $2 \cdot 18$ Fermi. This means that the neutron and

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the proton forming the deuteron are very loosely bound together. It is therefore quite probable that when deuteron strikes against a target, one of its constituent nucleons (either the proton or the neutron) may be captured by the target nucleus and the other simply scattered which is pictorially depicted below :

Qualitatively, deuteron stripping mechanism may be understood as follows:

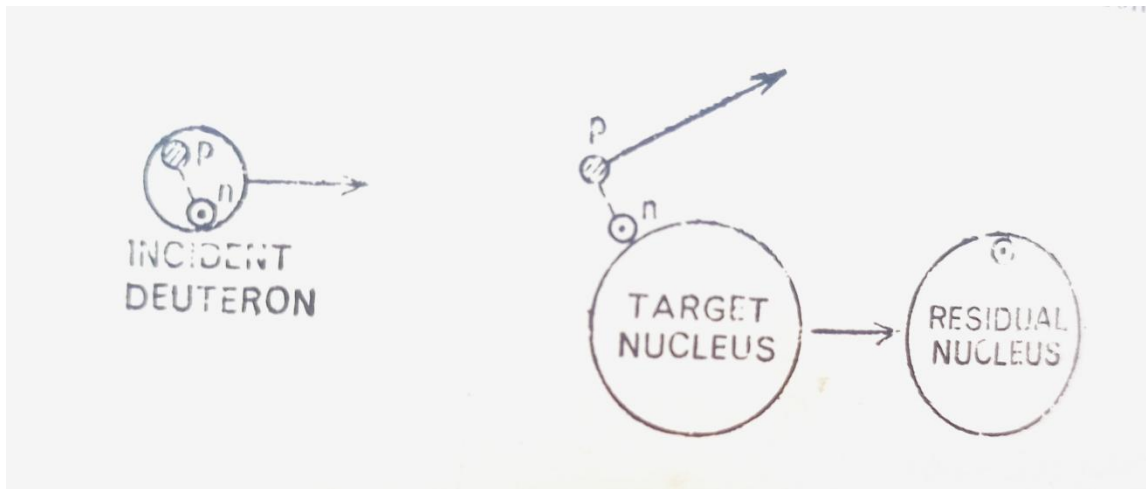


Fig.2: . Pictorial depiction of deuteron stripping.

Because of the finite size of the deuteron, it may happen that one of its constituent nucleons (either the proton or the neutron) comes in contact with the target nuclear surface before the other. Since nuclear interaction energies are much higher than the low binding energy of only 2.225MeV per nucleon for deuteron, the nucleon arriving first at the nuclear surface may be quickly separated from its partner and form a new residual nucleus. If the second nucleon, hits the target nuclear surface an instant later, compound nucleus formation takes place which decays at a later instant as described earlier. However, if the second nucleon misses the target nuclear surface, the stripping process, which is a direct process, takes place. The stripping process, is more probable at low energies of the deuteron projectile since then the Coulomb repulsion between the target nucleus and the

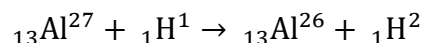
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proton keeps the proton away from the target nucleus. The nucleon that misses the target, proceeds with its original speed plus whatever is added to it during the breakup (stripping) process.

The first experimental studies of the (d, p) stripping reactions at low deuteron energies revealed that the reaction cross-section was much higher than anticipated. The theory of low energy deuteron stripping was first put forth by Oppenheimer and Phillips.

At very high deuteron projectile energies ($> 100\text{MeV}$), the stripping process has proved to be of considerable practical use in producing nearly mono-energetic beams of fast neutrons. For example, stripping of 190MeV deuterons from the Berkley Synchro-cyclotron, produced near mono-energetic beam of 90MeV neutrons confined to a narrow cone in the forward direction.

The inverse of the stripping reaction is called a pick-up reaction such as the following (p, d) reaction :



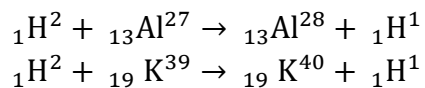
Other examples of pick-up reactions are (p, α) and (n, d) reactions. The (p, d) pick-up reaction takes place the following way :

As the incoming proton approaches the target nuclear surface, one of the surface neutrons is plucked out from the nuclear surface by the proton-neutron $(p - n)$ nuclear force and the two (the incident proton and the plucked neutron) travel on as bound deuteron. In the (n, d) pick-up reaction, a proton is picked up from the nuclear surface and the incoming neutron and this picked up proton travel on as deuteron. Similarly (p, α) pick-up reaction involves the pick-up of a triton out of the target nucleus.

Another reaction produced by deuteron is the so-called knock-out reaction in which deuteron projectile while colliding with the target nucleus enters the target nuclear surface but in the process, it may collide with a proton which is knocked out of the target nucleus and moves away from it. Following are the examples of the knock-out reactions:

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From what has been said above, it is seen that the theory of stripping and pick-up reactions is very much similar. A general theory of stripping and pick-up reactions was put forward by S.T. Butler. For simplicity we have a semi-classical description of the (d, p) stripping reaction.

Let $\mathbf{K}_d\hbar$ be the momentum of deuteron projectile and $\mathbf{K}_p\hbar$ the momentum of the proton just after stripping and θ_p be the angle between the incident deuteron direction and the direction of travel of the stripped

$$\mathbf{K}_p\hbar = \frac{1}{2}\mathbf{K}_d\hbar + \mathbf{K}_0\hbar \dots\dots\dots(18)$$

where $\mathbf{K}_p\hbar$ is the contribution to the momentum of the stripped proton from the internal momentum of deuteron at the instant of stripping. The neutron from the deuteron approaches the nuclear surface with a momentum $\mathbf{K}_n\hbar$ which is given by

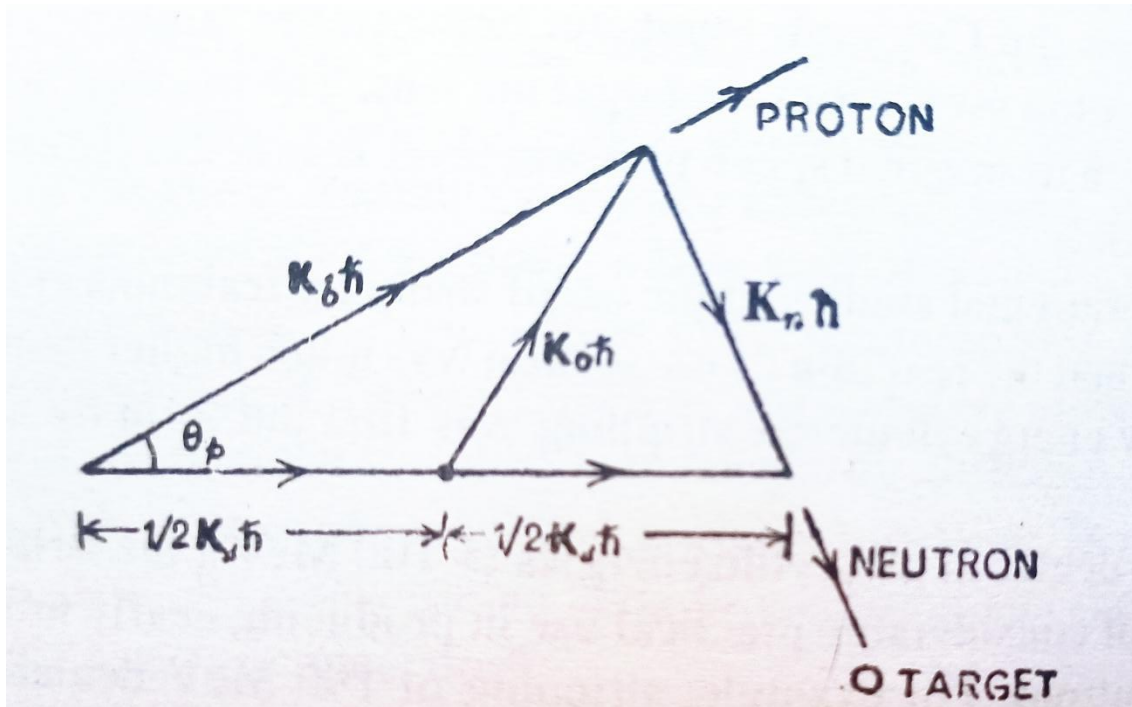


Fig.3.: Momentum relations in (d, p) stripping reactions.

$$\begin{aligned}
 \mathbf{K}_n \hbar &= \mathbf{K}_d \hbar - \mathbf{K}_p \hbar \\
 &= \mathbf{K}_d \hbar - \frac{1}{2} \mathbf{K}_d \hbar - \mathbf{K}_0 \hbar \\
 &= \frac{1}{2} \mathbf{K}_d \hbar - \mathbf{K}_0 \hbar \dots \dots \dots (19)
 \end{aligned}$$

In crude approximation $K_0 \hbar = |\mathbf{K}_0 \hbar|$ can be taken to be

$$K_0 \hbar = \sqrt{(M_n E_B)} \dots \dots \dots (20)$$

where M_n = neutron mass and E_B = the binding energy of deuteron,

Now from fig. (3)

$$\begin{aligned}
 (K_0 \hbar)^2 &= (K_p \hbar)^2 + \left(\frac{1}{2} K_d \hbar\right)^2 - K_p K_d \hbar^2 \cos \theta_p \\
 \text{or } K_0^2 &= K_p^2 + \frac{1}{4} K_d^2 - K_p K_d \cos \theta_p \\
 &= K_p^2 + \frac{1}{4} K_d^2 - K_p K_d (1 - 2 \sin^2 \theta_p / 2) \\
 &= \left(K_p - \frac{1}{2} K_d\right)^2 + 2 K_p K_d \sin^2 \theta_p / 2
 \end{aligned}$$

or

$$K_0 = \left[\left(K_p - \frac{1}{2} K_d\right)^2 + 2 K_p K_d \sin^2 \theta_p / 2 \right]^{1/2} \dots \dots \dots (21)$$

and

$$\begin{aligned}
 (K_n \hbar)^2 &= (K_p \hbar)^2 = (K_d \hbar)^2 - 2 K_p K_d \hbar^2 \cos \theta_p \\
 K_n^2 &= K_p^2 + K_d^2 - 2 K_p K_d \cos \theta_p \\
 &= K_p^2 + K_d^2 - 2 K_p K_d (1 - 2 \sin^2 \theta_p / 2) \\
 &= (K_p - K_d)^2 + 4 K_p K_d \sin^2 \theta_p / 2 \\
 \therefore K_n &= \left[(K_p - K_d)^2 + 4 K_p K_d \sin^2 \theta_p / 2 \right]^{1/2} \dots \dots \dots (22)
 \end{aligned}$$

If the neutron has an impact parameter R on the target nucleus, then the orbital angular momentum of the captured neutron is given by

or

$$\begin{aligned}
 K_n \hbar R &= \sqrt{[l(l+1)]} \hbar \\
 K_n \hbar R &= \sqrt{[l(l+1)]} \cong l \dots \dots \dots (23)
 \end{aligned}$$

In order to satisfy this equation K_n must have certain discrete values. For given values of K_p and K_d , this means that the possible values of θ_p are restricted by equations (21) and (22). This is a semi-classical argument and the uncertainty principle relaxes this rather stringent condition. Nevertheless the emergent protons are expected to be peaked in a direction θ_p . From equation (21) we find that for given values of K_p and K_d , K_0 increases

with θ_p . For S -wave neutrons ($l = 0$), we can take $\theta_p = 0$ meaning thereby that the protons are expected to be peaked in the forward direction.

11.6 RESONANCE SCATTERING AND REACTION CROSS-SECTIONS .

Breit-Wigner Dispersion Formula For $l = 0$ neutrons.

When target nuclei are bombarded by means of projectiles (both charged and uncharged), the phenomenon of resonance in nuclear reaction and scattering cross-sections is observed. The resonance is characterized by a sharp increase in the cross-sections. We are familiar with the phenomenon of resonance in classical physics. Resonance corresponds to driving a physical system capable of vibrations at one of its natural frequencies and always corresponds to a sharp rise in the induced amplitude. In quantum systems, resonance corresponds to driving a physical system having a large number of energy levels, at a frequency appropriate to induce transitions from the original state of the system represented by an energy E , to another state of energy E' . The resonance frequency here is given by $h\nu = E' - E_0$ or $\hbar\omega = E' - E_0$ and the resonant state or resonance refers to such an excited state.

In a quantised system, we have a ground state with a well defined energy say E_0 and a number of excited states each of which is unstable and as such does not possess a well defined energy but has an energy spread characterised by a width Γ and a central energy $E_1 = \hbar\omega_1$. The resonant energy i.e., the energy corresponding to maximum scattering is thus given by $\hbar\omega = E_1 - E_0$.

If we have a large probability for the formation of the excited state which has a number of decay modes then we may expect maxima in the cross-section for inelastic reactions at the resonant energy or the elastic reactions. The criterion for the existence of resonance then is sharp peaks in the cross-sections for both elastic and inelastic reactions.

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On entering the nucleus an individual nucleon may collide with nucleons in the bound state (target nuclei) raising these nucleons to excited states and thus forming a many nucleons excited state of the compound nucleus. For a discussion of the resonances observed in scattering and absorption cross-sections, we start with the assumption that the nucleus has a well-defined surface of radius R . Outside this radius R , the incident particle experiences no nuclear potential at all. Inside the nuclear surface, an incident particle is subjected to a negative time-independent potential so that it moves with a kinetic energy much larger than that it had outside the nuclear surface. As a consequence, its wavelength changes but it still retains its original identity. We further assume that inside the nuclear surface, the incident particle is subjected to strong interaction, so that it rapidly shares its energy with other nucleons present within the nucleus.

The scattering coefficient η_l which determines the absolute values of the scattering and reaction cross-sections is itself determined by the logarithmic derivative of the modified wave function at the nuclear surface. Of course evaluation of η_l is a hopelessly difficult task but even then under some extreme conditions η_l is expected to behave in a simple and predictable manner. It is under this expectation that we have considered the nucleus possessing a well-defined surface of radius R .

Here, we consider only the S -wave neutrons viz, $l = 0$ neutrons which avoids complications through Coulomb or centrifugal barriers. We also refrain from incorporating the introduction of a statistical spin factor. We shall here derive the Breit-Wigner formula in the neighborhood of a single isolated resonance level. We define a modified wave function as follows, by assuming that the wave function of the net system comprising of a neutron and a scattering nucleus is determined only by the radial distance r from the interaction centre. The radial wave function in terms of the outgoing and incoming waves may be written as

$$U(r) = r\psi(r) = U_{in}(r) + U_{out}(r) \dots\dots(24)$$

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Therefore for $l = 0$ neutrons, we get the total wave function as : for $r \geq R$

$$[\psi(r)]_{l=0} = \frac{\pi^{1/2}}{kr} i [e^{-i(kr)} - \eta_0 e^{i(kr)}] Y_{0,0}(\theta) = [U(r)]_{l=0} Y_{0,0}(\theta) \dots (25)$$

\therefore The radial wave function $[U(r)]_{l=0}$ for $l = 0$ neutrons is

$$[U(r)]_{l=0} = \frac{\pi^{1/2}}{k} i [e^{-ikr} - \eta_0 e^{ikr}] \dots\dots\dots(26)$$

Comparison of equations (24) and (26) shows that

$$U_{in}(r) = r\psi_{in}(r) = \frac{\pi^{1/2}}{k} i e^{-ikr} \dots\dots\dots(27)$$

$$\text{and } U_{out} = r\psi_{out} = -\frac{\pi^{1/2}}{k} i \eta_0 e^{ikr} \dots\dots\dots(28)$$

We now require that at the nuclear surface ($r = R$) the radial wave functions $U_{in}(r)$, $U_{out}(r)$ and their first derivatives match each other i.e.,

$$[U_{in}(r)]_{r=R} = [U_{out}(r)]_{r=R} \text{ and } \left[\frac{dU_{in}(r)}{dr} \right]_{r=R} = \left[\frac{dU_{out}(r)}{dr} \right]_{r=R} \dots\dots\dots(29)$$

This means that

$$\left[\frac{1}{U_{in}(r)} \cdot \frac{dU_{in}(r)}{dr} \right]_{r=R} = \left[\frac{1}{U_{out}(r)} \cdot \frac{dU_{out}(r)}{dr} \right]_{r=R} \dots\dots\dots(30)$$

Let us now define a dimensionless quantity ' f ' as follows

$$f \equiv \text{Lim}_{r \rightarrow R} \left(\frac{RdU}{Udr} \right) \dots\dots\dots(31)$$

which is R times the logarithmic derivative of radial wave function $U \equiv U(r)$ evaluated at the nuclear surface. From equation (26), we have

$$\begin{aligned}
\left[\frac{dU}{dr}\right]_1 &= \frac{\pi^{1/2}}{k} i [(-ik)e^{-ikr} - (ik)\eta_0 e^{ikr}] \\
\therefore \left[\frac{dU}{dr}\right]_{r=R} &= \pi^{1/2} [e^{-ikR} + \eta_0 e^{ikR}] \\
f = \left[\frac{R}{U} \cdot \frac{dU}{dr}\right]_{r=R} &= \frac{kR}{\pi^{1/2}} \cdot \frac{\pi^{1/2} [e^{-ikR} + \eta_0 e^{ikR}]}{i [e^{-ikR} - \eta_0 e^{ikR}]} \\
f &= -ikR \left[\frac{e^{-ikR} + \eta_0 e^{ikR}}{e^{-ikR} - \eta_0 e^{ikR}} \right] \dots \dots \dots (32) \\
fe^{-ikR} - f\eta_0 e^{ikR} &= -ikRe^{-ikR} - ikR\eta_0 e^{ikR} \\
fe^{-ikR} + ikRe^{-ikR} &= f\eta_0 e^{ikR} - ikR\eta_0 e^{ikR} \\
(f + ikR)e^{-ikR} &= \eta_0 (f - ikR)e^{ikR} \\
\eta_0 &= \frac{[f + ikR]e^{-ikR}}{[f - ikR]e^{ikR}} = \frac{[f + ikR]}{[f - ikR]} e^{-2ikR} \dots \dots \dots (33)
\end{aligned}$$

We have the scattering crosssection for $l = 0$ neutrons as,

$$\begin{aligned}
\sigma_{sc,0} &= \pi(\lambda)^2 |1 - \eta_0|^2 \\
&= \pi(\lambda)^2 \left| 1 - \frac{f + ikR}{f - ikR} e^{-2ikR} \right|^2 \\
&= \pi(\lambda)^2 \left| \frac{(f - ikR)e^{2ikR} - (f + ikR)}{(f - ikR)e^{2ikR}} \right|^2 \\
&= \pi(\lambda)^2 \left| \frac{fe^{2ikR} - ikRe^{2ikR} - f - ikR}{(f - ikR)e^{2ikR}} \right|^2 \\
&= \pi(\lambda)^2 \left| \frac{fe^{2ikR} - ikRe^{2ikR} - f - 2ikR + ikR}{(f - ikR)e^{2ikR}} \right|^2 \\
&= \pi(\lambda)^2 \left| \frac{(fe^{2ikR} - 1)ikR(e^{2ikR} - 1) - 2ikR}{(f - ikR)e^{2ikR}} \right|^2 \\
&= \pi(\lambda)^2 \left| \frac{(e^{2ikR} - 1)(f - ikR) - 2ikR}{(f - ikR)e^{2ikR}} \right|^2 \\
&= \frac{\pi(\lambda)^2}{e^{2ikR}} \left| e^{2ikR} - 1 - \frac{2ikR}{f - ikR} \right|^2 \\
&= \pi(\lambda)^2 \left| e^{2ikR} - 1 - \frac{2ikR}{f - ikR} \right| \dots \dots \dots (34)
\end{aligned}$$

This is so since $e^{2ikR} = |\cos 2kR + i \sin 2kR|$

$$\begin{aligned} &= \sqrt{(\cos^2 2kR + \sin^2 2kR)} = 1 \\ \therefore \sigma_{sc,0} &= \pi(\pi)^2 |A_{pot} + A_{res}|^2 \dots \dots \dots (35) \end{aligned}$$

where $A_{res} = -\frac{2ikR}{f-ikR}$ and $A_{pot} = e^{2ikR} - 1$. A_{res} is called internal or the resonance scattering amplitude and A_{pot} is called the potential or hard sphere scattering amplitude. This origin of the names comes from the consideration of nucleus as perfectly reflecting sphere of radius R . This sphere forces the wave-function to vanish at $r = R$. According to the definition of $f = \text{Lim}_{r \rightarrow R} \left\{ \frac{RdU}{Udr} \right\}$, this implies that f becomes infinite at the nuclear surface and in that case A_{res} becomes zero for a hard sphere. Scattering then is determined only by A_{pot} .

The separation of the scattering amplitude into an internal and external part depends upon the assumption of the nucleus having a well defined surface of radius R . Such a separation has only theoretical meaning i.e. it can not be observed experimentally. The only experimentally measurable quantity is $A_{res} + A_{pot}$. In spite of these restrictions, the separation of the scattering amplitude into an internal and external part is of prime importance for the proper understanding of nuclear reaction theory.

Now if f is real, we find from equation (33) that $|\eta_0| = 0$ and then there is no reaction (absorption). Therefore only pure elastic scattering would result. However, in general f will be a complex quantity and therefore we can write it as

$$f = f_{Re} + if_{Im} \dots \dots \dots (36)$$

where the symbols Re and Im stand for real and imaginary parts of f . In order that $|\eta_0| < 1$, the imaginary part must be negative, as is evident from expression (11 · 140) giving the value of η_0 .

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Now

$$A_{\text{res}} = \frac{-2ikR}{f-ikR} = \frac{-2ikR}{f_{\text{Re}}+if_{\text{Im}}-ikR} \dots\dots\dots(37)$$

by means of eqn. (36)

Showing that A_{res} is maximum at $f = 0$

and

$$\begin{aligned} A_{\text{pot}} &= e^{2ikR} - 1 = e^{ikR}(e^{ikR} - e^{-ikR}) \\ &= 2ie^{ikR} \sin kR \quad \dots\dots\dots(38) \end{aligned}$$

Substituting for A_{res} and A_{pot} in the above equation (34), we get,

$$\begin{aligned} \sigma_{sc,0} &= \pi(\lambda)^2 \left| 2ie^{ikR} \sin kR + \frac{-2ikR}{f_{\text{Re}} + if_{\text{Im}} - ikR} \right|^2 \\ &= 4\pi(\lambda)^2 \left| e^{ikR} \sin kR + \frac{-kR}{f_{\text{Re}} + if_{\text{Im}} - ikR} \right|^2 \\ &= 4\pi(\lambda)^2 \left| e^{ikR} \sin kR + \frac{kR}{i(kR - f_{\text{Im}}) - f_{\text{Re}}} \right|^2 \dots\dots\dots(39) \end{aligned}$$

Similarly, by the value of η_0 from equations (33) with $l = 0$, we obtain the reaction cross-section,

$$\sigma_{a,1} = \pi(\lambda)^2(1 - |\eta_0|^-) = \pi(\lambda)^2 \left[1 - \left| \frac{f + ikR}{f - ikR} e^{-2ikR} \right|^2 \right]$$

$$\begin{aligned}
&= \pi(\lambda)^2 \left[1 - \left| \frac{f_{\text{Re}} + if_{\text{Im}} + ikR}{f_{\text{Re}} + if_{\text{Im}} - ikR} e^{-2ikR} \right|^2 \right] \\
&= \pi(\lambda)^2 \left(\left[1 - \left| \frac{f_{\text{Re}} + i(kR + f_{\text{Im}})^2}{f_{\text{Re}^2} - i(kR - f_{\text{Im}})^2} \right|^2 \right] \right) \\
&= \pi(\lambda)^2 \left[1 - \frac{f_{\text{Re}^2} + (kR + f_{\text{Im}})^2}{f_{\text{Re}^2} + (kR - f_{\text{Im}})^2} \right] \\
&= \pi(\lambda)^2 \left[\frac{(kR - f_{\text{Im}})^2 - (kR + f_{\text{Im}})^2}{f_{\text{Re}^2} + (kR - f_{\text{Im}})^2} \right] \\
&= \pi(\lambda)^2 \left[\frac{-4f_{\text{Im}}kR}{f_{\text{Re}^2} + (kR - f_{\text{Im}})^2} \right] \dots \dots \dots (40)
\end{aligned}$$

We again find that for real f , i.e. $f_{\text{Im}} = 0$, $\sigma_{a,0}$ vanishes and the scattering cross-section $\sigma_{sc,0}$ takes on a form that may be easily interpreted if $\sin kR$ is very large or very small compared with $\left| \frac{kR}{ikR - f_{\text{Re}}} \right|$. So,

(i) If $\sin kR$ is very large i.e. kR is very large, then $A_{pot} \gg A_{res}$. It will correspond to a situation far away from resonance, then from equations (11.141) and (11.145), $\sigma_{sc,0}$ is given by

$$\begin{aligned}
\sigma_{sc,0} &= \pi(\hbar)^2 |A_{pot}|^2 \\
&= \pi(\hbar)^2 |2ie^{ikR} \sin kR|^2 \\
&= 4(\pi)^2 |e^{ikR} \sin kR|^2 \\
&= 4\pi(\pi)^2 \sin^2 kR. \text{ Since } e^{|ikR|^2} = 1 \dots \dots \dots (41)
\end{aligned}$$

If $kR \ll 1$ viz. for low energy neutrons, we get

$$\sigma_{sc,0} \simeq 4\pi(\pi)^2 \cdot k^2 R^2 \simeq 4\pi R^2 \dots \dots \dots (42)$$

which is four times the geometrical cross-section. This is the hard sphere scattering formula and represents scattering from an impenetrable sphere of radius R . This holds good for low energies. Since potential scattering is a function of nuclear radius R , it can provide us information regarding the size of the nucleus.

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(ii) If sinkR is very small i.e kR is very small. This implies that $A_{\text{res}} \gg \gg A_{\text{pot}}$. This will correspond to a situation quite near to the resonance. In this case, the scattering cross-section $\sigma_{sc,0}$ as given quite near to the resonance

$$\begin{aligned}\sigma_{sc,0} &= \pi(\hbar)^2 |A_{\text{res}}|^2 = \pi(\hbar)^2 \left| \frac{-2ikR}{f_{\text{Re}} + if_{\text{Im}} - ikR} \right|^2 \\ &= 4\pi(\pi)^2 \left| \frac{kR}{i(kR - f_{\text{Im}}) - f_{\text{Re}}} \right|^2 \\ &= \frac{4\pi R^2}{(kR - f_{\text{Im}})^2 + f_{\text{Re}}^2} \dots \dots \dots (43)\end{aligned}$$

which is the resonance scattering formula. This will have maximum value when $f_{\text{Re}} = 0$, so that

$$[\sigma_{sc,0}]_{\text{max}} = 4\pi(\hbar)^2$$

This shows that σ_{sc} depends critically upon f_{Re} . Assume now that for a certain value E_0 of the energy of the incident particle, $f_{\text{Re}} = 0$, so that resonance occurs at an energy E_0 . To study the behaviour of the resonance amplitude A_{res} and $[\sigma_{sc,0}]_{\text{res}}$, the scattering cross-section for $l = 0$ neutrons at an energy E in the vicinity of resonance energy E_0 , we can expand f in a Taylor series,

$$f = f|_{E=E_0} + \left. \frac{\partial f}{\partial E} \right|_{E=E_0} (E - E_0) + \dots \dots \dots (44)$$

and use the so called linear approximation method i.e. neglect all derivatives of orders higher than the first while noting that $f|_{E=E_0}$ vanishes identically. If now we define a width $\Gamma_{sc,0}$ through the relation

$$\left. \frac{\partial f}{\partial E} \right|_{E=E_0} = -\frac{2kR}{\Gamma_{sc,0}} \dots \dots \dots (45)$$

We can write

$$f = -\frac{2kR}{\Gamma_{sc,0}}(E - E_0) \dots\dots\dots(46)$$

and hence under the approximation; (for real f)

$$f = f_{Re} = -\frac{2kR}{\Gamma_{sc,0}}(E - E_0) \text{ while } f_{Im} = 0 \dots\dots\dots(47)$$

Substituting this value in the expression (43), we obtain

$$\begin{aligned} [\sigma_{sc,0}]_{res} &= \frac{4\pi(\lambda)^2 \cdot (kR)^2}{(kR)^2 + \frac{4k^2R^2(E - E_0)^2}{\Gamma_{sc,0}^2}} \\ &= \frac{4\pi(\lambda)^2\Gamma_{sc,0}^2}{\Gamma_{sc,0}^2 + 4(E - E_0)^2} \\ &= \frac{\pi(\lambda)^2\Gamma_{sc,0}^2}{(\Gamma_{sc,0}/2)^2 + (E - E_0)^2} \dots\dots\dots(48) \end{aligned}$$

This is the famous Breit-Wigner Single-Level Formula For Scattering. The above equation shows that the maximum possible cross-section is $4\pi(\lambda)^2$, which corresponds to the case $E = E_0$. In a region in the vicinity of resonance energy E_0 , the scattering cross-section shows the typical resonance shape common to many a physical phenomena, notably the refractive index of a medium near an absorption line. The physical meaning of $\Gamma_{sc,0}$ is the full energy width of resonance peak at half its maximum value as shown in the fig. (4).

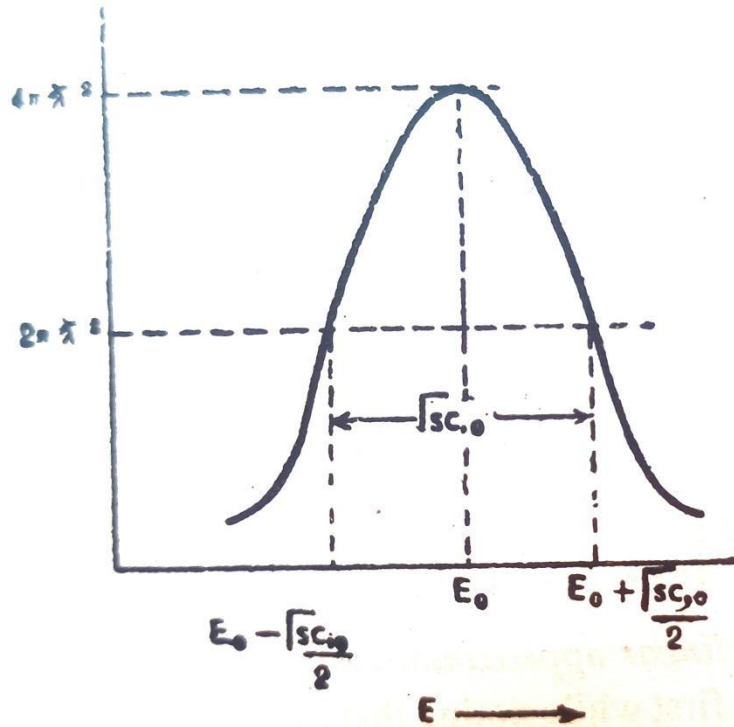


Fig.4: Schematic representation of the level width $\Gamma_{sc,0}$.

11.6.1 Reaction Cross-Section

For calculating this, we have to consider for the case where f is complex, in order to encompass the case where there are reactions besides elastic scattering. When reaction takes place f_{lm} is not zero. In this case we shall not be dealing with the ordinary stationary states of the Schrodinger's equation. The square of the modulus of the wavefunction of the nucleus shows time dependence. In order to include such time dependence, we formally introduce a complex value for energy.

Let

$$E = \epsilon - \frac{i\Gamma_{a,0}}{2} \dots\dots(49)$$

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There will be a value of the complex variable near E_0 for which f will be zero. We call this value $E_a - \frac{i\Gamma_{a,0}}{2}$. We now desire to have f in the vicinity of its zero value and for real values of the energy. Expanding f in a power series and applying the linear approximation i.e. taking the first term only, we get, since

$$f\left(E_a - \frac{i\Gamma_{a,0}}{2}\right) = 0$$

$$f(E) = \left(E - E_a + \frac{i\Gamma_{a,0}}{2}\right) \cdot \frac{\partial f}{\partial E}\left(E_a - \frac{i\Gamma_{a,0}}{2}\right) + \dots$$

The imaginary part of $\frac{\partial f}{\partial E}\left(E_a - \frac{i\Gamma_{a,0}}{2}\right)$ is small and so to a good approximation, we can consider only its real part. Therefore our expansion gives

$$f(E) = \left(E - E_a + \frac{i\Gamma_{a,0}}{2}\right) \cdot \frac{\partial f}{\partial E}\bigg|_{E=E_0}$$

Now from equation (45)

$$\frac{\partial f}{\partial E}\bigg|_{E=E_0} = -\frac{2kR}{\Gamma_{sc,0}}$$

$$\therefore f_{\text{Rc}} = -(E - E_a) \frac{2k\Gamma_{a,0}R}{\Gamma_{sc,0}} \dots\dots\dots(50)$$

and $f_{\text{Im}} = -\frac{k\Gamma_{a,0}}{\Gamma_{sc,0}} \dots\dots\dots(51)$

Substituting these values in the expression (40) for the reaction cross-section, we obtain

$$\begin{aligned}
\sigma_{a,0} &= \pi(\hbar)^2 \left[4k^2 R^2 \cdot \frac{\Gamma_{a,0}}{\Gamma_{sc,0}} \right] / \left[\left(kR + \frac{kR\Gamma_{a,0}}{\Gamma_{sc,0}} \right)^2 + (E - E_a)^2 \frac{4k^2 R^2}{\Gamma_{sc,0}} \right] \\
&= \pi(\lambda)^2 \frac{4\Gamma_{a,0}\Gamma_{sc,0}}{(\Gamma_{sc,0} + \Gamma_{a,0})^2 + 4(E - E_a)^2} \\
&= \pi(\lambda)^2 \frac{\Gamma_{a,0}\Gamma_{sc,0}}{\left[\frac{(\Gamma_{sc,0} + \Gamma_{a,0})}{2} \right]^2 + (E - E_a)^2} \dots \dots \dots (52) \\
&= \frac{\pi(\lambda)^2 \Gamma_{a,0}\Gamma_{sc,0}}{\frac{\Gamma^2}{4} + (E - E_a)^2} \quad (53)
\end{aligned}$$

where $E_a \sim E_0$ and $\Gamma_{a,0} + \Gamma_{sc,0} =$ total level width

Similarly from equation (43)

$$\begin{aligned}
\sigma_{sc,0} &= 4\pi R^2 \frac{1}{\left(kR + \frac{kR\Gamma_{a,0}}{\Gamma_{sc,0}} \right) + (E - E_a)^2 \frac{4k^2 R^2}{\Gamma_{sc,0}^2}} \\
&= 4\pi(\lambda)^2 \frac{\Gamma_{sc,0}^2}{(\Gamma_{sc,0} + \Gamma_{a,0})^2 + 4(E - E_a)^2} \dots \dots \dots (54) \\
&= \frac{\pi(\lambda)^2 \Gamma_{sc,0}^2}{\left(\frac{\Gamma_{sc,0} + \Gamma_{a,0}}{2} \right)^2 + (E - E_a)^2} \\
&= \frac{\pi(\lambda)^2 \Gamma_{sc,0}^2}{\frac{\Gamma^2}{4} + (E - E_a)^2}
\end{aligned}$$

Equations (53) and (54) are the Breit-Wigner Formula For Single Isolated Level For $l = 0$ Neutrons when f is complex. Equation (53) shows that resonance reaction is always accompanied by resonance scattering since $\sigma_{a,0}$ is zero when $\sigma_{sc,0}$ zero.

In order to trace the resonance curve for scattering cross-section in detail, we shall have to include A_{pot} also. Doing this, we get

$$\sigma_{sc,0} = \pi(\lambda)^2 \left| \frac{\Gamma_{sc,0}}{i \left(\Gamma_{sc,0} + \Gamma_{a,0} \right) + (E - E_0)} + 2e^{ikR} \sin kR \right|^2$$

If we assume that resonance shows scattering only, i.e., $\Gamma_{a,0} = 0$, and also that the neutron energy is low, i.e. $kR \ll 1$, then approximately

$$\sigma_{sc,0} = \pi(\lambda) \left| \frac{\Gamma_{sc,0}}{i\Gamma_{sc,0} + (E - E_a)} \right|^2 \dots\dots\dots(55)$$

As the energy E approaches the resonance energy E_a , $(E - E_a)$ diminishes and becomes exactly equal to zero at resonance, then

$$\begin{aligned} [\sigma_{sc,0}]_{res} &= \pi(\lambda)^2 \left| \frac{\Gamma_{sc,0}}{i\Gamma_{sc,0}} + 2kR \right|^2 \\ &= 4\pi(\lambda)^2 |-i| + kR|^2 \\ &= 4\pi(\lambda)^2 \text{ since } kR \text{ is small } \dots\dots\dots(56) \end{aligned}$$

For values of E smaller than E_a , i.e. for the negative values of the term $(E - E_a)$, the real part of the first term in the expression for the cross-section (55) becomes negative and so reduces the positive term kR .

For values of $E > E_a$, $(E - E_a)$ is positive, the first term decreases again and so also the cross-section. A plot of $\sigma_{sc,0}$ for $l = 0$ neutrons is shown in the fig. (11 · 10) below.

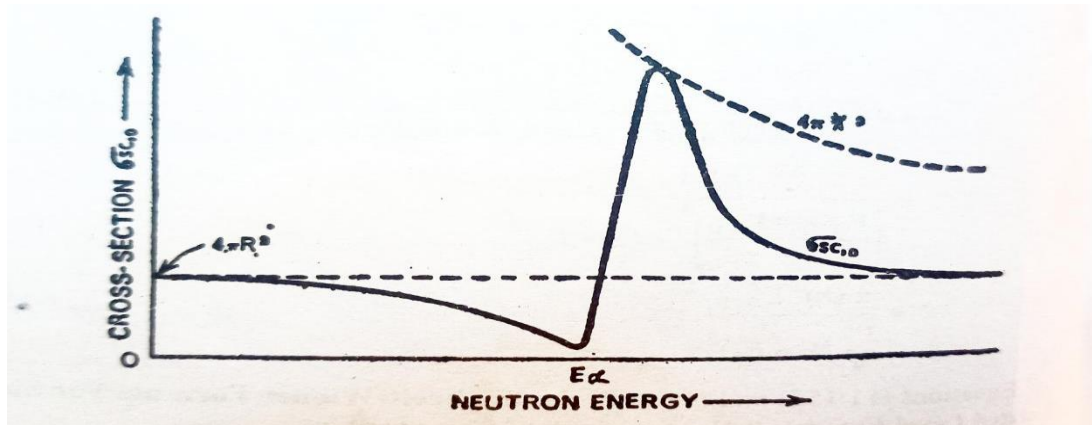


Fig.5: Elastic scattering cross-section for $l = 0$ neutrons

11.7 CONTINUUM THEORY OF NUCLEAR REACTIONS

If we investigate the interaction of neutrons with nuclei at different energies, starting from the lowest neutron energy, we find that at neutron energies below the lowest excitation energy of the compound nucleus, the only decay channel which is open, is the entrance channel. There can then be no energy loss and the only possible reaction is, elastic scattering. Further because of low incident energy, only $l = 0$ neutrons will be affected by the nucleus. As the energy of the incident neutrons increases, the number of available channels also increases. Let us consider the situation in which the number of available channels is very large. For medium and heavy nuclei, this will be the case when the energy of the incident neutrons is of the order of several MeV. The postulate, that a large number of the channels are available, simplifies the problem in that the probability of the incident particle leaving through the entrance channel becomes negligible. In this case no resonances in cross-section will be observed because the individual levels become broader and are very closely spaced. For this reason this theory is known as continuum theory of nuclear reactions. The continuum theory of nuclear cross-sections treats the individual levels not separately but as an average over many resonances. We now formulate the continuum theory for $l = 0$ neutrons.

We have seen that scattering and absorption cross-sections could be completely determined provided, the scattering coefficient η_l , introduced earlier, is known. This in turn requires the knowledge of f - the logarithmic derivative of the modified radial wave function $U(r)$. Hence the crux of the problem in nuclear reaction studies is the evaluation of η from some nuclear model.

Hence we start with the following model of the nucleus :

- (i) The nucleus possesses a well defined surface of radius R .
- (ii) As the incident neutrons enters the nuclear surface, it is subjected to a negative potential and hence it moves with a much larger kinetic energy than what it had outside the nuclear surface.

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(iii) The incident neutron is subjected to very strong interactions within the nucleus so that it rapidly shares its energy with other nucleons present within the nucleus.

Under these assumptions, the incident neutron having once entered the nuclear surface, has negligible probability of coming out of it. In other words, the amplitude of the outgoing wave, for $r < R$ would be of the outgoing wave, for $r < R$, would be negligibly small and hence the wave function inside the nuclear surface may be written as

$$U_0 = e^{-ik_0r} \text{ for } r \leq R \quad \dots\dots\dots(57)$$

Now from equation (31), we have

$$f = \text{Lim}_{r \rightarrow R} \left(\frac{R}{U} \frac{dU}{dr} \right). \text{ Hence in this case,}$$

$$f = \frac{R}{e^{-k_0R}} (-ik_0) e^{-ik_0R} = -ik_0 \wedge \dots\dots\dots(58)$$

Hence from equation (33),

$$\eta_0 = \left(\frac{f + ikR}{f - ikR} \right) e^{-2ikR} = \left(\frac{-ik_0R + ikR}{-ik_0R - ikR} \right) e^{-2ikR}$$

$$\text{or } \eta_0 = \left(\frac{k_0 - k}{k_0 + k} \right) e^{-2ikR}$$

$$\text{and so, } |\eta_0| = \left(\frac{k_0 - k}{k_0 + k} \right) \dots\dots\dots(59)$$

Hence the cross-section for the formation of the compound nucleus with $l = 0$ neutrons, is given by,

$$\begin{aligned}\sigma_{a,0} &= \frac{\pi}{k^2} \left[1 - |\eta_0|^2 \right] = \frac{\pi}{k^2} \left\{ 1 - \left(\frac{k_0 - k}{k_0 + k} \right)^2 \right\} \\ &= \frac{4\pi k_0}{k(k_0 + k)^2} \dots \dots \dots (60)\end{aligned}$$

This expression may be considered to be the product of two terms (i) π/k^2 and (ii) $4k_0k/(k_0 + k)^2$ of which the latter has the meaning of a 'transmission coefficient'. It has values between 0 and 1 when $k \ll k_0$ and $k \approx k_0$ respectively. k and k_0 are related as under

$$\frac{\hbar^2 k_0^2}{2M} = \frac{\hbar^2 k^2}{2M} + V_0 \dots \dots \dots (61)$$

where V_0 , represents the depth of the potential well.

According to the above formula, the cross-section becomes infinite for neutron energies approaching zero. So this theory fails for this case as it is only an approximate theory. For very small neutron energies $k \ll k_0$, the cross-section for the formation of the compound nucleus, as contained above, reduces to

$$\sigma_{a,0} = \frac{4\pi k_0}{kk_0} = \frac{4\pi}{kk_0} \dots \dots \dots (62)$$

11.8 OPTICAL MODEL THEORY OF NUCLEAR REACTIONS

A high degree of precision in understanding nucleon-nucleon interaction is a prerequisite for understanding the free nucleon-nuclei interaction. Even with such knowledge, it may be extremely difficult to establish a theory of nucleon-nucleon or nucleus-nucleus interactions due to pure mathematical challenges. Therefore, rather than focusing on the forces that exist between specific single nucleons, we try to understand these interactions by looking at the overall behaviour of the system, which is the nucleus. The optical

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model of the nucleus was created as a result of this. We already know that the simplest sort of two-nucleon potential is a square-well, which only consists of a real part. In order to fit in with the experimental data, the depth and width of the well had to be selected in an appropriate manner. However this model predicts, (i) the values of the neutron absorption cross-section that are far too low, (ii) that energy variation of the neutrons has but a little effect on these cross-sections, (iii) widely spaced resonances, which are all quite contrary to most reliable experimental evidences. This model therefore, was replaced by another known as the compound nucleus model which envisages the nuclear reactions to proceed by way of the formation of an intermediate state - the compound nucleus, first put forward by Bohr. This strong absorption model is capable of explaining successfully the existence of sharp, narrow resonances in the elastic scattering cross-section of slow neutrons by atomic nuclei, but it fell short of providing a satisfactory quantitative explanation of large variations (giant resonances) in the cross-sections with energy as also quite marked forward peaking in elastic scattering for medium and fast neutrons. These giant resonances were observed to have cross-sections with broad peaks with a large width of the order of 1MeV, varying regularly with neutron energy and mass number of target nuclei. This is in an obvious contradiction with the compound nucleus theory which ascribes sharp and less broader peaks in resonance cross-section. This then led to the possibility of nuclear reactions proceeding by way of direct interaction processes which do away with the intermediate stage of compound nucleus formation. The direct interaction model gained importance chiefly because of its ability to explain successfully deuteron stripping.

However the compound nucleus and direct interaction models are incomplete unless we specify the interaction potential. These two models provide two extremes - compound nucleus envisages strong absorption and the direct interaction no absorption. To account for the experimental evidence of the presence of scattering along with absorption, a compromise between the two extremes was needed. This led to a form of the interaction potential that contained both a real and an imaginary part - the imaginary part to provide

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an adequate explanation for the absorption of nucleons in nuclear matter. So a semi-transparent (cloudy crystal ball) model of the nucleus known as the optical model gained prominence chiefly because of its ability to explain both the existence and location of giant resonances in neutron cross-sections, both at low and high neutron energies. The model was put forth by Bethe, Feshbach, Porter and Weisskopf. In this model, the nucleus is not taken to be completely black to the incident nucleons but rather translucent to the particle waves *i.e.* the particle of a given kinetic energy may have a mean free path in the nuclear matter of such a magnitude that they may and may not be absorbed in the nucleus to form a compound nucleus. This fact may be introduced in mathematical analysis by taking a complex potential in which both the real and imaginary parts are energy-dependent. All that was needed, was to choose the complex potential parameters so as to produce the required mean free path of the incident particle in the target nucleus at various energies of the incident nucleon. A small value of the imaginary part means long free path.

The simplest form of the optical model complex potential is the simple square-well type which may be written as :

$$V(r) = \begin{cases} -(V_0 + iW) = -V_0(1 - i\zeta) & \text{for } r \leq R \\ 0 & \text{for } r > R \end{cases} \dots\dots\dots(63)$$

where $R = R_0 A^{1/3}$, the nuclear radius and both V_0, W are positive and real numbers.

Impressive gross fits to the elastic scattering cross-sections of low energy neutrons (from 0.05 to 3MeV) and target mass number upto $A = 238$, were obtained by using this square-well complex potential by Freshbach, Porter and Weisskopf with the following values of the well-parameters :

$$V_0 = 42\text{MeV}, W = 1.26\text{MeV}, R = 1.45 A^{1/3}\text{Fermi}$$

$$\zeta = \frac{W}{V_0} = 0.03$$

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To gain insight into the inherent physical content of the complex potential, we write the one-dimensional Schrodinger wave equation incorporating this potential into it.

$$\nabla^2\psi + \frac{2M}{\hbar}(E + V_0 + iW)\psi = 0 \dots\dots(64)$$

Let us find out plane wave solutions of this equation of the form

$$\psi = e^{\pm ikr} \dots\dots(65)$$

wherein k is now a complex wave number and plus sign betokens incoming waves. This complex wave number k is given by

$$\begin{aligned} k &= \left\{ \frac{2M}{\hbar^2} (E + V_0 + iW) \right\}^{1/2} \dots\dots\dots(66) \\ &= \frac{(2M)^{1/2}}{\hbar} (E + V_0)^{1/2} \left\{ 1 + \frac{iW}{E + V_0} \right\}^{1/2} \dots\dots\dots(67) \end{aligned}$$

If we assume that W is small (long mean free path) compared with $(E + V_0)$, then the above expression may be written as (expanding binomially and applying first order approximation)

$$\begin{aligned} k &= \frac{(2M)^{1/2}}{\hbar} (E + V_0)^{1/2} \left\{ \frac{iW}{2(E + V_0)} \right\} \dots\dots\dots(68) \\ &= \frac{(2M)^{1/2}}{\hbar} (E + V_0)^{1/2} + \frac{iW(2M)^{1/2}(E + V_0)^{1/2}}{2\hbar} (E + V_0) \end{aligned}$$

The complex wave number k may be written as

$$k \rightarrow k_r + ik_i \dots\dots\dots(69)$$

where k_r stands for the real part of the wave number and k_i for its imaginary part . Then comparing (68) and (69), we can write for the real and imaginary part

$$k_r = \frac{1}{\lambda} \left\{ \frac{2M}{\hbar^2} (E + V_0) \right\}^{1/2} \dots \dots \dots (70)$$

$$k_i = \frac{W((2M)^{1/2}(E + V_0)^{1/2})}{2(E + V_0)\hbar} = \frac{W}{2(E + V_0)} k_r \dots \dots \dots (71)$$

Now since $(E + V_0) > W$ i.e. $\frac{W}{E+V_0} < 1$, therefore we conclude that within the potential-well, real part of the potential (k_r) is much larger than the imaginary part (k_i). Equation (70) gives the real part of the wave number (k_r) within the potential well and outside the well the wave number $k_0 = \left(\frac{2ME}{\hbar^2}\right)^{1/2}$. We thus find that the real part of the wave number *viz*, k_r within the well (i.e. within the nucleus) is larger than that outside the nucleus. ($k_r > k_0$). This means that the wave-length and consequently the velocity of the incident particle within the nucleus is smaller than that outside the nucleus : The real part of the refractive index of the nuclear matter in this model is

$$n_r = \frac{c_0}{c_r} = \frac{k_r}{k_0} = \left(\frac{E+V_0}{E}\right)^{1/2} \dots \dots \dots (72)$$

If we take $E = 10\text{MeV}$ and $V_0 = 40\text{MeV}$

$$n_r = \sqrt{((10 + 40)/10)} = \sqrt{5} \cong 2.24$$

which is much greater than 1.

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However the actual refractive index is given by

$$n = \left(\frac{E + V_0 + iW}{E} \right)^{1/2} \dots\dots\dots(73)$$

where $E = 10\text{MeV}$, $W = 10\text{MeV}$ and $V_0 = 40\text{MeV}$,

$$k_r = 1.55\text{fm}^{-1} \text{ and } \therefore (\text{\AA})^2 = 0.645\text{fm}$$

and

$$k_i = \frac{k_r W}{2(E + V_0)} = \frac{k_r \times 10}{2(10 + 40)} = \frac{k_r}{10} = 0.155\text{fm}^{-1}$$

Now, the plane wave solution for the outgoing wave is found to be

$$\psi = e^{ikx} = e^{i(k_r + ik_i)x} = e^{ik_r x} e^{-k_i x} \dots\dots\dots(74)$$

which obviously represents an exponentially decaying (attenuated) wave. It therefore represents stream of particles, some of which are being absorbed. The wave number k_i has the character of an absorption coefficient. The probability of compound nucleus formation depends upon the square of the amplitude of the wave function.

An incident neutron of energy E , has an energy $E + V_0$ within the well, therefore velocity within the well is

$$C_{\text{well}} = \left\{ \frac{2(E + V_0)}{M} \right\}^{1/2}$$

and mean life time $\tau = \frac{\hbar}{2W}$. Therefore the mean distance the neutron traverses within the nucleus before it loses its energy in an inelastic collision i.e. Mean decay length = Λ is

$$\begin{aligned} \Lambda = C_{\text{well}} \tau &= \left\{ \frac{2(E + V_0)}{M} \right\}^{1/2} \cdot \frac{\hbar}{2W} = \frac{\hbar}{W} \frac{(E + V_0)}{[2M(E + V_0)]^{1/2}} \\ &= \frac{E + V_0}{W \cdot K_r} \text{ since } k_r = \left\{ \frac{2M}{\hbar^2} (E + V_0) \right\}^{1/2} \\ &= \left(\frac{E + V_0}{W} \right) \lambda \dots\dots\dots(75) \end{aligned}$$

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with $E = 10\text{MeV}$, $V_0 = 40\text{MeV}$ and $W = 10\text{MeV}$, $\Lambda = 3.226\text{fm}$, which is not small compared to nuclear dimensions. This lends credence to the hypothesis of compound nucleus formation in which the particle is trapped within the nucleus for such a long time that its initial energy is effectively shared with the entire ensemble of nucleons and it loses its pristine identity.

One of the most unexpected features of the analysis is the large free path of neutrons in nuclear matter. Strong interaction between nucleons should be revealed by a mean free path much smaller than the nuclear radius. A larger free path corresponds to a small value of the imaginary part of the potential, to account for strong absorption. Bethe considered a larger imaginary term. At low energies (long mean free path), we may expect that the attenuation of the incident beam is predominant at the nuclear surface. This may be due to the fact that imaginary part of the potential near the nuclear surface may be larger than its value deeper inside the nucleus. (The nucleus may be regarded as a Fermi gas, the Pauli's exclusion principle then prevents nucleons being scattered into energy levels already occupied and therefore nucleons only near the top of the Fermi energy distribution, can interact with the incoming low energy neutrons). However, as the energy of the incident neutrons is increased, this effect becomes less important and more and more neutrons may be scattered as there is sufficient energy to lift them to empty states above the top of the Fermi level. This we expect and do find that the imaginary part of the potential increases with increase in energy i.e. the absorption of the incident beam may take place uniformly throughout the whole volume of the nucleus.

One important aspect of the optical potential model is that it can never represent a black nucleus, even if $W \rightarrow \infty$, because one cannot invent a potential well with boundary condition such that all the incoming partial waves are completely absorbed. So it is a grey sphere partly absorbing and partly transmitting.

Complex square well potential gives too large a cross-section for elastic scattering process in the backward direction. A much better fit to the experimental cross-sections is obtained by a modified form of the potential introduced by Woods and Saxon which

replace the square-well potential by a smoothed out potential. This form of the potential is

$$V(r) = -(V_0 + iW)[1 + e^{(r-R)/a}]^{-1} \dots\dots\dots(76)$$

A plot of the real part of this potential is shown in the fig.6 below.

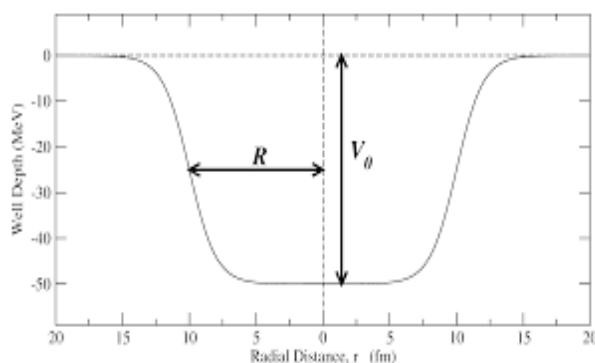


Fig 6 : Woods and Saxon Potential diagram(<http://inside.mines.edu>)

R in this expression is the potential radius at half the well depth. Parameter 'a' is a measure of the amount of tapering of the potential near the nuclear surface, a large 'a' meaning a diffuse nuclear surface. Typical values of 'a' range between 0.50 and 0.65fm.

The parameters W and V_0 are dependent upon the energy and type of the incident particles.

Recently other forms of the optical model potentials have been introduced to take into account the deformation of nuclei, the spin orbit coupling and also the energy region in which experiments are being performed. With the development of giant electronic computers, more and more sophisticated optical model calculations have been made with more and more complex potentials. For charged particle interactions, coulomb potential has been introduced. Such a complex potential used by Rosen, Berry, Goldhaber and Auerbach which includes spin dependence also, is

$$V(r) = V_{\text{coulomb}} - V[1 + e^{(r-R)/a}]^{-1} + 4iWb \left(\frac{d}{dr}\right) [1 + e^{(r-R)/b}]^{-1} - V_S(\hbar\pi)^2 \left(\frac{1}{r}\right) \left|\frac{df(r)}{dr}\right| l \cdot \vec{v} \dots (76)$$

where λ_π is the pion compton wavelength and $f(r) = [1 + e^{(r-R)/a}]^{-1}$.

The optical model, which is so successful in explaining the course of nuclear reactions, has however a serious drawback in that, very little is known about the nuclear surface. The imaginary part W is expected to have a maximum value ther the nuclear surface because it is in this region that there are maximum value near available for the scattered nucleons.

In addition to the theory of neutron-nuclear interaction, the optical model has α -particles and other heavier nuclei by including V_{coulomb} term in the imaginary potential as above

11.9 NUCLEAR FISSION

11.9.1 Discovery of Nuclear Fission

Nuclear fission is a special type of nuclear reaction in which an excited compound nucleus breaks up generally into two fragments of comparable mass numbers and atomic numbers. Fission usually occurs amongst the isotopes of the heaviest elements known, e.g., uranium, thorium etc.

Nuclear fission was discovered by the two German chemists Otto Hahn and F. Strassmann in 1939. It happened to be one of the most important discoveries in nuclear physics, since it paved the way for the utilization of the internal energy of the nucleus for practical purposes.

We have seen earlier that Enrico Fermi and his associates in Rome investigated the neutron capture (n, γ) reactions by various nuclei using neutrons, slowed down to very low energies (1934). Along with other elements, when they bombarded uranium ($Z = 92$), the last naturally occurring element in the periodic table by slow neutrons they found

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evidence for the production of a few β^- active isotopes of short half-lives. One of these was the $23\text{min}^{239}\text{U}$ isotope, which by β^- emission should transform into the isotope ^{239}Np of the first transuranic element neptunium ($Z = 93$). Though this transformation was later actually observed from the radioactivity of ^{239}Np , Fermi and his associates failed to establish definitely that such transformation had actually taken place.

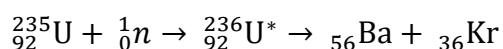
Subsequently, Hahn and Strassmann, along with Lise Meitner in Germany and independently L. Curie-Joliot and L. Savich in France, tried to identify chemically the radioactive products produced by the neutron bombardment on uranium. The former group observed that one of the products was chemically similar to the element barium ($Z = 56$), while the latter group found evidence that one of the products was similar to the element lanthanum ($Z = 57$).

Since the mass number and atomic number of barium or lanthanum are much smaller than those of uranium, it was not possible to explain the above observations on the basis of the general ideas about nuclear reactions prevalent in those days. It was believed that during a nuclear reaction, generally a nucleon (e.g., a proton or a neutron) or a group of them comprising a light nucleus like a deuteron or an α -particle, was emitted. So the residual nucleus produced in a reaction could differ from the target nucleus by only a few units in A and Z . So it was at first thought that the bariumlike element, identified in the experiments of Hahn and his associates, was in reality an element heavier than barium and chemically similar to the latter. A glance at the periodic table shows that the element radium with $Z = 88$ falls in the same vertical column as barium (Group II-B). Hence that the chemically similar. An isotope of radium could conceivably be produced from uranium bombarded with neutrons by the emission of two α -particles.

Soon afterwards, Hahn and Strassmann, by very careful chemical analysis, definitely established that it was barium, and not radium, that was produced as one of the reaction-products, when uranium was bombarded with slow neutrons. They also found lanthanum and cerium ($Z = 58$) amongst the reaction-products. Subsequently, Meitner and her nephew Otto Frisch, working in Sweden (to which country they had escaped to save themselves from Nazi persecution in Germany) provided the correct explanation of Hahn

and Strassmann's results by suggesting that the uranium nucleus bombarded with neutrons broke up into two large fragments. They gave the name nuclear fission to this new phenomenon. Since the atomic number of barium is 56, the other fragment produced in the fission of uranium should have the atomic number $(92 - 56)$ or 36. It should thus be the nucleus of an isotope of krypton, which was subsequently identified. The two fragments produced in fission are known as fission fragments.

The nuclear fission discovered by Hahn and Strassmann can be symbolically written as



Because of the uncertainties in assigning the mass numbers to the fission fragments, these are not shown in the above equation.

Since the fission fragments are heavy and carry positive charges, which are many times the electronic charge (high Z) they are expected to produce intense ionization in gases. This was confirmed by Frisch, who observed large electrical pulses produced by them in an ionization chamber connected to a linear amplifier.

11.9.2 Energy Release in Fission

Nuclear fission is a highly exoergic reaction. Various experiments have established that the total kinetic energy of the two fission fragments is about 167MeV, which shows the enormity of the energy release in the fission process, compared to the energy release in an ordinary nuclear reactions. Besides, the fission fragments, some energy is also carried by the γ -rays and a few prompt neutrons emitted along with the fragments during fission. To these must be added the energies of the β^- particles and the antineutrinos emitted by the fission fragments, which are usually radioactive, as also the energies of the γ -rays associated with the β -disintegrations of the fragments. Thus the total energy evolved during nuclear fission is higher than the value given above. In Table.1 there are shown the distribution of the energies carried by the different components in nuclear fission.

Table.1

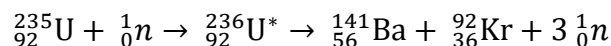
Components	Energy (MeV)
Kinetic energy of the fission fragments Kinetic energy of the prompt neutrons	167
Energy of the prompt γ -rays	5
Energy of the β^- -particles emitted by the fission fragments	6
Energy of the antineutrinos emitted by the fission fragments	8
Energy of the γ -rays emitted by the fission fragments	12
Total energy release in fission	604

The energy released during nuclear fission can be measured by bombarding a piece of uranium with thermal neutrons, which is found to be heated due to the absorption of the fission fragments and some of the other products. The heat thus generated can be measured by calorimetric method, which gives a value of about 186MeV per uranium nucleus fissioned. This falls short of the value given in the above table. This is due to the fact that the antineutrinos and γ -rays produced have very high penetrability and hence escape from the uranium piece.

The enormous quantity of energy release in nuclear fission can be understood qualitatively with the help of the binding fraction (f_B) curve given in Ch. II (see Fig. 2.2). A heavy nucleus like uranium has a value of $f_B = B/A \approx 7.6$ MeV per nucleon. The fragments produced in its fission have mass numbers near the middle of the periodic table and hence the values of f_B for them are 8.5MeV per nucleon. Thus, during the fission process, about 0.9MeV energy per nucleon is released, so that the total energy release is around $238 \times 0.9 = 212$ MeV.

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We can determine the energy release in fission quantitatively from the known atomic masses of the nuclei involved. If we assume that three prompt neutrons are released when the fission occurs, we can write in a typical case



From the mass energy equivalence principle, we then get

$$\begin{aligned} Q &= M({}^{235}\text{U}) + M_n - M({}^{141}\text{Ba}) - M({}^{92}\text{Kr}) - 3M_n \\ &= 235.04278 + 1.00866 - 140.9129 - 91.89719 - 3 \times 1.00866 \\ &= 0.21537\text{u} = 200.6\text{MeV} \end{aligned}$$

Energies of more or less the same order of magnitude are released in the fission of other nuclei.

Because of this enormous energy release during fission, it is possible to obtain very large quantity of energy by the nuclear fission of a small amount of uranium. For example if 1 g of ${}^{235}\text{U}$ is completely fissioned, we can calculate the energy released from the above estimate of the Q value: The number of atoms of ${}^{235}\text{U}$ per kilogram is

$$n = \frac{6.025 \times 10^{23} \times 10^3}{235} = 2.564 \times 10^{24}$$

Hence the energy release per gram of ${}^{235}\text{U}$ is

$$\begin{aligned} E &= \frac{nQ}{10^3} = \frac{2.564 \times 10^{24}}{10^3} \times 200.6 \times 1.6 \times 10^{-13} \\ &= 8.229 \times 10^{10} \\ &= 2.29 \times 10^4 \text{kWh} \end{aligned}$$

A thermal power generator having a capacity of 1MW (heat) would have to be run for 229 hours to generate this amount of energy. The mass of coal which must be burnt to produce an equivalent amount of energy can be estimated as follows. Since the energy release in the chemical process of burning of coal ($\text{C} + \text{O}_2 = \text{CO}_2$) is 4eV per atom of carbon, the quantity of energy released when 1 kg of carbon is completely burnt is

$$\begin{aligned}\varepsilon &= \frac{4 \times 1.6 \times 10^{-19} \times 6.025 \times 10^{23} \times 10^3}{12} \\ &= 3.213 \times 10^7 \text{ J} \\ &= 8.926 \text{ kWh}\end{aligned}$$

So mass of carbon required is

$$m = \frac{2.29 \times 10^4}{8.926} = 2.56 \times 10^3 \text{ kg}$$

The above estimates clearly demonstrate the advantage of using uranium for power generation.

11.10 NUCLEAR FUSION

It is an accepted fact that the energy created during nuclear fission can be used practically for both constructive and destructive purposes. Extensive attempts are currently being conducted to put the energy created in another sort of nuclear reaction to use for peaceful reasons. The fusion reaction is the name given to this process. As the name suggests, two (or more) light nuclei combine to form a heavier nucleus in this sort of reaction. Such reactions are typically exoergic for very light nuclei, which can be deduced qualitatively from the binding fraction curve. For very light nuclei, the binding fraction f_B is a rapidly rising function of A which means that a nucleus produced as a result of the fusion of two lighter nuclei may have greater binding energy than the combined binding energies of the latter.

For an example, if we consider the fusion of two deuterons to produce an α -particle according to the equation ${}^2\text{H} + {}^2\text{H} \rightarrow {}^4\text{He}$, we get the Q value of the reaction as

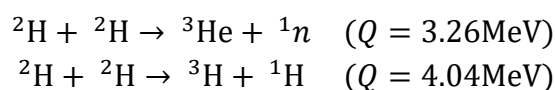
$$\begin{aligned}Q &= B_\alpha - 2B_d = 4f_{B\alpha} - 2 \times (2f_{Bd}) \\ &= 28.3 - 2 \times 2.225 = 23.84 \text{ MeV}\end{aligned}$$

Thus the energy released per nucleon in this reaction is $23.84/4 = 5.96 \text{ MeV}$. This is much larger than the energy released per nucleon in fission. The latter is about (200/

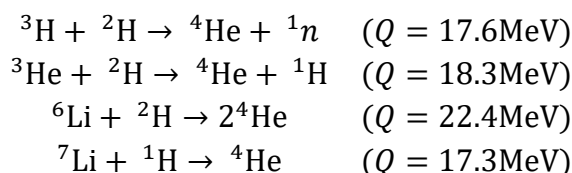
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238) or 0.84MeV. So mass for mass, the fusion reaction usually gives more energy than the fission reaction.

It may be noted that the fusion reaction given in the above example does not actually occur. Due to the huge quantity of energy release, the ^4He nucleus has so great an excitation energy that it breaks up by the emission of a proton or a neutron, as soon as it is formed, giving rise to the following reactions:



Other examples of exoergic fusion reactions are



The energy released in the last four reactions is much greater than in the first two. The energy released in the $d - t$ reaction is 17.6MeV or 3.5MeV per nucleon.

The fusion reactions in the examples given above can occur only if the incident projectile (usually the deuteron) has sufficiently high energy so that it can come into close contact with the target nucleus by over-coming the electrostatic repulsion. For two deuterons the electrostatic with the when in contact ($R_d = 4.3 \times 10^{-15}$ m) is

$$V_b = \frac{e^2}{8\pi\epsilon_0 R_d} = 0.17\text{MeV}$$

Classically the fusion can take place only if the incident particle has energy at least equal to V_s . However, quantum mechanically there may be tunnelling through the barrier, so that the reaction may take place at much lower energy (\sim a few keV). Of course the probability of penetration through the barrier will be higher at higher energies.

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Reactions using charged particles as projectiles are carried out in the laboratory, using particle accelerators. However, there is another way in which the above fusion reactions may be made to occur. The incident particles (p , d etc.) can acquire fairly high energies if the gases e.g., hydrogen or deuterium are heated to very high temperatures (several hundred million degrees). According to the kinetic theory of gases, the mean thermal energy of the gas molecules increases as the temperature increases. At the temperature T K, the mean thermal energy is of the order

$$E = kT = 0.87 \times 10^{-4} T \text{ eV}$$

Here $k = 1.38 \times 10^{-23}$ J/degree is the Boltzmann constant. For $T \approx 10^7$ K, E is of the order of a keV,

If a gas is heated to such a high temperature, many changes will take place in the structure of the molecules and atoms of the gas. At the temperature of a few thousand degrees kelvin, the molecules of the gas dissociate and the bare atoms move about freely at random. If the temperature is raised to 10^4 K or higher, then the electrons in the orbits of the atoms begin to separate from the latter, thereby ionising them. The process, known as thermal ionization is governed by Saha ionization equation deduced by the Indian physicist M.N. Saha. shows that the energies of the atoms are then of the order of a few electrons volts, i.e., comparable to the ionization energy of the atoms. Thus, at these temperatures, the positively charged atomic ions and the negative electrons move about freely. They constitute what is known as a plasma, which is regarded as the fourth state of matter. The plasma as a whole is electrically neutral. So unless electric or magnetic fields are applied, the only force acting on the plasma is the gravitational force.

As the temperature of the plasma increases all the electrons in the atoms get loose from the latter and we get a mixture of bare nuclei and electrons. As the temperature further rises, the mean thermal energy of the nuclei and the electrons increases. When the temperature is 10^7 K or higher, this energy is of the order of kilo electron volt or higher. The energy distribution of the plasma particles is similar to the Maxwellian distribution. Hence the plasma contains some particles which have much higher energies than the

mean thermal energy kT for a particular temperature T . At higher temperatures, the number of the high energy particles increases. Such higher energy particles (nuclei) have finite probability of overcoming the mutual Coulomb repulsion between them which makes possible for them to penetrate through the potential barrier and produce fusion reaction. This is known as the thermo-nuclear reaction.

11.10.1 Source of Energy in Stars

It is known that the sun is an average sized star, emitting 4×10^{26} J energy per second. Its mass is 2×10^{30} kg, which means that the mean energy production in the sun is 2×10^{-4} J/kg · s. Actually the rate of energy liberation in the central region, where the energy is produced, must be larger than the above figure. The energy liberation in the sun is going on at the above rate for at least 4×10^9 years.

Neither chemical reactions nor gravitational energy changes can account for the above huge rate of energy release for such a long time. For example, if the sun consisted of carbon and oxygen only and the solar energy were generated due to the burning of carbon, then all of it would burn up in only 1500 years. That leaves nuclear energy as the only possible source of solar (and stellar) energy. The sun is known to be mainly made up of hydrogen and helium (90%) in about equal proportions. If by some suitable series of nuclear reactions, four hydrogen nuclei combine to produce one helium nucleus, then the energy release for each such fusion will be

$$\begin{aligned} E &= 4M_H - M_{\text{He}} = 4 \times 1.007825 - 4.002603 \\ &= 0.028697\text{u} = 26.73\text{MeV} = 4.28 \times 10^{-12} \text{ J} \end{aligned}$$

Since each kilogram of hydrogen contains about 6×10^{26} protons, the energy content of such a source will be about 2.4×10^{15} J per kg, which could liberate energy at the rate of 2×10^{-4} J/kg.s for 10^{12} y. So the nuclear reactions leading to the fusion of four hydrogen nuclei to produce one helium

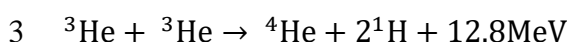
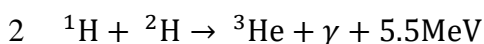
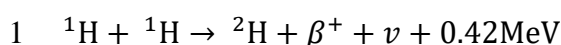
R. Atkinson and F. Houtermans (1928) were the first to suggest that the successive capture of four protons by some light nuclei to produce an α -particle might be the

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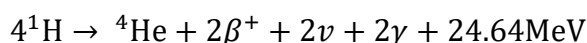
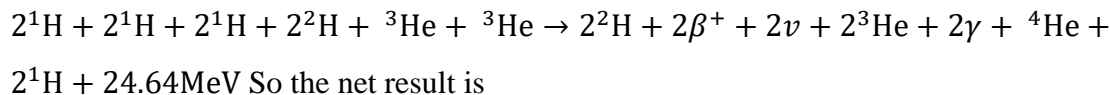
processes which would release energy at reasonable rates for the sun to continue burning for such a long time. They had suggested the idea of cyclic nuclear reactions, the importance of which was proved later.

Two such thermonuclear reaction cycles have been suggested for the production of a helium nucleus by the fusion of four protons.

(A) Proton-proton cycle:



Two reactions, each of (1) and (2), must occur for each reaction (3) to take place. When these are written out in detail and all the reactions are added, we get



Since the neutrinos escape, we are left with about 24.3MeV energy after the fusion of four protons to produce one helium nucleus. Thus the result is the gradual depletion of the protons and the building up of the helium concentration.

The mean reaction rates for the different thermonuclear reactions can be theoretically calculated. For the three reactions considered above, the mean reaction times under the conditions prevailing at the centre of the sun (Density $\rho = 10^5 \text{ kg/m}^3$; $T = 2 \times 10^7 \text{ K}$; Hydrogen concentration = 0.35) are:

$$1.5 \times 10^{10} \text{ y for reaction (1); } 6\text{s for reaction (2); } 9 \times 10^5 \text{ y for reaction (3)}$$

Since the mean time for the $p - p$ reaction is much longer than the other two, we must suppose that the sequence of reactions to be in equilibrium. The first reaction has never

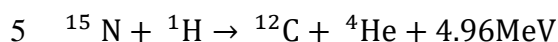
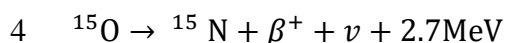
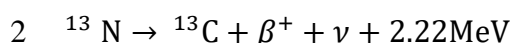
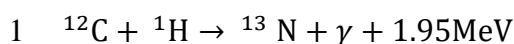
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been observed in the laboratory. It is actually the β^+ transformation of the excited ${}^2\text{He}^*$ produced as the compound nucleus in the reaction. The above estimated life-times tell us that the concentration of hydrogen in the sun will fall to $1/e$ times its present value in a time about 5 times the present age of the sun ($5 \times 10^9\text{y}$)

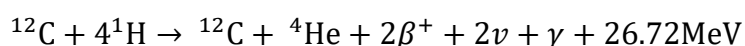
If the β^+ transformation in reaction (1) is allowed, then this reaction would be of primary importance in the sun and more so in less massive stars.

(B) Carbon cycle:

This cycle was proposed by H.A. Bethe (1939) and comprises of the following reactions:



The net result is



Notice that the net result is the fusion of four protons to produce one ${}^4\text{He}$ a presence of ${}^{12}\text{C}$, which must be present, but is not destroyed in produce one ${}^4\text{He}$ nucleus in the Since about 1.7MeV energy is carried away by the neutrinos which escape, the it acts like a catalyst. 25.02MeV. The reaction period of the cycle is determined essentially by the reaction release is reactions (1) and (4) which are $2.5 \times 10^6\text{y}$ and $3 \times 10^8\text{y}$ respectively by the reaction periods of the

The present evident are $2.5 \times 10^6\text{y}$ and $3 \times 10^8\text{y}$ respectively.

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main part of the energy production is due to the with masses between 0.4 to 2.5 solar masses, the massive stars, the situation is reversed. Since according to the $p - p$ cycle rather than the $p - p$ cycle. For less solar masses form the bulk of the stellar population in our galaxy, we can say stars of 0.4 or lower cycle probably goes on in some very bright stars, the in our galaxy, we can say that whereas carbon the $p - p$ cycle.

Thermo-nuclear fusion opens up immense possibilities for the peaceful utilization of nuclear energy, in which the sea water may serve as the main and an almost inexhaustible source of fuel.

11.11 SUMMARY

After studying the unit learners will be able to

- Explain Compound Nucleus
- Describe Direct Reactions
- Understand the Theory of Stripping and Pick-up Reactions
- Explain Resonance Scattering and Reaction Cross-sections
- Discuss the Continuum Theory of Nuclear Reactions
- Describe Optical Model Theory of Nuclear Reactions
- Explain Nuclear Fission
- Discuss the Nuclear Fusion

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11.13 TERMINAL QUESTIONS

1. Explain the theory of Compound Nucleus in detail
2. Describe direct reactions with some suitable examples.
3. Explain about the resonance scattering and reaction cross-sections
4. Discuss the continuum theory of nuclear reactions in detail.
5. Describe optical model theory of nuclear reactions .
6. Explain the Nuclear Fission and Nuclear Fusion processes.